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SALAGEAN-TYPE HARMONIC UNIVALENT FUNCTIONS WITH FIXED FINITELY MANY COEFFICIENTS

SAURABH PORWAL - BASEM A. FRASIN - REKHA KUREEL

In the present paper, authors introduced and study a new class of Salagean-type harmonic univalent functions that have fixed finitely many coefficients. We obtain coefficient conditions, extreme points, convolution condition, convex combinations for the above class of harmonic univalent functions.

1. Introduction

A continuous complex-valued function $f = u + iv$ defined in a simply connected complex domain D is said to be harmonic in D if both u and v are real harmonic in D. In any simply connected domain we can write $f = h + \overline{g}$, where h and g are analytic in D. We call h the analytic part and g the co-analytic part of f. A necessary and sufficient condition for f to be locally univalent and sense-preserving in D is that $|h'(z)| > |g'(z)|$, $z \in D$, (see Clunie and Sheil-Small [4]). For more basic results on harmonic functions one may refer following standard introductory text book by Duren [8], (see also Ahuja [1] and Ponnusamy and Rasila [13, 14]).

Denote by S_H the class of functions $f = h + \overline{g}$ that are harmonic univalent and sense-preserving in the open unit disc $U = \{z : |z| < 1\}$ for which $f(0) =$

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$f_z(0)-1=0$. Then for $f=h+\overline{g}\in S_H$, we may express the analytic functions h and g as

$$h(z)=z+\sum_{k=2}^{\infty}a_kz^k,\quad g(z)=\sum_{k=1}^{\infty}b_kz^k,\quad |b_1|<1. \tag{1}$$

Note that the family S_H reduces to the class S of normalized analytic univalent functions whenever the co-analytic part of $f=h+\overline{g}$ is zero; i.e. $g\equiv 0$.

For $f=h+\overline{g}$ given by (1), Jahangiri et al. [10] defined the modified Salagean operator of f as

$$D^mf(z)=D^mh(z)+(-1)^m\overline{D^mg(z)}, \tag{2}$$

$$\text{where}\quad D^mh(z)=z+\sum_{k=2}^{\infty}k^ma_kz^k,$$

$$D^mg(z)=\sum_{k=1}^{\infty}k^mb_kz^k$$

and D^m stands for the differential operator introduced by Salagean [15].

For $0\le\alpha<1,\ m\in N,\ n\in N_0,\ m>n$ and $z\in U$, let $S_H(m,n,\alpha,\lambda)$ denote the family of harmonic functions f of the form (1) satisfying the condition

$$Re\left\{\frac{D^mf(z)}{\lambda D^mf(z)+(1-\lambda)D^nf(z)}\right\}>\alpha, \tag{3}$$

where D^mf is defined by (2).

Further, let the subclass $\overline{S}_H(m,n,\alpha,\lambda)$ of $S_H(m,n,\alpha,\lambda)$ consisting of harmonic functions $f_m=h+\overline{g}_m$ in $\overline{S}_H(m,n,\alpha,\lambda)$ so that h and g_m are of the form

$$h(z)=z-\sum_{k=2}^{\infty}|a_k|z^k,\ \ g_m(z)=(-1)^{m-1}\sum_{k=1}^{\infty}|b_k|z^k. \tag{4}$$

These classes $S_H(m,n,\alpha,\lambda)$ and $\overline{S}_H(m,n,\alpha,\lambda)$ were extensively studied by Dixit and Porwal [6].

Now, we introduce a new subclass $\overline{S}_H(m,n,\alpha,\lambda,c_i)$ of $\overline{S}_H(m,n,\alpha,\lambda)$ consisting of functions of the form

$$f_m(z)=h(z)+\overline{g_m(z)} \tag{5}$$

where

$$h(z)=z-\sum_{i=2}^{l}\frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}z^i-\sum_{k=l+1}^{\infty}|a_k|z^k,$$

$$g_m(z)=(-1)^{m-1}\sum_{k=1}^{\infty}|b_k|z^k$$

where $0 \le c_i \le 1$ and $0 \le \sum_{i=2}^{l} c_i \le 1$.

Several authors such as ([2], [5], [7], [9], [11], [12] and [16]) studied the analytic functions with fixed finitely many coefficients. Recently Ahuja and Jahangiri [3] studied the analogues results on harmonic univalent functions with fixed second coefficients. Motivated with these works, we have study the above mentioned new class of harmonic univalent functions with fixed finitely many coefficients.

In this paper, we prove several interesting and useful results for functions belonging to the class $\overline{S}_H(m,n,\alpha,,\lambda,c_i)$.

To prove our main results we shall require the following lemma due to Dixit and Porwal [6].

Lemma 1.1. *A function $f_m(z)$ of the form* (4) *is in $\overline{S}_H(m,n,\alpha,\lambda)$, if and only if*

$$\sum_{k=2}^{\infty} \frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}|a_k| + \sum_{k=1}^{\infty} \frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}|b_k| \le 1, \tag{6}$$

where $m \in N$, $n \in N_0$, $m > n$, $0 \le \alpha < 1$ and $0 \le \lambda < 1$.

2. Main Results

In our first theorem, we introduce a necessary and sufficient coefficient bound for harmonic functions in $\overline{S}_H(m,n,\alpha,\lambda,c_i)$.

Theorem 2.1. *Let the function $f_m(z)$ be defined by* (5) *belonging to the class $\overline{S}_H(m,n,\alpha,\lambda,c_i)$, if and only if*

$$\sum_{k=l+1}^{\infty} \frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}|a_k| + \sum_{k=1}^{\infty} \frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}|b_k| \le 1-\sum_{i=2}^{l} c_i, \tag{7}$$

where $0 \le \alpha < 1$, $0 \le \lambda < 1$, $m \in N, n \in N_0$, $m > n$, $0 \le c_i \le 1$ and $0 \le \sum_{i=2}^{l} c_i \le 1$.

The result is sharp.

Proof. Putting

$$|a_i| = \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}, \qquad (i = 2,3,\dots l), \tag{8}$$

in Lemma 1.1, we have

$$\sum_{i=2}^{l} c_i + \sum_{k=l+1}^{\infty} \frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}|a_k| \\ + \sum_{k=1}^{\infty} \frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}|b_k| \leq 1, \quad (9)$$

which clearly implies (7).

Further by taking the function $f(z)$ of the form

$$f(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i \\ - \sum_{k=l+1}^{\infty} \frac{1-\alpha}{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}|x_k| z^k \\ + (-1)^{m-1} \sum_{k=1}^{\infty} \frac{1-\alpha}{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}|y_k| \overline{z}^k, \quad (10)$$

where $\sum_{k=l+1}^{\infty} |x_k| + \sum_{k=1}^{\infty} |y_k| = 1 - \sum_{i=2}^{l} c_i,$

we can easily verify that the result (7) is sharp. □

In the following theorem, we examine the extreme points of the closed convex hull of $\overline{S}_H(m,n,\alpha,\lambda,c_i)$ which is denoted by clco $\overline{S}_H(m,n,\alpha,\lambda,c_i)$.

Theorem 2.2. *$f_m \in clco\, \overline{S}_H(m,n,\alpha,\lambda,c_i)$, if and only if*

$$f_m(z) = \sum_{k=l}^{\infty} x_k h_k(z) + \sum_{k=1}^{\infty} y_k g_{m_k}(z) \quad (11)$$

where

$$h_l(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i,$$

$$h_k(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i \\ - \frac{(1-\sum_{i=2}^{l} c_i)(1-\alpha)z^k}{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}, \quad (k = l+1, l+2, \ldots),$$

$$g_{m_k}(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i \\ + \frac{(-1)^{m-1}(1-\sum_{i=2}^{l} c_i)(1-\alpha)}{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n} \overline{z}^k, \quad (k = 1, 2, 3, \ldots),$$

$$x_k \geq 0,\ y_k \geq 0 \textit{ and } \sum_{k=l}^{\infty} x_k + \sum_{k=1}^{\infty} y_k = 1.$$

In particular, the extreme points of $\overline{S}_H(m,n,\alpha,\lambda,c_i)$ *are* $\{h_k\}$ *and* $\{g_{m_k}\}$.

Proof. Suppose $f_m(z)$ is expressed by (11). Then

$$\begin{aligned}
f_m(z) &= \sum_{k=l}^{\infty} x_k h_k(z) + \sum_{k=1}^{\infty} y_k g_{m_k}(z) \\
&= z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i - \sum_{k=l+1}^{\infty} \frac{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n} x_k z^k \\
&\quad + (-1)^{m-1} \sum_{k=1}^{\infty} (1-\alpha) \frac{(1-\sum_{i=2}^{l} c_i)}{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n} y_k \overline{z}^k.
\end{aligned}$$

Therefore, $f_m \in \operatorname{clco} \overline{S}_H(m,n,\alpha,\lambda,c_i)$, since

$$\begin{aligned}
&\sum_{i=2}^{l} c_i + \sum_{k=l+1}^{\infty} \frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha} \left[\frac{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n} \right] x_k \\
&+ \sum_{k=1}^{\infty} \frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha} \left[\frac{(1-\alpha)(1-\sum_{i=2}^{l} c_i)}{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n} \right] y_k \\
&= \sum_{i=2}^{l} c_i + \sum_{k=l+1}^{\infty} (1-\sum_{i=2}^{l} c_i) x_k + \sum_{k=1}^{\infty} (1-\sum_{i=2}^{l} c_i) y_k \\
&= \sum_{i=2}^{l} c_i + (1-\sum_{i=2}^{l} c_i)(1-x_l) \\
&= 1 - x_l(1-\sum_{i=2}^{l} c_i) \leq 1.
\end{aligned}$$

Conversely, assume that $f_m(z) = h(z) + \overline{g}_m(z) \in \operatorname{clco} \overline{S}_H(m,n,\alpha,\lambda,c_i)$, where $h(z)$ and $g_m(z)$ are given by

$$h(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n} z^i - \sum_{k=l+1}^{\infty} |a_k| z^k$$

and

$$g_m(z) = (-1)^{m-1} \sum_{k=1}^{\infty} |b_k| \overline{z}^k.$$

Since

$$|a_k| \leq \frac{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n} \quad (k = l+1, \ldots),$$

$$|b_k| \le \frac{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n} \quad (k=1,2,\ldots),$$

we may set

$$x_k = \frac{|a_k|\{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n\}}{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}, \quad (k=l+1,\ldots),$$

$$y_k = \frac{|b_k|\{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n\}}{(1-\sum_{i=2}^{l} c_i)(1-\alpha)}, \quad (k=1,2,\ldots)$$

and define $x_l = 1-\sum_{k=l+1}^{\infty} x_k - \sum_{k=1}^{\infty} y_k$.

Then the proof is complete by noting that

$$\begin{aligned}
f(z) &= z-\sum_{i=2}^{l}\frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}z^i - \sum_{k=l+1}^{\infty}|a_k|z^k + \sum_{k=1}^{\infty}|b_k|\overline{z}^k \\
&= h_l(z)+\sum_{k=l+1}^{\infty}(h_k(z)-h_l(z))x_k+\sum_{k=1}^{\infty}(g_k(z)-h_l(z))y_k \\
&= x_l h_l(z)+\sum_{k=l+1}^{\infty}x_k h_k(z)+\sum_{k=1}^{\infty}y_k g_k(z) \\
&= \sum_{k=l}^{\infty}x_k h_k(z)+\sum_{k=1}^{\infty}y_k g_k(z).
\end{aligned}$$

□

For our next theorem, we need to define the convolution of two harmonic functions. For the harmonic functions of the form

$$f_m(z) = z-\sum_{k=2}^{\infty}|a_k|z^k+(-1)^{m-1}\sum_{k=1}^{\infty}|b_k|\overline{z}^k$$

and

$$F_m(z) = z-\sum_{k=2}^{\infty}|A_k|z^k+(-1)^{m-1}\sum_{k=1}^{\infty}|B_k|\overline{z}^k$$

we define their convolution

$$(f_m * F_m)(z) = f_m(z) * F_m(z) = z-\sum_{k=2}^{\infty}|a_kA_k|z^k+(-1)^{m-1}\sum_{k=1}^{\infty}|b_kB_k|\overline{z}^k. \quad (12)$$

Theorem 2.3. *The family* $\overline{S}_H(m,n,\alpha,\lambda,c_i)$ *is closed under convolution.*

Proof. For $f_m, F_m \in \overline{S}_H(m,n,\alpha,\lambda,c_i))$, we may write

$$(f_m * F_m)(z) = z - \sum_{i=2}^{l} \frac{c_i^2(1-\alpha)^2}{\{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n\}^2} z^i - \sum_{k=l+1}^{\infty} |a_k A_k| z^k + (-1)^{m-1} \sum_{k=1}^{\infty} |b_k B_k| \overline{z}^k$$

where

$$f_m(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{\{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n\}} z^i - \sum_{k=l+1}^{\infty} |a_k| z^k + (-1)^{m-1} \sum_{k=1}^{\infty} |b_k| \overline{z}^k$$

and

$$F_m(z) = z - \sum_{i=2}^{l} \frac{c_i(1-\alpha)}{\{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n\}} z^i - \sum_{k=l+1}^{\infty} |A_k| z^k + (-1)^{m-1} \sum_{k=1}^{\infty} |B_k| \overline{z}^k.$$

Since $F_m(z) \in \overline{S_H}(m,n,\alpha,\lambda,c_i)$, we note that $|A_k| \le 1$, $(k = l+1, l+2, \dots)$ and $|B_k| \le 1, (k = 1,2,\dots)$.

Now

$$\begin{aligned}
&\sum_{i=2}^{l} \left(\frac{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}{1-\alpha} \right) \left(\frac{c_i^2(1-\alpha)^2}{\{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n\}^2} \right) \\
&+ \sum_{k=l+1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n\}}{1-\alpha} |a_k A_k| \\
&+ \sum_{k=1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n\}}{1-\alpha} |b_k B_k| \\
&\le \sum_{i=2}^{l} \frac{(1-\alpha)c_i^2}{\{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n\}} + \sum_{k=l+1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n\}}{1-\alpha} |a_k| \\
&+ \sum_{k=1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)\}}{1-\alpha} |b_k| \\
&\le \sum_{i=2}^{l} c_i + \sum_{k=l+1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n\}}{1-\alpha} |a_k| \\
&\quad + \sum_{k=1}^{\infty} \frac{\{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)\}}{1-\alpha} |b_k| \le 1,
\end{aligned}$$

since $f_m \in \overline{S}_H(m,n,\alpha,\lambda,c_i)$. Thus $f_m * F_m \in \overline{S}_H(m,n,\alpha,\lambda,c_i)$. □

Next, we discuss the convex combination of the class $\overline{S}_H(m,n,\alpha,\lambda,c_i)$.

Theorem 2.4. *The family $\overline{S}_H(m,n,\alpha,\lambda,c_i)$ is closed under convex combination.*

Proof. For $j=1,2,3,\ldots$, let $f_{j_m}(z)\in\overline{S_H}(m,n,\alpha,\lambda;c_i)$, where f_{j_m} is given by

$$f_{j_m}(z)=z-\sum_{i=2}^{l}\frac{c_i(1-\alpha)}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}z^i-\sum_{k=l+1}^{\infty}|a_{jk}|z^k+(-1)^{m-1}\sum_{k=1}^{\infty}|b_{jk}|\overline{z}^k.$$

Then by Theorem 2.1, we have

$$\sum_{k=l+1}^{\infty}\frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}|a_{k_j}|$$
$$+\sum_{k=1}^{\infty}\frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}|b_{k_j}|\le 1-\sum_{i=2}^{l}c_i. \quad (13)$$

For $\sum_{j=1}^{\infty}t_j=1$, $0\le t_j\le 1$, the convex combination of f_{j_m} may be written as

$$\sum_{j=1}^{\infty}t_jf_{j_m}(z)=z-\sum_{i=2}^{l}\frac{c_i(1-\alpha)z^i}{i^m(1-\alpha\lambda)-\alpha(1-\lambda)i^n}-\sum_{k=l+1}^{\infty}\left(\sum_{j=1}^{\infty}t_j|a_{jk}|z^k\right)$$
$$+(-1)^{m-1}\sum_{k=1}^{\infty}\left(\sum_{j=1}^{\infty}t_j|b_{jk}|\overline{z}^k\right).$$

Using the condition (7), we obtain

$$\begin{aligned}
&\sum_{i=2}^{l}c_i+\sum_{k=l+1}^{\infty}\frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}\sum_{j=1}^{\infty}t_j(|a_{jk}|)\\
&+\sum_{k=1}^{\infty}\frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}\sum_{j=1}^{\infty}t_j(|b_{jk}|)\\
&=\sum_{i=2}^{l}c_i+\sum_{j=1}^{\infty}t_j\left(\sum_{k=l+1}^{\infty}\frac{k^m(1-\alpha\lambda)-\alpha(1-\lambda)k^n}{1-\alpha}|a_{jk}|\right.\\
&\left.+\sum_{k=1}^{\infty}\frac{k^m(1-\alpha\lambda)-(-1)^{m-n}\alpha(1-\lambda)k^n}{1-\alpha}|b_{jk}|\right)\\
&\le\sum_{i=2}^{l}c_i+\sum_{j=1}^{\infty}t_j\left(1-\sum_{i=2}^{l}c_i\right)=\sum_{i=2}^{l}c_i+\left(1-\sum_{i=2}^{l}c_i\right)\sum_{j=1}^{\infty}t_j=1.
\end{aligned}$$

This is the condition required by Theorem 2.1 and so
$\sum_{j=1}^{\infty}t_jf_{j_m}(z)\in\overline{S_H}(m,n,\alpha,\lambda,c_i)$. □

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SAURABH PORWAL
Department of Mathematics
U.I.E.T., C.S.J.M. University, Kanpur-208024,
(U.P.), India
e-mail: saurabhjcb@rediffmail.com

BASEM A. FRASIN
Faculty of Science
Department of Mathematics
Al al-Bayt University,- 130095
Mafraq, Jordan
e-mail: bafrasin@yahoo.com

REKHA KUREEL
Department of Mathematics
College of Education and Technology
Devpalpur, Chaubeypur-209203, Kanpur
(U.P.), India
e-mail: rekhakureel76@gmail.com