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NORMAL FILTERS IN RESIDUATED LATTICES

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Residuated lattices play an important role in the study of fuzzy logic. In the present paper, we introduce the notion of a normal filter in a residuated lattice and give some characterizations of them. We state and prove some theorems and exampels which determine the relationship between this notion and the other types of filters of a residuated lattice. Finally we investigate the relation between the set of dense elements and normal filters of a residuated lattice.

1. Introduction

Non-classical logic has become a formal and useful tool for computer science to deal with uncertain information and fuzzy information. The algebraic counterparts of some non-classical logics satisfy residuation and those logics can be considered in a frame of residuated lattices [16]. For example, Hajek's *BL* (basic logic [7]), Lukasiewicz's *MV* (many-valued logic [4]) and *MTL* (monoidal t-norm based logic [5]) are determined by the class of *BL*-algebras, *MV*-algebras and *MTL*-algebras, respectively [15, 18]. All of these algebras have lattices with residuation as a common support set. Thus it is very important to investigate properties of algebras with residuation. Residuated lattices were introduced by Ward and Dilworth in [16]. The filter theory of the logical algebras

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plays an important role in the studying of these algebras and the completeness of the corresponding non-classical logics. From a logical point of view, various filters correspond to various sets of provable formulas. At present, the filter theory of residuated lattice has been widely studied, and some important results have been obtained. Turunen in [13] proposed the notions of implicative filters and Boolean filters (Turunen called them implicative deductive systems and Boolean deductive systems, respectively) of BL-algebras, and proved that implicative filters are equivalent to Boolean filters in BL-algebras. Boolean filters are important filters because the quotient algebras induced by Boolean filters are Boolean algebras. In [20] Zhu and Xu observed that those still hold in general residuated lattices. Therefore, the implicative filters and Turunen's implicative deductive systems are also equivalent in general residuated lattices. In addition they defined positive implicative filter, G-filter, fantastic filter and MV-filter in residuated lattice. It has been proven that positive implicative filters and G-filters and also fantastic filters and MV-filters are equivalent. Boroumand and Pourkhatoun in [2] proposed the notion of obstinate filters in residuated lattices and obtained some results.
Normal filters in BL-algebra were defined in paper [1]. Borzooei and Paad [3] proved normal filters and fantastic filters are coincident in BL-algebras. The main purpose of this paper is to define normal filters in residuated lattices and investigate relationship between normal filters and the other types of filters in residuated lattices. For examples, we show that fantastic and normal filters do not coincide in residuated lattices.
In the following, some preliminary theorems and definitions are stated from [2, 6, 12, 14, 16]. In section 3, we define normal filters in a residuated lattice and prove some theorems that determine relationship between this notion and the other types of filters. For example we show that every obstinate filter is a normal filter.

2. Preliminaries

At first we recall the definition of a residuated lattice.
By a residuated lattice, we mean an algebraic structure $L = (L, \wedge, \vee, \odot, \rightarrow, 0, 1)$, where
(LR_1) $(L, \wedge, \vee, 0, 1)$ is a bounded lattice,
(LR_2) $(L, \odot, 1)$ is a commutative monoid with a unit element 1,
(LR_3) For all $a, b, c \in L$, $c \leq a \rightarrow b$ if and only if $a \odot c \leq b$.
For any element x of a residuated lattice, we denote: $x^- = x \rightarrow 0$.
Let L be a residuated lattice. We have the following results.

Theorem 2.1. *The following properties hold for all $x, y, z \in L$:*
(lr_1) $x \rightarrow x = 1, 1 \rightarrow x = x$,

(lr_2) $x \to y \leq (z \to x) \to (z \to y)$,
(lr_3) $x \to y \leq (y \to z) \to (x \to z)$,
(lr_4) $x \leq y \Leftrightarrow x \to y = 1$,
(lr_5) $x \to (y \to z) = y \to (x \to z) = (x \odot y) \to z$,
(lr_6) $x \odot (x \to y) \leq y$ *and* $x \leq y \to x$,
(lr_7) *If* $x \leq y$, *then* $y \to z \leq x \to z$ *and* $z \to x \leq z \to y$,
(lr_8) $x \leq y \Rightarrow y^- \leq x^-$,
(lr_9) $x \leq x^{--}, 1^- = 0, 0^- = 1, x^{---} = x^-$ *and* $x^{--} \leq x^- \to x$.

The following definitions are stated from [8, 10–13, 15, 18, 19, 21].

Let L be a residuated lattice, $F \subseteq L$, and $x, y, z \in L$. For convenience, we enumerate some conditions which will be used in the following study:

$(F1) : x, y \in F \Rightarrow x \odot y \in F$.
$(F2) : x \in F, x \leq y \Rightarrow y \in F$.
$(F3) : 1 \in F$.
$(F4) : x, x \to y \in F \Rightarrow y \in F$.
$(F5) : z, z \to ((x \to y) \to x) \in F \Rightarrow x \in F$.
$(F5)' : x \vee x^- \in F$.
$(F5)'' : x \to (z^- \to y), y \to z \in F \Rightarrow x \to z \in F$.
$(F6) : z \to (x \to y), z \to x \in F \Rightarrow z \to y \in F$.
$(F6)' : x^2 \to y \in F \Rightarrow x \to y \in F$. $(F6)'' : x \to x^2 \in F$.
$(F7) : z, z \to (y \to x) \in F \Rightarrow ((x \to y) \to y) \to x \in F$.
$(F7)' : y \to x \in F \Rightarrow ((x \to y) \to y) \to x \in F$.
$(F7)'' : ((x \to y) \to y) \to ((y \to x) \to x) \in F$.
$(F8) : x \to y \in F$ or $y \to x \in F$.
$(F8)' : x \vee y \in F \Rightarrow x \in F$ or $y \in F$.
$(F8)'' : (x \to y) \vee (y \to x) \in F$.
$(F9) : x \in F$ or $x^- \in F$.
$(F9)' : x, y \notin F \Rightarrow x \to y \in F$ and $y \to x \in F$.
$(F10) : x^{--} \to x \in F$.

• A nonempty subset F of L is called a filter of L if it satisfies in the conditions $(F1)$ and $(F2)$, for all $x, y \in L$. The set of all filters in L is denoted by $F(L)$. We have $F \in F(L)$ if and only if it satisfies in the conditions $(F3)$ and $(F4)$, for all $x, y \in L$.

• A subset F of L is called an implicative filter of L if it satisfies in the conditions $(F3)$ and $(F5)$, for all $x, y, z \in L$.

• A subset F of L is called a Boolean filter (BF) of L if it is a filter of L that satisfies in the condition $(F5)'$, for all $x \in L$.

Let F be a subset of L. Then F is an implicative filter of L if and only if F is a Boolean filter of L.

• A subset F of L is called a Boolean filter of the second kind $(BF2)$ of L if it is a filter of L that satisfies the condition $(F9)$, for all $x \in L$.

• A subset F of L is called an obstinate filter of L if it is a filter of L that satisfies in the condition $(F9)'$, for all $x \in L$.

Let F be a subset of L. Then F is an obstinate filter of L if and only if F is a Boolean filter of the second kind $(BF2)$ of L.

• A subset F of L is called a positive implicative filter of L if it satisfies in the conditions $(F3)$ and $(F6)$, for all $x, y, z \in L$.

• A subset F of L is called a G-filter of L if it is a filter of L that satisfies in the condition $(F6)'$, for all $x, y \in L$.

Let F be a subset of L. Then F is a positive implicative filter of L if and only if F is a G-filter of L.

• A subset F of L is called a fantastic filter of L if it satisfies in the conditions $(F3)$ and $(F7)$, for all $x, y, z \in L$.

• A subset F of L is called an MV-filter of L if it is a filter of L that satisfies in the condition $(F7)'$, for all $x, y \in L$.

Let F be a filter of L. Then F is an MV- filter of L if and only if it satisfies in the condition $(F7)''$, for all $x, y \in L$.

Let F be a subset of L. Then F is a fantastic filter of L if and only if F is an MV-filter of L.

• A subset F of L is called a prime filter (PF) of L if it is a filter of L that satisfies in the condition $(F8)$, for all $x, y \in L$.

• A subset F of L is called a prime filter of the second kind $(PF2)$ of L if it is a filter of L that satisfies in the condition $(F8)'$, for all $x, y \in L$.

• A subset F of L is called a prime filter of the third kind $(PF3)$ of L if it is a filter of L that satisfies in the condition $(F8)''$, for all $x, y \in L$.

If filter F is PF, then F is $PF2$ and $PF3$. Also filter F of L is $PF2$ if and only if for all $F_1, F_2 \in F(L)$, $F = F_1 \cap F_2$ implies $F = F_1$ or $F = F2$,

• A subset F of L is called an involution filter of L if it is a filter of L that satisfies in the condition $(F10)$, for all $x \in L$.

A proper filter M of L is maximal if it is not contained in any other proper filter of L. We shall denote by $Max(L)$ the set of all maximal filters of L.

Theorem 2.2 ([2]). *Let F be a filter of L. Then the following conditions are equivalent:*
(i) *F is a maximal and a Boolean filter,*
(ii) *F is a prime filter of the second kind and a Boolean filter,*
(iii) *F is an obstinate filter.*

Theorem 2.3 ([2]). *Let F be a filter of L. For any $x, y \in L$, the following conditions are equivalent:*

(i) *F is a maximal and an implicative filter,*
(ii) *F is a maximal and a positive implicative filter,*
(iii) *F is an obstinate filter.*

Theorem 2.4 ([20]). *Let F be a filter of L. Then the following assertions are equivalent, for all $x,y \in L$:*
(1) F is a Boolean filter of L.
(2) $(x \to y) \to x \in F \Rightarrow x \in F$.
(3) $(\bar{x} \to x) \to x \in F$.
(4) The quotient residuated lattice L/F is a Boolean algebra.

Theorem 2.5 ([20]). *Let F be a filter of L. Then the following assertions are equivalent, for all $x,y,z \in L$:*
(1) F is a G-filter of L.
(2) $z \to (y \to x) \in F \Rightarrow (z \to y) \to (z \to x) \in F$.
(3) $z, z \to (y \to (y \to x)) \in F \Rightarrow y \to x \in F$.
(4) $x \to x^2 \in F$.
(5) The quotient residuated lattice L/F is a G-algebra.

Theorem 2.6 ([20]). *In any residuated lattice L, the following conditions are equivalent, for all $x,y \in L$:*
(1) L is an MV-algebra.
(2) Any filter of L is an MV-filter of L.
(3) $\{1\}$ is an MV-filter of L.
(4) $((x \to y) \to y) \to x = y \to x$.
(5) $(x \to y) \to y = (y \to x) \to x$.

3. Normal filter

Definition 3.1. Let F be a filter of L. F is called a normal filter of L if it satisfies:

$$z \in F \text{ and } z \to ((y \to x) \to x) \in F \Rightarrow (x \to y) \to y \in F, \quad \forall x,y,z \in L.$$

Theorem 3.2. *Let F be a filter of L. Then F is a normal filter of L if and only if $(y \to x) \to x \in F \Rightarrow (x \to y) \to y \in F$, for all $x,y \in L$.*

Proof. Let F be a normal filter of L and $(y \to x) \to x \in F$. Since $1 \to ((y \to x) \to x) = (y \to x) \to x \in F$ and $1 \in F$, thus $(x \to y) \to y \in F$.
Conversly, let $z \in F$ and $z \to ((y \to x) \to x) \in F$. Since F is a filter, we have $(y \to x) \to x \in F$, so by hypothesis $(x \to y) \to y \in F$. $\square$

Example 3.3. Let $A = [0,1]$ (unit real interval). Define $\odot$ and $\to$ as follows, for all $x,y \in A$,

$$x \odot y = \min\{x, y\}, \qquad x \to y = \begin{cases} 1 & \text{if} \quad x \leq y \\ y & \text{if} \quad x > y \end{cases}$$

Then $(A, \wedge, \vee, \odot, \to, 0, 1)$ is a residuated lattice and all filters of A are in the form of $[x, 1]$, for $x \in [0, 1]$. We show that A does not have any proper normal filter. Consider $F = [x, 1]$ where $0 < x$. Let $z_1, z_2 \in (0, x)$ and $z_1 < z_2$. Then $(z_2 \to z_1) \to z_1 = 1 \in F$ while $(z_1 \to z_2) \to z_2 = z_2 \notin F$.

Theorem 3.4. *Let F be an implicative filter of L. Then F is a normal filter.*

Proof. Let $(x \to y) \to y \in F$. By Theorem 2.1, we can conclude that

$$x \leq (y \to x) \to x \Rightarrow ((y \to x) \to x) \to y \leq x \to y.$$

On the other hand we have $y \leq (y \to x) \to x$, so by Theorem 2.1 and hypotheses we get that

$$(x \to y) \to y \leq (((y \to x) \to x) \to y) \to ((y \to x) \to x).$$

Since $(x \to y) \to y \in F$, we obtain $(((y \to x) \to x) \to y) \to ((y \to x) \to x) \in F$. Therefore $(y \to x) \to x \in F$, since F is an implicative filter. $\square$

By the following example we show that the converse of the above theorem is not true in general.

Example 3.5. Let $L = \{0, a, b, 1\}$ with $0 < b < a < 1$. L becomes a residuated lattice relative to the following operations:

$\odot$	0	a	b	1
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
1	0	a	b	1

$\to$	0	a	b	1
0	1	1	1	1
a	a	1	a	1
b	a	1	1	1
1	0	a	b	1

$\{1\}$ is a normal filter of L but it is not an implicative filter, since $(a \to 0) \to a = 1 \in F$ while $a \notin F$.

By the following theorem, we find a condition under which the converse of Theorem 3.4 holds:

Theorem 3.6. *F is an implicative filter if and only if F is a positive implicative and normal filter.*

Proof. Let F be a positive implicative and normal filter. Assume $(x \to y) \to x \in F$, by Theorem 2.1, we have $(x \to y) \to x \leq (x \to y) \to ((x \to y) \to y)$. Thus by $F \in F(L)$, we get that $(x \to y) \to ((x \to y) \to y) \in F$ and, we conclude $(x \to y) \to y \in F$, since F is a positive implicative filter. So $(y \to x) \to x \in F$, since F is a normal filter. On the other hand, by Theorem 2.1 we have $y \leq x \to y$, thus $(x \to y) \to x \leq y \to x$, and so by hypotheses we get that $y \to x \in F$. Therefore $(y \to x) \to x \in F$ and $y \to x \in F$ imply that $x \in F$.
The converse follows from Theorem 3.4 and [[8] , Theorem 3.11]. □

The next example shows that normality is different from the other properties of filters introduced on page 4 of the present paper.

Example 3.7. (i) Consider residuated lattice in the Example 3.5, $F = \{1\}$ is a normal filter while it is not a G-filter (positive implicative filter), since $a^2 \to b = 1 \in F$ but $a \to b = a \notin F$. Also F is not an involution filter, since $b^{--} \to b = a \notin F$.
(ii) Let $L = \{0, a, b, c, 1\}$ with $0 < a, b < c < 1$, but a and b are incomparable. L becomes a residuated lattice relative to the following operations:

$\odot$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	0	a	a
b	0	0	b	b	b
c	0	a	b	c	c
1	0	a	b	c	1

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	b	1	1
b	a	a	1	1	1
c	0	a	b	1	1
1	0	a	b	c	1

$F = \{1\}$ is a positive implicative filter (G-filter) and $PF2$ while it is not a normal filter, since $(c \to 0) \to 0 = 1 \in F$ but $(0 \to c) \to c = c \notin F$. Furthermore we can check that $G = \{c, 1\}$ is a normal filter, while it is not PF since $a \to b = b \notin F$ and $b \to a = a \notin F$. Also G is neither $PF2$ nor a maximal filter.
(iii) Let $L = \{0, a, b, c, 1\}$ with $0 < a < b, c < 1$, but c and b are incomparable. L becomes a residuated lattice relative to the following operations:

$\odot$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	a	a	a
b	0	a	b	a	b
c	0	a	a	c	c
1	0	a	b	c	1

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	0	1	1	1	1
b	0	c	1	c	1
c	0	b	b	1	1
1	0	a	b	c	1

$F = \{c, 1\}$ is PF and $PF3$ while it is not a normal filter, since $(a \to 0) \to 0 = 1 \in F$ but $(0 \to a) \to a = a \notin F$. Also $G = \{a, c, 1\}$ is a maximal filter while is not a normal filter, since $(b \to 0) \to 0 = 1 \in F$ but $(0 \to b) \to b = b \notin F$.

(iv) Let $L = \{a,b,c,d,e,f,1\}$ with $0 < d < c < b < a < 1$ and $0 < d < e < f < a < 1$ but $\{b,f\}$ and $\{c,e\}$ are incomparable. L becomes a residuated lattice relative to the following operations:

$\odot$	0	a	b	c	d	e	f	1
0	0	0	0	0	0	0	0	0
a	0	c	c	c	0	d	d	a
b	0	c	c	c	0	0	d	b
c	0	c	c	c	0	0	0	c
d	0	0	0	0	0	0	0	d
e	0	d	0	0	0	d	d	e
f	0	d	d	0	0	d	d	f
1	0	a	b	c	d	e	f	1

$\rightarrow$	0	a	b	c	d	e	f	1
0	1	1	1	1	1	1	1	1
a	d	1	a	a	f	f	f	1
b	e	1	1	a	f	f	f	1
c	f	1	1	1	f	f	f	1
d	a	1	1	1	1	1	1	1
e	b	1	a	a	a	1	1	1
f	c	1	a	a	a	a	1	1
1	0	a	b	c	d	e	f	1

$F = \{1\}$ is an involution filter while it is not a normal filter, since $(a \rightarrow f) \rightarrow f = 1 \in F$ but $(f \rightarrow a) \rightarrow a = a \notin F$.

Lemma 3.8. *Let F be a normal filter of L and $a \in L$. Then $a \in F$ if and only if $a^{--} \in F$.*

Proof. Let $a \in F$. We have $a \leq a^{--}$, by $F \in F(L)$ we get that $a^{--} \in F$. Conversely, let $a^{--} \in F$. Then $(a \rightarrow 0) \rightarrow 0 \in F$. Since F is a normal filter, we have $a = (0 \rightarrow a) \rightarrow a \in F$, that is $a \in F$. □

The converse of the above lemma may not hold.

Example 3.9. Consider $F = \{1\}$ in the residuated lattice L in the Example 3.7 part (iv). We have $a \in F \Leftrightarrow a^{--} \in F$, for all $a \in L$ while F is not a normal filter.

Theorem 3.10. *Every MV-filter of L is a normal filter of L.*

Proof. Let F be an MV-filter of L and $(x \rightarrow y) \rightarrow y \in F$. By definition of MV-filter we have $((x \rightarrow y) \rightarrow y) \rightarrow ((y \rightarrow x) \rightarrow x) \in F$ for all $x, y \in L$. Since F is a filter, we conclude that $(y \rightarrow x) \rightarrow x \in F$. □

By the following example we show that the converse of the above theorem may be not true.

Example 3.11. Consider the residuated lattice L in Example 3.5. $F=\{1\}$ is a normal filter while it is not an MV-filter, since $((b\to 0)\to 0)\to((0\to b)\to b)=a\notin F$

Theorem 3.12. *Every obstinate filter of L is a normal filter. The converse may not hold.*

Proof. Let F be an obstinate filter and $(x\to y)\to y\in F$. We prove that $(y\to x)\to x\in F$. We consider four cases for $x,y\in L$.
Case 1:If $x,y\notin F$, then by hypotheses $x\to y\in F$ and $y\to x\in F$. So by $F\in F(L)$, we get that $y\in F$, that is a contradiction.
Case 2: If $x,y\in F$, then $(y\to x)\to x\in F$, by $F\in F(L)$.
Case 3: If $x\in F$ and $y\notin F$, since $x\le(y\to x)\to x$, then $(y\to x)\to x\in F$.
Case 4: If $x\notin F$ and $y\in F$, since $y\le(y\to x)\to x$, then $(y\to x)\to x\in F$.
Consider the residuated lattice L in Example 3.7 part (iii), $\{b,1\}$ is a normal filter while it is not an obstinate filter, since $a,0\notin F$ but $a\to 0=0\notin F$. $\square$

The theorem below follows from Theorems 2.2, 2.3 and 3.12:

Theorem 3.13. *Let F be a filter of L. Then under each of the following conditions F is a normal filter:*
1. F is a maximal filter and BF.
2. F is a $PF2$ and BF.
3. F is a maximal filter and positive implicative filter.

Let $x\in F$. We denote the upset of x by $A(x)$, $A(x)=\{y\in L|x\le y\}$. It is easy to see that $A(x)\in F(L)$ if and only if $x^2=x$. Also by considering the residuated lattice L in Example 3.7 part (iii), $A(c)=\{c,1\}$ is not a normal filter, hence $A(x)$ is not necessarily a normal filter.

Theorem 3.14. *In any residuated lattice L, $A(x)$ is a normal filter, for all $x\in L$ if and only if $(x\to y)\to y=(y\to x)\to x$, for all $x,y\in L$ and $x^2=x$, for all $x\in L$.*

Proof. Let $A(z)$ is a normal filter, for all $z\in L$ and $x,y\in L$. We have $(x\to y)\to y\in A((x\to y)\to y)$, and so $(y\to x)\to x\in A((x\to y)\to y)$. Thus $(x\to y)\to y\le((y\to x)\to x)$. By replacing x to y, $((y\to x)\to x)\le(x\to y)\to y$. Therefore $(x\to y)\to y=(y\to x)\to x$.
The proof of the converse is clear. $\square$

The set of dense elements of L is denoted by $D_s(L) = \{a \in L | a^- = 0\}$. $D_s(L)$ is a filter of L and need not be normal. Consider the residuated lattice L in Example 3.7 part (iv), $D_s(L) = \{1\}$ and $(a \to f) \to f \in D_s(L)$ while $(f \to a) \to a = a \notin D_s(L)$.

Theorem 3.15. *$D_s(L) \subseteq F$, for every normal filter F of L.*

Proof. Let $a \in D_s(L)$. Then $(a \to 0) \to 0 = 1 \in F$. Since F is a normal filter, thus $(0 \to a) \to a = a \in F$. Therefore $D_s(L) \subseteq F$. □

It is easy to see that, if $\{F_i\}_{i \in I}$ is a family of normal filters of L, then $\bigcap_{i \in I} F_i$ is a normal filter of L.

Corollary 3.16. *$D_s(L)$ is a normal filter if and only if $D_s(L)$ is the intersection of all normal filters of L.*

Corollary 3.17. *If $F = \{1\}$ is a normal filter, then $D_s(L) = \{1\}$.*

The converse of the above corollary may not be true. Consider the residuated lattice L in Example 3.7 part (iv), $D_s(L) = \{1\}$, while $F = \{1\}$ is not a normal filter.

Lemma 3.18. *If $D_s(L) = L - \{0\}$, then $D_s(L)$ is the only proper normal filter of L.*

Proof. Let $x, y \in L$. We prove that $(x \to y) \to y, (y \to x) \to x \in D_s(L)$. Consider four cases for $x, y \in L$.
Case 1. $x = y = 0$. Then $(x \to y) \to y = (y \to x) \to x = 1 \in D_s(L)$.
Case 2. $x \neq 0$ and $y = 0$. Thus $(x \to y) \to y = (x \to 0) \to 0 = 0 \to 0 = 1 \in D_s(L)$ and $(y \to x) \to x = x \in D_s(L)$.
Case 3. $x = 0$ and $y \neq 0$. Similar to case 2, we can obtain $(x \to y) \to y, (y \to x) \to x \in D_s(L)$.
Case 4. $x \neq 0$ and $y \neq 0$. Then $y, x \leq (x \to y) \to y$ and $y, x \leq (y \to x) \to x$, imply that $(x \to y) \to y, (y \to x) \to x \in D_s(L)$. □

The converse of the above theorem is not true in general.

Example 3.19. In the residuated lattice L in Example 3.5, $D_s(L)$ is the only normal filter of L while $D_s(L) \neq L - \{0\}$.

Conclusion and future research

Filters theory play an important role in the studying of logical systems and the related algebraic structures. In this paper, we have introduced the notion of

normal filters in residuated lattices and established properties of normal filters in residuated lattices. Then we have investigated the relationships between normal filters and other types of filters in residuated lattices. For example, we have proved that implicative (Boolean, obstinate, $BF2$, fantastic and MV)-filters are normal filters and the converse is not true in general.
There are some open problems:
1. Does extension property for normal filters in residuated lattice hold?
2. Is every normal filter, prime filter of kind third $(PF3)$?
We hope above research would serve as a fundation for further on study the structure of residuated lattices and corresponding many valued logical systems.

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