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# SOME PROPERTIES OF A NEW SUBCLASS OF ANALYTIC UNIVALENT FUNCTIONS

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The purpose of the present paper is to study the integral operator of the form

$$\int_0^z \left\{ \frac{D^n f(t)}{t} \right\}^\delta dt$$

where $f$ belongs to the subclass $C(n,\alpha,\beta)$ and $\delta$ is a real number. We obtain integral characterization for the subclass $C(n,\alpha,\beta)$ and also prove distortion, rotation and radii theorem for this class. Relevant connections of the results presented here with various known results are briefly indicated.

## 1. Introduction

Let $A$ denote the class of functions $f$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1}$$

which are analytic in the open unit disk $U = \{z : z \in C \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Let $S$ be the subclass of $A$ consisting of functions of the form (1) which are also univalent in $U$.

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A function $f$ of $S$ is said to be starlike of order $\alpha (0 \leq \alpha < 1)$, denoted by $f \in S^*(\alpha)$, if and only if

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \quad z \in U,$$

and is said to be convex of order $\alpha (0 \leq \alpha < 1)$, denoted by $f \in K(\alpha)$, if and only if

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, \quad z \in U.$$

The classes $S^*$ and $K$ of starlike and convex functions, respectively, are identified by $S^*(0) \equiv S^*$ and $K(0) \equiv K$.

In 1983, Salagean [17], introduced a derivative operator known as Salagean operator which is defined as follows:

Let $f(z) \in A$ and be of the form (1). Then we define :

$$\begin{aligned}
&D^0 f(z) = f(z)\\
&D^1 f(z) = zf'(z)\\
&\quad \cdots\cdots\cdots\cdots\\
&D^n f(z) = D(D^{n-1} f(z)), \quad n \in N.
\end{aligned}$$

Thus

$$D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k. \tag{2}$$

A function $f$ of $A$ belongs to the class $S(n,\alpha)$ of functions of the form (1) satisfying the condition

$$\Re\left\{\frac{D^{n+1} f(z)}{D^n f(z)}\right\} > \alpha, \ z \in U, \tag{3}$$

where $D^n$ stands for the Salagean operator.

The class $S(n,\alpha)$ was first introduced by Salagean [17] and further studied by Kadioğlu [2].

It should be worthy to note that $S(0,\alpha) = S^*(\alpha)$ and $S(1,\alpha) = K(\alpha)$.

A function $f$ of $A$ belongs to the class $C(n,\alpha,\beta)$ if there exists a function $F \in S^*(\alpha)$ such that

$$\left|\arg\frac{D^n f(z)}{F(z)}\right| < \frac{\beta\pi}{2}, \quad z \in U,$$

where $n \in N_0, \ 0 \leq \alpha < 1, \ 0 < \beta \leq 1$.

By specializing the parameters in $C(n,\alpha,\beta)$ we obtain the following known subclasses of $A$ studied earlier by various researchers.

(1) $C(0,\alpha,\beta) \equiv CS^{*}(\alpha,\beta)$ studied by Mishra [7].

(2) $C(1,\alpha,\beta) \equiv C(\alpha,\beta)$ studied by Mishra [7].

(3) $C(0,0,\beta) \equiv CS^{*}(\beta)$ studied by Reade [14].

(4) $C(1,0,\beta) \equiv C(\beta)$ studied by Kaplan [3].

(5) $C(0,0,1) \equiv S^{*}$ studied by Roberston [15], (see also [1], [19]).

(6) $C(1,0,1) \equiv K$ studied by Roberston [15], (see also [1], [19]).

In the present paper, we study the integral operator

$$h(z) = \int_0^z \left\{ \frac{D^n f(t)}{t} \right\}^{\delta} dt \tag{4}$$

where $n \in N_0$ and $\delta$ is a real number. For $n = 0$ and $n = 1$ this integral operator was studied by Kim [4], Merkes and Wright [6], Mishra [7], Nunokawa ([8], [9]), Pfaltzgraff [11], Royster [16], Patil and Thakare [10] and Shukla and Kumar [18], (see also [13]).

To prove our main results, we shall require the following definition and lemmas.

**Definition 1.1.** Let $P(\alpha)$ denote the class of functions of the form $P(z) = 1 + \sum_{k=1}^{\infty} p_k z^k$ which are regular in $U$ and satisfy $\Re\{P(z)\} > \alpha,\ z \in U$.

**Lemma 1.2.** *Let* $P(z) = 1 + \sum\limits_{k=1}^{\infty} p_k z^k$ *be analytic in* $U$. *If* $\Re\{P(z)\} > \alpha$ *in* $U$, *then*

$$\alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{P(re^{i\theta})\right\} d\theta < 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1), \tag{5}$$

*where* $0 \le \theta_1 < \theta_2 \le 2\pi$, $z = re^{i\theta}$ *and* $0 \le r < 1$.

*Proof.* Since

$$\Re\{P(z)\} > \alpha$$

it is easy to see that

$$\left(\Re\{P(z)\} - \alpha\right)\big|_{z=0} = 1 - \alpha.$$

Then by mean value theorem, we have

$$0 \le \int_{\theta_1}^{\theta_2} \left(\Re\left\{P(re^{i\theta})\right\} - \alpha\right) d\theta \le \int_0^{2\pi} \left(\Re\left\{P(re^{i\theta})\right\} - \alpha\right) d\theta = 2\pi(1-\alpha).$$

or, equivalently

$$0 \leq \int_{\theta_1}^{\theta_2} \left( \Re\left\{P(re^{i\theta})\right\}\right) d\theta - \alpha(\theta_2 - \theta_1) \leq 2\pi\,(1-\alpha)\,,$$

or

$$\alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{P(re^{i\theta})\right\} d\theta < 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1).$$

□

The following lemma is a direct consequence of Lemma 1.2, and improves a result of Patil and Thakare ([10, Lemma 2.2]).

**Lemma 1.3.** *If $f \in S^*(\alpha)$, then*

$$\alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{\frac{zf'(z)}{f(z)}\right\} d\theta < 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1), \tag{6}$$

*where $0 \leq \theta_1 < \theta_2 \leq 2\pi$, $z = re^{i\theta}$ and $0 \leq r < 1$.*

In the following lemma, we obtain integral characterization for the class $C(n, \alpha, \beta)$.

**Lemma 1.4.** *If $f \in C(n, \alpha, \beta)$, then*

$$-\beta\pi + \alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\} d\theta < \beta\pi + 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1), \tag{7}$$

*where $0 \leq \theta_1 < \theta_2 \leq 2\pi$, $z = re^{i\theta}$ and $0 \leq r < 1$. Conversely, let $f$ be analytic and satisfying $D^n f(z) \neq 0$ in $U$, if*

$$\int_{\theta_1}^{\theta_2} \Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\} d\theta > -\beta\pi + \alpha(\theta_2 - \theta_1)$$

*then $f \in C(n, \alpha, \beta)$.*

*Proof.* $f \in C(n, \alpha, \beta)$ implies that there exists a function $F \in S^*(\alpha)$ such that

$$\left|\arg \frac{D^n f(z)}{F(z)}\right| < \frac{\beta\pi}{2}, \quad z \in U.$$

Therefore

$$-\frac{1}{2}\beta\pi < \arg D^n f(z) - \arg F(z) < \frac{1}{2}\beta\pi.$$

Let $0 \leq \theta_1 < \theta_2 \leq 2\pi$. Then with $z = re^{i\theta_2}$, we have

$$-\frac{1}{2}\beta\pi < \arg D^n f(re^{i\theta_2}) - \arg F(re^{i\theta_2}) < \frac{1}{2}\beta\pi. \tag{8}$$

and with $z = re^{i\theta_1}$, we have

$$-\frac{1}{2}\beta\pi < -\arg D^n f(re^{i\theta_1}) + \arg F(re^{i\theta_1}) < \frac{1}{2}\beta\pi. \tag{9}$$

Combining (8) and (9), we obtain

$$\begin{aligned} -\beta\pi + \arg F(re^{i\theta_2}) - \arg F(re^{i\theta_1}) \\ < \arg D^n f(re^{i\theta_2}) - \arg D^n f(re^{i\theta_1}) \\ < \beta\pi + \arg F(re^{i\theta_2}) - \arg F(re^{i\theta_1}), \end{aligned}$$

or

$$-\beta\pi + \int_{\theta_1}^{\theta_2} d\arg F(re^{i\theta}) < \int_{\theta_1}^{\theta_2} d\arg D^n f(re^{i\theta}) < \beta\pi + \int_{\theta_1}^{\theta_2} d\arg F(re^{i\theta}),$$

or

$$\begin{aligned} -\beta\pi + \int_{\theta_1}^{\theta_2} \Re\left\{\frac{zF'(z)}{F(z)}\right\} d\theta \\ < \int_{\theta_1}^{\theta_2} \Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\} d\theta \\ < \beta\pi + \int_{\theta_1}^{\theta_2} \Re\left\{\frac{zF'(z)}{F(z)}\right\} d\theta. \end{aligned} \tag{10}$$

But $F \in S^*(\alpha)$, then using Lemma 1.3 in (10), we have

$$-\beta\pi + \alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\} d\theta < \beta\pi + 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1)$$

and this completes the proof of direct part of the lemma.

To prove the converse part, we follow the techniques of Kaplan [3] and Patil and Thakare [10] and can obtain the desired result. □

**Remark 1.5.** If we put $n = 1$ in Lemma 1.4, we obtain the following result

If $f \in C(\alpha, \beta)$, then

$$-\beta\pi + \alpha(\theta_2 - \theta_1) < \int_{\theta_1}^{\theta_2} \Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} d\theta < \beta\pi + 2\pi(1-\alpha) + \alpha(\theta_2 - \theta_1), \tag{11}$$

where $0 \le \theta_1 < \theta_2 \le 2\pi$, $z = re^{i\theta}$ and $0 \le r < 1$. Conversely, let $f$ be analytic and satisfying $f'(z) \neq 0$ in $U$, if

$$\int_{\theta_1}^{\theta_2} \Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} d\theta > -\beta\pi + \alpha(\theta_2 - \theta_1) \tag{12}$$

then $f \in C(\alpha, \beta)$.

## 2. Main Results

**Theorem 2.1.** *If $f \in C(n,\alpha,\beta)$, then $h \in C(\eta,\gamma)$, provided*

$$\frac{-\gamma}{\beta+2(1-\alpha)} \leq \delta \leq \frac{\gamma+2(1-\eta)}{\beta+2(1-\alpha)}. \tag{13}$$

*The result is sharp when (i) $\gamma = 0$ (ii) $\eta = 0, \gamma = 1$.*

*Proof.* From relation (3) we have

$$h'(z) = \left\{\frac{D^n f(z)}{z}\right\}^{\delta}.$$

Applying logarithmic differentiation and then taking real parts of both sides, we obtain

$$\Re\left\{1+\frac{zh''(z)}{h'(z)}\right\} = \delta\Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\} + (1-\delta).$$

For $\delta > 0$, using Lemma 1.4, we get

$$\begin{aligned}\int_{\theta_1}^{\theta_2}\Re\left\{1+\frac{zh''(z)}{h'(z)}\right\}d\theta &= \delta\int_{\theta_1}^{\theta_2}\Re\left\{\frac{D^{n+1}f(z)}{D^n f(z)}\right\}d\theta + (1-\delta)(\theta_2-\theta_1)\\ &> \delta[-\beta\pi+\alpha(\theta_2-\theta_1)] + (1-\delta)(\theta_2-\theta_1)\\ &= -\beta\delta\pi + [1-(1-\alpha)\delta](\theta_2-\theta_1).\end{aligned}$$

To prove that $h \in C(\eta,\gamma)$, we have to show that the right hand side of the above inequality is not less than $-\gamma\pi+\eta(\theta_2-\theta_1)$, provided

$$0 \leq \delta \leq \frac{\gamma+2(1-\eta)}{\beta+2(1-\alpha)}. \tag{14}$$

Similarly, for $\delta < 0$, using Lemma 1.4, we get

$$\int_{\theta_1}^{\theta_2}\Re\left\{1+\frac{zh''(z)}{h'(z)}\right\}d\theta > \delta\left[\beta\pi+2(1-\alpha)+\alpha(\theta_2-\theta_1)\right] + (1-\delta)(\theta_2-\theta_1).$$

To show that $h \in C(\eta,\gamma)$, we have to prove that the right-hand side of the above inequality is not less than $-\gamma\pi+\eta(\theta_2-\theta_1)$, provided

$$\frac{-\gamma}{\beta+2(1-\alpha)} \leq \delta \leq 0. \tag{15}$$

Combining (14) and (15), we obtain (13).

Thus the proof of Theorem 2.1 is established.

To show the sharpness, let us take the function $f(z)$ defined by the relation

$$D^n f(z) = \frac{z}{(1-z)^{2(1-\alpha)+\beta}}, \tag{16}$$

then it is easy to see that this function belongs to $C(n,\alpha,\beta)$ with respect to the function $\frac{z}{(1-z)^{2(1-\alpha)}}$ belonging to $S^*(\alpha)$. Then

$$h(z) = \int_0^z \frac{dt}{(1-t)^{[2(1-\alpha)+\beta]\delta}} \tag{17}$$

and from condition (12) this functions belongs to $C(0,1)$ if and only if

$$\frac{-1}{2(1-\alpha)+\beta} \leq \delta \leq \frac{3}{2(1-\alpha)+\beta}.$$

Again for $\gamma = 0$, from (17) we have

$$1 + \frac{zh''(z)}{h'(z)} = \frac{1 + \left[1 - 2\left(1 - \frac{\{2(1-\alpha)+\beta\}\delta}{2}\right)\right] z}{1-z}$$

and $\Re\left\{1 + \frac{zh''(z)}{h'(z)}\right\} > \eta$ if and only if

$$1 - \frac{\{2(1-\alpha)+\beta\}\,\delta}{2} \geq \eta \;\Rightarrow\; 0 \leq \delta \leq \frac{2(1-\eta)}{\beta + 2(1-\alpha)}. \qquad \square$$

**Remark 2.2.** The undermentioned results are particular cases of Theorem 2.1.

(i) If we put $n = 0$ and $n = 1$ in Theorem 2.1 we obtain the corresponding results of Mishra [7].

(ii) If we put $n = 1, \beta = 0, \gamma = 0$ we obtain a result of Patil and Thakare [10].

(iii) If we put $n = 1, \beta = 0, \eta = 0$ we obtain a result of Patil and Thakare [10].

(iv) If we put $n = 1, \alpha = 0, \eta = 0$ we obtain a result of Patil and Thakare [10].

(v) If we put $n = 0, \beta = 0, \eta = 0$ we obtain a result of Patil and Thakare [10].

(vi) If we put $n = 1, \alpha = 0, \beta = 0, \eta = 0$ and $\gamma = 1$ we obtain a result of Nunokawa [9] as well as that of Merkes and Wright [6].

(vii) If we put $n = 0, \alpha = 0, \beta = 0, \eta = 0$ and $\gamma = 1$ we obtain a result of Nunokawa [9] as well as that of Merkes and Wright [6].

(viii) If we put $n = 1, \alpha = 0, \beta = 1, \eta = 0$ and $\gamma = 1$ we obtain a result of Nunokawa [9] as well as that of Merkes and Wright [6].

(ix) If we put $n = 0, \alpha = 0, \eta = 0$ we obtain a result of Shukla and Kumar [18].

(x) If we put $n = 0, \alpha = 0, \beta = 1, \eta = 0$ and $\gamma = 1$ we obtain a result of Kim [4].

(xi) If we put $n = 0, \alpha = 1/2, \beta = 0, \eta = 0$ and $\gamma = 1$ we obtain a result of Nunokawa [9] as well as that of Merkes and Wright [6].

**Theorem 2.3.** *Let $f \in C(n, \alpha, \beta)$. Then for $|z| = r$*

$$\frac{r(1-r)^{\beta}}{(1+r)^{\beta+2(1-\alpha)}} \leq |D^n f(z)| \leq \frac{r(1+r)^{\beta}}{(1-r)^{\beta+2(1-\alpha)}}$$

*The result is sharp.*

*Proof.* By definition $f \in C(n, \alpha, \beta)$ if and only if there exists a function $P \in P(0)$ and $F(z) \in S^*(\alpha)$ such that

$$\frac{D^n f(z)}{F(z)} = [P(z)]^{\beta}.$$

Therefore

$$|D^n f(z)| = |P(z)|^{\beta} |F(z)|.$$

Now using the well-known inequalities (see [1])

$$\frac{1-r}{1+r} \leq |P(z)| \leq \frac{1+r}{1-r}$$

and

$$\frac{r}{(1+r)^{2(1-\alpha)}} \leq |F(z)| \leq \frac{r}{(1-r)^{2(1-\alpha)}},$$

we obtain the required inequalities.

Sharpness follows if we take $f(z)$ connected by the relation

$$D^n f(z) = \frac{z(1+z)^{\beta}}{(1-z)^{\beta+2(1-\alpha)}}$$

and

$$F(z) = \frac{z}{(1-z)^{2(1-\alpha)}}.$$

□

**Theorem 2.4.** *If $f \in C(n,\alpha,\beta)$, then*

$$\left|\arg \frac{D^n f(z)}{z}\right| \leq \beta \sin^{-1} \frac{2r}{1+r^2} + 2(1-\alpha)\sin^{-1} r.$$

*The result is sharp.*

*Proof.* If $f \in C(n,\alpha,\beta)$, then

$$\frac{D^n f(z)}{F(z)} = [P(z)]^{\beta},$$

for some $P(z) \in P(0)$ and $F(z) \in S^*(\alpha)$.

Thus

$$\left|\arg \frac{D^n f(z)}{z}\right| \leq \beta \left|\arg P(z)\right| + \left|\arg \frac{F(z)}{z}\right|. \tag{18}$$

Now using the well-known results

$$\left|\arg P(z)\right| \leq \sin^{-1} \frac{2r}{1+r^2} \tag{19}$$

and a result of Pinchuk [12]

$$\left|\arg \frac{F(z)}{z}\right| \leq 2(1-\alpha)\sin^{-1} r, \tag{20}$$

using (19) and (20) in (18) we get the required result.

Sharpness follows if we take $f(z)$ to be the same as in Theorem 2.3. □

**Theorem 2.5.** *If $f \in C(n,\alpha,\beta)$, then $f \in S(n,0)$ for $|z| < r_0$, where*

$$r_0 = \frac{(1+\beta-\alpha) - \sqrt{\alpha^2 - 2\beta\alpha + \beta(2+\beta)}}{1-2\alpha}, \quad \text{when } \alpha \neq \frac{1}{2}$$

*and*

$$r_0 = \frac{1}{1+2\beta}, \quad \text{when } \alpha = \frac{1}{2}.$$

*The result is sharp.*

*Proof.* $f \in C(n,\alpha,\beta)$ if and only if there there exists a function $P \in P(0)$ and $F(z) \in S^*(\alpha)$ such that

$$\frac{D^n f(z)}{F(z)} = [P(z)]^{\beta}.$$

$$D^n f(z) = [P(z)]^{\beta} F(z). \tag{21}$$

Logarithmic differentation of (21) yields

$$\frac{z(D^n f(z))'}{D^n f(z)} = \beta \frac{zP'(z)}{P(z)} + \frac{zF'(z)}{F(z)}.$$

Now by a result of MacGregor [5], we know that

$$\left|\frac{zP'(z)}{P(z)}\right| \leq \frac{2r}{1-r^2}.$$

Therefore

$$\begin{aligned}\Re\left\{\frac{z(D^n f(z))'}{D^n f(z)}\right\} \geq \Re\left\{\frac{zF'(z)}{F(z)}\right\} - \beta\left|\frac{zP'(z)}{P(z)}\right| \\ &\geq \frac{1-(1-2\alpha)r}{1+r} - \beta\left(\frac{2r}{1-r^2}\right) \\ &= \frac{(1-2\alpha)r^2 - 2(1+\beta-\alpha)r+1}{1-r^2}.\end{aligned}$$

The right hand side of the above inequality is not less than or equal to zero provided $|z| = r < r_0$, where $r_0$ is as given in the statement of theorem. Sharpness follows if we take $f(z)$ to be the same as in Theorem 2.3. □

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