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NEW GENERALIZATIONS OF SOME INEQUALITIES FOR k-SPECIAL AND q,k-SPECIAL FUNCTIONS

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In this work, we establish some new inequalities involving some k-special and q,k-special functions, by using the technique of A. McD. Mercer [11].

1. Introduction

Let L be a positive linear functional defined on a subspace $C^*(I) \subset C(I)$, where I is the interval $(0,a)$ with $a > 0$ or $(0,+\infty)$.
Let f and g be two functions continuous on I which are strictly increasing and strictly positive on I.
In [11], A. Mcd. Mercer, posed the following result:
Supposing that $f,g \in C^*(I)$ such that $f(x) \to 0$, $g(x) \to 0$ as $x \to 0^+$ and $\frac{f}{g}$ is strictly increasing, we define the function ϕ by

$$\phi = g\frac{L(f)}{L(g)},$$

and let F be a function defined on the ranges of f and g such that the compositions $F(f)$ and $F(g)$ each belong to $C^*(I)$.
a) If F is convex then

$$L[F(f)] \geq L[F(\phi)]. \tag{1}$$

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b) If F is concave then

$$L[F(f)] \leq L[F(\phi)]. \tag{2}$$

The objective of this paper is to use the technique of A. McD. Mercer [11] to develop some new inequalities for k-Gamma, k-Beta and k-Zeta functions then q,k-Gamma and q,k-Beta functions which are a generalization of some inequalities studied in [11, 13].
Note that for $\alpha \in \mathbb{R}$, the function

$$F(t) = t^{\alpha}$$

is convex if $\alpha < 0$ or $\alpha > 1$ and concave if $0 < \alpha < 1$.
Then, for f and g satisfying the conditions (1) and (2), we have:
$L(f^{\alpha}) > L(\phi^{\alpha})$ if $\alpha < 0$ or $\alpha > 1$ and $L(f^{\alpha}) < L(\phi^{\alpha})$ if $0 < \alpha < 1$.
Substituting for ϕ this reads:

$$\frac{[L(g)]^{\alpha}}{L(g^{\alpha})} > (resp. <) \frac{[L(f)]^{\alpha}}{L(f^{\alpha})},$$

if $\alpha < 0$ or $\alpha > 1$ (resp. $0 < \alpha < 1$). In particular, if we take $f(x) = x^{\beta}$ and $g(x) = x^{\delta}$ with $\beta > \delta > 0$, we obtain the following useful inequality:

$$\frac{[L(x^{\delta})]^{\alpha}}{L(x^{\alpha\delta})} \gtrless \frac{[L(x^{\beta})]^{\alpha}}{L(x^{\alpha\beta})}, \tag{3}$$

where $\gtrless$ correspond to the case ($\alpha < 0$ or $\alpha > 1$) and $(0 < \alpha < 1)$ respectively.

In recent years, many authors have studied Gamma and Beta functions. For more information see [1], [2], [3], [5], [6], [10], [13].

2. Basic Results

Throughout this paper, we will fix $q \in (0,1)$. For the convenience of the reader, we provide in this section a summary of the mathematical notions and definitions used in this paper (see [7], [9] , [12]). We write for $a \in \mathbb{C}$,

$$[a]_q = \frac{1-q^a}{1-q}.$$

The q-Jackson integrals from 0 to a and from 0 to ∞ are defined by (see[8])

$$\int_0^a f(x) d_q x = (1-q)a \sum_{n=0}^{\infty} f(aq^n) q^n,$$

$$\int_0^\infty f(x)d_qx = (1-q)\sum_{n=-\infty}^{\infty} f(q^n)q^n,$$

provided the sums converge absolutely.
For $k>0$, the Γ_k function is defined by (see[4], [10])

$$\Gamma_k(x) = \lim_{n\to\infty} \frac{n!k^n(nk)^{\frac{x}{k}-1}}{(x)_{n,k}}, x \in \mathbb{C}\setminus k\mathbb{Z}^-,$$

where $(x)_{n,k} = x(x+k)(x+2k)\dots(x+(n-1)k)$.
The above definition is a generalization of the definition of $\Gamma(x)$ function.
For $x\in\mathbb{C}$ with $\Re(x)>0$, the function $\Gamma_k(x)$ is given by the integral (see [4])

$$\Gamma_k(x) = \int_0^\infty t^{x-1}e^{-\frac{t^k}{k}}dt,$$

and satisfies the following properties: (see [5], [10])

1. $\Gamma_k(x+k) = x\Gamma_k(x)$
2. $(x)_{n,k} = \dfrac{\Gamma_k(x+nk)}{\Gamma_k(x)}$
3. $\Gamma_k(k) = 1$

Definition 2.1. Let $x,y,s,t\in\mathbb{R}$ and $n\in\mathbb{N}$, we note by

1. $(x+y)^n_{q,k} := \prod_{j=0}^{n-1}(x+q^{jk}y)$
2. $(1+x)^\infty_{q,k} := \prod_{j=0}^{\infty}(1+q^{jk}x)$
3. $(1+x)^t_{q,k} := \dfrac{(1+x)^\infty_{q,k}}{(1+q^tx)^\infty_{q,k}}$.

We have $(1+x)^{s+t}_{q,k} = (1+x)^s_{q,k}(1+q^{ks}x)^t_{q,k}$.

We recall the two q,k-analogues of the exponential functions (see [5])

$$E^x_{q,k} = \sum_{n=0}^{\infty} q^{\frac{kn(n-1)}{2}}\frac{x^n}{[n]_{q^k}!} = (1+(1-q^k)x)^\infty_{q,k}$$

and

$$e^x_{q,k} = \sum_{n=0}^{\infty}\frac{x^n}{[n]_{q^k}!} = \frac{1}{(1-(1-q^k)x)^\infty_{q,k}}.$$

These q,k-exponential functions satisfy the following relations:

$$D_{q^k} e^x_{q,k} = e^x_{q,k}, \qquad D_{q^k} E^x_{q,k} = E^{q^k x}_{q,k} \qquad \text{and} \qquad E^{-x}_{q,k} e^x_{q,k} = e^x_{q,k} E^{-x}_{q,k} = 1.$$

The q,k-Gamma function is defined by [5]

$$\Gamma_{q,k}(x) = \frac{(1-q^k)^{\infty}_{q,k}}{(1-q^x)^{\infty}_{q,k}(1-q)^{\frac{x}{k}-1}}, x > 0.$$

When $k = 1$ it reduces to the known q-Gamma function Γ_q.
It satisfies the following functional equation:

$$\Gamma_{q,k}(x+k) = [x]_q \Gamma_{q,k}(x), \qquad \Gamma_{q,k}(k) = 1$$

and it has the following integral representation (see[5])

$$\Gamma_{q,k}(x) = \int_0^{\left(\frac{[k]_q}{(1-q^k)}\right)^{\frac{1}{k}}} t^{x-1} E_{q,k}^{-\frac{q^k t^k}{[k]_q}} d_q t, \quad x > 0.$$

The k-Beta function is defined by (see [4])

$$B_k(t,s) = \int_0^{\infty} x^{t-1}(1+x^k)^{-\frac{t+s}{k}} dx, \qquad t,s > 0.$$

By using the following change of variable $x^k = u$ and $u = \dfrac{y}{1-y}$, we obtain

$$B_k(t,s) = \frac{1}{k}\int_0^1 u^{\frac{t}{k}-1}(1-u)^{\frac{s}{k}-1} du, \qquad s,t,k > 0.$$

It is well-known that

$$B_k(s,t) = \frac{\Gamma_k(s)\Gamma_k(t)}{\Gamma_k(s+t)}.$$

The q,k-Beta function is defined by (see [5])

$$B_{q,k}(t,s) = [k]_q^{-\frac{t}{k}} \int_0^{[k]_q^{\frac{1}{k}}} x^{t-1}\left(1-q^k\frac{x^k}{[k]_q}\right)^{\frac{s}{k}-1}_{q,k} d_q x, \quad s > 0, t > 0.$$

By using the following change of variable $u = \dfrac{x}{[k]_q^{\frac{1}{k}}}$, the last equation becomes

$$B_{q,k}(t,s) = \int_0^1 u^{t-1}(1-q^k u^k)^{\frac{s}{k}-1}_{q,k} d_q u, \quad s > 0, t > 0.$$

It verifies the relation

$$B_{q,k}(t,s) = \frac{\Gamma_{q,k}(t)\Gamma_{q,k}(s)}{\Gamma_{q,k(t+s)}}, \quad s,t > 0.$$

The function $B_{q,k}$ satisfies the following formulas for $s,t>0$

1. $B_{q,k}(t,\infty)=(1-q)^{\frac{t}{k}}\Gamma_{q,k}(t)$
2. $B_{q,k}(t+k,s)=\frac{[t]_q}{[s]_q}B_{q,k}(t,s+k)$
3. $B_{q,k}(t,s+k)=B_{q,k}(t,s)-q^s B_{q,k}(t+k,s)$
4. $B_{q,k}(t,s+k)=\frac{[s]_q}{[s+t]_q}B_{q,k}(t,s)$
5. $B_{q,k}(t,k)=\frac{1}{[t]_q}$.

Definition 2.2. We define the function ζ_k as (see [10])

$$\zeta_k(s)=\frac{1}{\Gamma_k(s)}\int_0^{\infty}\frac{t^{s-k}}{e^t-1}dt,\ s>k.$$

Note that when $k=1$ we obtain the known Riemann Zeta function $\zeta(s)$.

3. Main results

We start with the following theorem:

Theorem 3.1. *Let f be the function defined by*

$$f(x)=\frac{[\Gamma_k^{(2n)}(k+x)]^{\alpha}}{\Gamma_k^{(2n)}(k+\alpha x)}$$

then for all $\alpha>1$ (resp.$0<\alpha<1$) f is decreasing (resp. increasing) on $(0,\infty)$.

Proof. The k-Gamma function is infinitely differentiable on $(0,\infty)$ and we have

$$\Gamma_k^{(n)}(x)=\int_0^{\infty}t^{x-1}[\log(t)]^n e^{-\frac{t^k}{k}}dt,\qquad n\in\mathbb{N}.$$

We consider the subspace $C^*(I)$ obtained from $C(I)$ by requiring its members to satisfy:

(i) $\omega(x)=O(x^{\theta})$ (for any $\theta>-k$) as $x\to 0$,

(ii) $\omega(x)=O(x^{\varphi})$ (for any finite φ) as $x\to+\infty$.

Then, for $\omega\in C^*(I)$ we define

$$L(\omega)=\int_0^{\infty}\omega(t)t^{k-1}(\log(t))^{2n}e^{-\frac{t^k}{k}}dt.$$

The operator L is well-defined on $C^*(I)$ and it is a positive linear functional on $C^*(I)$.
Using the inequality (3), we obtain for $\beta > \delta > 0$

$$\frac{[\Gamma_k^{(2n)}(k+\delta)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha\delta)} \geqslant \frac{[\Gamma_k^{(2n)}(k+\beta)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha\beta)}.$$

Theorem 3.1 is thus proved. □

Corollary 3.2. *For all $x \in [0,k]$, we have:*

$$\frac{[\Gamma_k^{(2n)}(2k)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha k)} \leq \frac{[\Gamma_k^{(2n)}(k+x)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha x)} \leq [\Gamma_k^{(2n)}(k)]^{\alpha-1}, \qquad \alpha \geq 1,$$

and

$$[\Gamma_k^{(2n)}(k)]^{\alpha-1} \leq \frac{[\Gamma_k^{(2n)}(k+x)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha x)} \leq \frac{[\Gamma_k^{(2n)}(2k)]^\alpha}{\Gamma_k^{(2n)}(k+\alpha k)}, \qquad 0 < \alpha \leq 1.$$

Corollary 3.3. *For all $x \in [0,k]$, we have*

$$\frac{k^\alpha}{\Gamma_k(\alpha k+k)} \leq \frac{[\Gamma_k(k+x)]^\alpha}{\Gamma_k(k+\alpha x)} \leq 1, \qquad \alpha \geq 1,$$

and

$$1 \leq \frac{[\Gamma_k(k+x)]^\alpha}{\Gamma_k(k+\alpha x)} \leq \frac{k^\alpha}{\Gamma_k(\alpha k+k)}, \qquad 0 < \alpha \leq 1.$$

Theorem 3.4. *Let f be the function defined by*

$$f(x) = \frac{[\Gamma_{q,k}^{(2n)}(k+x)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha k)}$$

then for all $\alpha > 1$ (resp. $0 < \alpha < 1$) f is decreasing (resp. increasing) on $(0,\infty)$.

Proof. Since that $\Gamma_{q,k}$ is an infinitely differentiable function on $(0,+\infty)$, we have

$$\Gamma_{q,k}^{(n)}(x) = \int_0^{\left(\frac{[k]_q}{(1-q^k)}\right)^{\frac{1}{k}}} t^{x-1}(\log(t))^n E_{q,k}^{-\frac{q^k t^k}{[k]_q}} d_q t, \qquad x > 0,\ n \in \mathbb{N}.$$

We consider $I = (0, (\frac{[k]_q}{(1-q^k)})^{\frac{1}{k}})$ and the subspace $C^*(I)$ obtained from $C(I)$ by requiring its members to satisfy:

(i) $\omega(x) = O(x^\theta)$ (for any $\theta > -k$) as $x \to 0$,

(ii) $\omega(x) = O(1)$ as $x \to (\frac{[k]_q}{(1-q^k)})^{\frac{1}{k}}$.
Then we have,

$$L(\omega) = \int_0^{(\frac{[k]_q}{(1-q^k)})^{\frac{1}{k}}} \omega(t) t^{k-1} (\log(t))^{2n} E_{q,k}^{-\frac{q^k t^k}{[k]_q}} d_q t.$$

The operator L is a positive linear functional on $C^*(I)$.
By applying the inequality (3), we obtain for $\beta > \delta > 0$

$$\frac{[\Gamma_{q,k}^{(2n)}(k+\delta)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha\delta)} \gtrless \frac{[\Gamma_{q,k}^{(2n)}(k+\beta)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha\beta)}.$$

Theorem 3.4 is thus proved. □

In particular, we have the following results

Corollary 3.5. *For all $x \in [0,k]$, we have:*

$$\frac{[\Gamma_{q,k}^{(2n)}(2k)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha k)} \leq \frac{[\Gamma_{q,k}^{(2n)}(k+x)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha x)} \leq [\Gamma_{q,k}^{(2n)}(k)]^{\alpha-1}, \qquad \alpha \geq 1,$$

and

$$[\Gamma_{q,k}^{(2n)}(k)]^{\alpha-1} \leq \frac{[\Gamma_{q,k}^{(2n)}(k+x)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha x)} \leq \frac{[\Gamma_{q,k}^{(2n)}(2k)]^\alpha}{\Gamma_{q,k}^{(2n)}(k+\alpha k)}, \qquad 0 < \alpha \leq 1.$$

Corollary 3.6. *For all $x \in [0,k]$, we have*

$$\frac{k^\alpha}{\Gamma_{q,k}(\alpha k+k)} \leq \frac{[\Gamma_{q,k}(k+x)]^\alpha}{\Gamma_{q,k}(k+\alpha x)} \leq 1, \qquad \alpha \geq 1,$$

and

$$1 \leq \frac{[\Gamma_{q,k}(k+x)]^\alpha}{\Gamma_{q,k}(k+\alpha x)} \leq \frac{k^\alpha}{\Gamma_{q,k}(\alpha k+k)}, \qquad 0 < \alpha \leq 1.$$

Remark 3.7. Applying Theorem 3.1 and Theorem 3.4 for $k = 1$, we obtain the Theorem 2.1 and Theorem 3.2 in [13].

Theorem 3.8. *For $s > 0$, let f be the function defined by*

$$f(x) = \frac{[B_k(k(x+1),s)]^\alpha}{B_k(k(\alpha x+1),s)}$$

then for all $\alpha > 1$ (resp. $0 < \alpha < 1$) f is decreasing (resp. increasing) on $(0,\infty)$.

Proof. The k-Beta function is defined by

$$B_k(t,s) = \frac{1}{k}\int_0^1 x^{\frac{t}{k}-1}(1-x)^{\frac{s}{k}-1}dx.$$

We consider the interval $I=(0,1)$ and the subspace $C^*(I)$ obtained from $C(I)$ by requiring its members to satisfy:

(i) $\omega(x) = O(x^\theta)$ (for $\theta > -1$) as $x \to 0$

(ii) $\omega(x) = O(1)$ as $x \to 1$.

For $\omega \in C^*(I)$, we define

$$L(\omega) = \int_0^1 \omega(x)(1-x)^{\frac{s}{k}-1}dx.$$

L is a positive linear functional on $C^*(I)$.
Applying the inequality (3), we obtain for $\beta > \delta > 0$

$$\frac{[B_k(k(\delta+1),s)]^\alpha}{B_k(k(\alpha\delta+1),s)} \geqslant \frac{[B_k(k(\beta+1),s)]^\alpha}{B_k(k(\alpha\beta+1),s)}.$$

Theorem 3.8 is thus proved. □

Corollary 3.9. *For $x \in [0,k]$ and $s > 0$, we have*

$$\frac{k^{2(\alpha-1)}(\alpha k^2+s)}{\alpha(k^2+s)^\alpha}\frac{[B_k(k^2,s)]^\alpha}{B_k(\alpha k^2,s)} \le \frac{[B_k(k(x+1),s)]^\alpha}{B_k(k(\alpha x+1),s)} \le [B_k(k,s)]^{\alpha-1} \qquad \alpha \ge 1.$$

Theorem 3.10. *For $s > 0$, let f be the function defined by*

$$f(x) = \frac{[B_{q,k}(k+x,s)]^\alpha}{B_{q,k}(k+\alpha x,s)}$$

then for all $\alpha > 1$ (resp. $0 < \alpha < 1$) f is decreasing (resp. increasing) on $(0,\infty)$.

Proof. The q,k-Beta function is defined by

$$B_{q,k}(t,s) = \int_0^1 x^{t-1}(1-q^k x^k)_{q,k}^{\frac{s}{k}-1} d_q x.$$

We consider the interval $I=(0,1)$ and the subspace $C^*(I)$ obtained from $C(I)$ by requiring its members to satisfy:

(i) $\omega(x) = O(x^\theta)$ (for any $\theta > -k$) as $x \to 0$

(ii) $\omega(x) = O(1)$ as $x \to 1$

Then we put,

$$L(\omega) = \int_0^1 \omega(x) x^{k-1}(1-q^k x^k)_{q,k}^{\frac{s}{k}-1} d_q x.$$

L is defined on $C^*(I)$ and it is a positive linear functional on $C^*(I)$.
Applying the inequality (3), we obtain for $\beta > \delta > 0$

$$\frac{[B_{q,k}(k+\delta,s)]^{\alpha}}{B_{q,k}(k+\alpha\delta,s)} \lesseqgtr \frac{[B_{q,k}(k+\beta,s)]^{\alpha}}{B_{q,k}(k+\alpha\beta,s)}$$

Theorem 3.10 is thus proved. □

Corollary 3.11. *For $x \in [0,k]$ and $s > 0$, we have*

$$\frac{[k]_q^{\alpha}[\alpha k+s]_q}{[k+s]_q^{\alpha}[\alpha k]_q}\frac{B_{q,k}^{\alpha}(k,s)}{B_{q,k}(\alpha k,s)} \leq \frac{[B_{q,k}(k+x,s)]^{\alpha}}{B_{q,k}(k+\alpha x,s)} \leq [B_{q,k}(k,s)]^{\alpha-1} \qquad \alpha \geq 1.$$

Theorem 3.12. *Let f be the function defined by*

$$f(x) = \frac{[\zeta_k(x+k+1)\Gamma_k(x+k+1)]^{\alpha}}{\zeta_k(\alpha x+k+1)\Gamma_k(\alpha x+k+1)}$$

then for all $\alpha > 1$ (resp. $0 < \alpha < 1$) f is decreasing (resp. increasing) on $(0,\infty)$.

Proof. From the definition of Zeta function, we can write

$$\zeta_k(s)\Gamma_k(s) = \int_0^{\infty} x^{s-k}\frac{1}{e^x-1}dx, \qquad s > k.$$

We consider the subspace $C^*(I)$ obtained from $C(I)$ by requiring its members to satisfy:

(i) $\omega(x) = O(x^{\theta})$ (for any $\theta > -1$) as $x \to 0$

(ii) $\omega(x) = O(x^{\varphi})$ (for any finite φ) as $x \to +\infty$.

Then we have,

$$L(\omega) = \int_0^{\infty} \omega(x)\frac{x}{e^x-1}dx.$$

The linear functional L is well-defined on $C^*(I)$ and it is positive.
Applying the inequality (3), we obtain $\beta > \delta > 0$

$$\frac{[\zeta_k(\delta+k+1)\Gamma_k(\delta+k+1)]^{\alpha}}{\zeta_k(\alpha\delta+k+1)\Gamma_k(\alpha\delta+k+1)} \lesseqgtr \frac{[\zeta_k(\beta+k+1)\Gamma_k(\beta+k+1)]^{\alpha}}{\zeta_k(\alpha\beta+k+1)\Gamma_k(\alpha\beta+k+1)}$$

Theorem 3.12 is thus proved. □

Remark 3.13. Applying Theorem 3.12 for $k = 1$, we obtain the inequality for ζ function proved in [11].

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