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SYMMETRIZATION OF A MATHEMATICAL MODEL OF CHARGE TRANSPORT IN SEMICONDUCTORS

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A mathematical model of charge transport in semiconductors is considered. The model is a quasilinear system of differential equations. A problems of finding an additional entropy conservation law and system symmetrization are solved.

Introduction.

It is well known that *hydrodynamic models* are widely used in mathematical modelling of various physical phenomena. This is typical, for example, for semiconductor physics. In the last years a lot of new mathematical models of hydrodynamic type for description of charge transport in semiconductors were proposed. It is necessary to note, however, that mathematical justification of such models is far from perfect in most cases.

Below we consider the mathematical model of charge transport in semiconductors, which has recently been proposed in the article [1]. The model is a quasilinear system of rather complex equations (see section 1). We simplify this system using reliable reasons; next, for this simpler system, we formulate and solve the problem of finding the *additional entropy conservation law*. Moreover, we discuss symmetrization of this simpler equations system.

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1. Preliminary information.

Now, we briefly describe the mathematical model suggested in [1]. Some related mathematical and calculational aspects are discussed in [2]. Following [3], we write down a dimensionless quasilinear system of hydrodynamic equations, which describe this model, in *one-dimensional non-stationary case* (reduction of the system to a dimensionless form is detailed in [7]):

$$R_\tau + J_s^{(1)} = 0, \tag{1.1}$$

$$J_\tau^{(1)} + J_s^{(2)} = \mathcal{F}^{(2)}, \tag{1.2}$$

$$e_\tau + J_s^{(3)} = \mathcal{F}^{(3)}, \tag{1.3}$$

$$\sigma_\tau + J_s^{(4)} = \mathcal{F}^{(4)}, \tag{1.4}$$

$$J_\tau^{(3)} + J_s^{(5)} = \mathcal{F}^{(5)}. \tag{1.5}$$

Here τ is the time; s, the spacial coordinate;
R, the electron gas density;
$J^{(1)} = Ru$, the electron flow, i.e. a variable which characterizes the electric current in a semiconductor;
u, the electron gas velocity;
$J^{(2)} = Ru^2 + \mathcal{P}$, the flow in the momentum balance equation (1.2);
$\mathcal{P} = P + \Sigma$, the single component of the stress tensor in the one-dimensional case;
$P = R\vartheta$, the pressure in the electron gas;
ϑ, the temperature;
$e = R\widetilde{E} = R\Big(\frac{u^2}{2} + \frac{3}{2}\vartheta\Big)$, the volume density of the total energy;
$J^{(3)} = J^{(1)}(\widetilde{E} + \vartheta) + \Theta + \Sigma u$, the energy flow in the energy conservation law (1.3);
Θ, the heat flow;

$\sigma = \frac{2}{3}Ru^2 + \Sigma$;
$J^{(4)} = u\sigma + \frac{4}{3}u\mathcal{P} + \frac{8}{15}\Theta$;
$J^{(5)} = u^2 e + \frac{5}{2}u^2\mathcal{P} + \frac{5}{2}P\vartheta + \frac{7}{2}\Sigma\vartheta + \frac{16}{5}\Theta u$, the flow of the energy flow;
$\mathcal{F}^{(2)} = RQ - \frac{J^{(1)}}{\tau_P}$;

$\mathcal{F}^{(3)} = J^{(1)}Q - R\frac{\widetilde{E}-3/2}{\tau_W}$;
$\mathcal{F}^{(4)} = \frac{4}{3}J^{(1)}Q - \frac{\Sigma}{\tau_\sigma}$;
$\mathcal{F}^{(5)} = \left(\frac{3}{2}Ru^2 + \frac{5}{2}P + \Sigma\right)Q - \frac{J^{(3)}}{\tau_q}$ are the source terms in (1.2)–(1.5);
$Q = \varphi_s$, φ is the electric potential;
$\tau_P = \tau_P(\widetilde{E})$, the relaxation time in (1.2);
$\tau_W = \tau_W(\widetilde{E})$, the relaxation time in (1.3);
$\tau_\sigma = \tau_\sigma(\widetilde{E})$, the relaxation time in the equation (1.4) for the stress Σ;
$\tau_q = \tau_q(\widetilde{E})$, the relaxation time in the equation (1.5) for the heat flow Q.

The system (1.1)–(1.5) has to be completed with *one-dimensional Poisson equation* for the electric potential $\varphi(\tau, s)$ (see [7]), but since it is unrelated to the problem of symmetrization discussed in this article it is not placed here.

Remark 1.1. Now we briefly describe the method to obtain system of the hydrodynamic type in the theory of charge transport in semiconductors. Mostly, the *Boltzmann transport equation* is basic (see [1]), for which it is possible to write down, generally speaking, an infinite system of the so called *momentum correlations*. In order to obtain a system with a finite number of equations, some realistic reasoning (the so called *closing procedures*) is used. While analyzing the system of moment correlations, it is easy to understand that only the first three equations (in the one-dimensional case) are physically clear. They are the mass conservation law, the momentum conservation law, and the energy conservation law. The other moment correlations have no such a physical meaning; essentially they are differential equations for various additions in the impulse and energy flows. So, in the system (1.1)–(1.5), the equations (1.4), (1.5) are differential equations in additions Σ, Θ which appear in the flows $J^{(2)}$, $J^{(3)}$.

After simple but cumbersome calculations, the system (1.1)–(1.5) can be rewritten in a *non-divergent form*:

$$U_\tau + BU_s = F, \tag{1.6}$$

where $B = uI_5 + B_0$,

$$U = \begin{pmatrix} R \\ u \\ P \\ \Sigma \\ \Theta \end{pmatrix}, \qquad B_0 = \begin{pmatrix} 0 & R & 0 & 0 & 0 \\ 0 & 0 & R^{-1} & R^{-1} & 0 \\ 0 & \lambda & 0 & 0 & 2/3 \\ 0 & \mu & 0 & 0 & 8/15 \\ b_{51} & b_{52} & b_{53} & b_{54} & 0 \end{pmatrix}, \qquad F = \begin{pmatrix} 0 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{pmatrix}$$

$$\lambda = \frac{5P + 2\Sigma}{3}, \quad \mu = \frac{4P + 7\Sigma}{3}$$

$$b_{51} = -\frac{5P + 7\Sigma}{2R^2}P, \quad b_{52} = \frac{16}{5}\Theta, \quad b_{53} = \frac{5\mathcal{P}}{2R}, \quad b_{54} = \frac{P - \Sigma}{R},$$

$$F_2 = Q - \frac{u}{\tau_P}, \quad F_3 = \frac{2}{3}Rg, \quad g = u^2\Big(\frac{1}{\tau_P} - \frac{1}{2\tau_W}\Big) + \frac{3(1-\vartheta)}{2\tau_W},$$

$$F_4 = \frac{4}{3}\frac{Ru^2}{\tau_P} - \frac{\Sigma}{\tau_\sigma},$$

$$F_5 = -\frac{\Theta}{\tau_q} + \Sigma u\Big(\frac{1}{\tau_\sigma} + \frac{1}{\tau_P} - \frac{1}{\tau_q}\Big) + \frac{5}{2}Pu\Big(\frac{1}{\tau_P} - \frac{1}{\tau_q}\Big) +$$

$$+ \frac{Ru^3}{2}\Big(\frac{5}{3\tau_W} - \frac{1}{\tau_q} - \frac{1}{\tau_P}\Big) - \frac{5}{2}J^{(1)}\frac{1-\vartheta}{\tau_W},$$

I_5 is an unitary matrix of order 5. We note that *hyperbolicity* of the system (1.6) has been discussed in [2]. As shown, it becomes hyperbolic under essential restrictions on components of the vector U. In our opinion, this makes difficulties for using numerical methods to find solutions to (1.6).

At the same time in [6, 7] the so called *gas-dynamical model* of charge transport in semiconductors has been considered. This model has been derived from (1.1)–(1.5) by turning Σ and Θ into zero and truncating the last two equations. The gas-dynamical model in a non-divergent form looks like the following:

$$U_\tau + BU_s = F. \tag{1.7}$$

Here

$$U = \begin{pmatrix} R \\ u \\ P \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & R & 0 \\ 0 & 0 & R^{-1} \\ 0 & 5P/3 & 0 \end{pmatrix} + uI_3, \qquad F = \begin{pmatrix} 0 \\ F_2 \\ F_3 \end{pmatrix};$$

the aggregates F_2, F_3 are described above. The system (1.7) is indeed a gas-dynamical equations system (with right parts) for a polytropic gas with the adiabatic exponent $\gamma = 5/3$. Results of numerical calculations (see [6]) have shown that this model adequately depict peculiarities of test tasks in semiconductor physics.

Remark 1.2. Actually, turning Σ and Θ into zero and truncating the differential equations for them is an element of the closing procedure (see. Remark 1.1.) while obtaining the gas-dynamical model of charge transport in semiconductors.

We note that for the system (1.7) it is easy to write down an *additional entropy law* (see [6, 7]):

$$(RS)_\tau + (RuS)_s = \tilde{g}, \tag{1.8}$$

where S is the *mass entropy* connected with the density R and the pressure P by the *state equation*

$$P = R^{5/3} \exp\{(2/3)S\}, \tag{1.9}$$

$$\tilde{g} = \frac{R}{\vartheta} g \quad \text{(the aggregate } g \text{ is described above).}$$

In this article, we consider a simpler system which is obtained from (1.1)–(1.5) by turning Θ into zero and truncating the last equation (1.5). The system in a non-divergent form looks as follows:

$$U_\tau + BU_s = F. \tag{1.10}$$

Here

$$U = \begin{pmatrix} R \\ u \\ P \\ \Sigma \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & R & 0 & 0 \\ 0 & 0 & R^{-1} & R^{-1} \\ 0 & \lambda & 0 & 0 \\ 0 & \mu & 0 & 0 \end{pmatrix} + uI_4, \qquad F = \begin{pmatrix} 0 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix};$$

the aggregates F_2, F_3, F_4 are described above. Under some natural physical restrictions

$$R > 0, \quad \mathcal{P} > 0, \tag{1.11}$$

the system (1.10) is hyperbolic. The aim of this article is to find an *additional entropy conservation law* for the system (1.10)

$$\eta_\tau + \Phi_s = G, \tag{1.12}$$

(where η, Φ, G are some functions in components of the vector U), which would hold at any smooth solutions to the system (1.1)–(1.4). Here η is the *entropy function*; Φ, the *entropy flow*; G, the *entropy production*.

2. On finding the entropy law for the system (1.10).

As well know (see [5]), existence of the additional entropy conservation law (1.12) means existence of functions

$$q_i = q_i(U), \qquad i = \overline{1, 4},$$

called the *Lagrange multipliers* (or *canonical variables* (see [3])) such that the following relations are valid:

$$d\eta = q_1 dR + q_2 dJ^{(1)} + q_3 de + q_4 d\sigma, \tag{2.1}$$

$$d\Phi = \sum_{i=1}^{4} q_i dJ^{(i)}, \tag{2.2}$$

$$G = \sum_{i=2}^{4} q_i \mathcal{F}^{(i)}. \tag{2.3}$$

In what follows we assume that the entropy function η is of the form

$$\eta = RS + f(R, u, \vartheta, \Sigma). \tag{2.4}$$

Moreover, the addition f must satisfy the following condition:

$$f\big|_{\Sigma=0} = 0. \tag{2.5}$$

Taking into account the relation (1.9), we easily obtain from (2.4):

$$d\eta = (S - 1 + f_R)dR + f_u du + \Big(\frac{3R}{2\vartheta} + f_\vartheta\Big)d\vartheta + f_\Sigma d\Sigma. \tag{2.6}$$

On the other hand, we deduce from (2.1):

$$d\eta = \tilde{q}_1 dR + \tilde{q}_2 dJ^{(1)} + q_3 de + q_4 d\Sigma = \tag{2.7}$$

$$= (\tilde{q}_1 + u\tilde{q}_2 + q_3\widetilde{E})dR + (\tilde{q}_2 + uq_3)Rdu + \frac{3}{2}Rq_3 d\vartheta + q_4 d\Sigma,$$

where

$$\tilde{q}_1 = q_1 - \frac{2}{3}u^2 q_4,$$

$$\tilde{q}_2 = q_2 + \frac{4}{3} u q_4.$$

Comparing (2.6) and (2.7), we finally have:

$$\begin{cases} q_1 = S - 1 + f_R - (u/R) f_u + (u^2 - \widetilde{E})\Delta + \frac{2}{3} u^2 f_\Sigma, \\ q_2 = R^{-1} f_u - u\Delta - \frac{4}{3} u f_\Sigma, \\ q_3 = \Delta, \\ q_4 = f_\Sigma. \end{cases} \tag{2.8}$$

Here $\Delta = \frac{1}{\vartheta} + \frac{2}{3R} f_\vartheta$.

Now, we compose the aggregate

$$L = q_1 R + q_2 J^{(1)} + q_3 e + q_4 \sigma - RS - f = -R + R f_R + \Sigma f_\Sigma - f. \tag{2.9}$$

Following the classic article [9], we call the aggregate L by the *generating function* (see also [3, 4]).

We rewrite (2.2) as follows:

$$\begin{aligned} d\Phi = {} & u(q_1 dR + q_2 dJ^{(1)} + q_3 de + q_4 d\sigma) + \\ & + (q_1 R + q_2 J^{(1)} + q_3 e + q_4 \sigma) du + \\ & + \Big(q_2 + u\Big(q_3 + \frac{4}{3} q_4\Big)\Big) d\mathcal{P} + \mathcal{P}\Big(q_3 + \frac{4}{3} q_4\Big) du. \end{aligned} \tag{2.10}$$

In a view of (2.1), (2.8), (2.9), the expression (2.10) transforms into:

$$d\Phi = d(u\eta) + \frac{1}{R} f_u d\mathcal{P} + \Big\{ R f_R + \frac{2\mathcal{P}}{3R} f_\vartheta + \mu f_\Sigma + \frac{\Sigma}{\vartheta} - f \Big\} du. \tag{2.10$'$}$$

The aggregate μ is described above. The right hand expression in (2.10′) is a total differential if $f_u = 0$, i.e. $f = f(R, \vartheta, \Sigma)$, and

$$R f_R + \frac{2\mathcal{P}}{3R} f_\vartheta + \mu f_\Sigma = f - \frac{\Sigma}{\vartheta}. \tag{2.11}$$

We consider (2.11) as a differential equation for finding the function f. We note, however, that the solution to (2.11) can be found in the form

$$f = f(R, r), \quad r = \frac{\Sigma}{\vartheta}, \quad f\big|_{r=0} = 0. \tag{2.12}$$

In this connection, (2.11) can be rewritten as:

$$R f_R + \Big(\frac{5}{3}r - \frac{2r^2}{3R} + \frac{4}{3}R\Big) f_r = f - r. \tag{2.11$'$}$$

We find the general solution to (2.11$'$). For this purpose we form the so called concomitant system of ordinary differential equations (see [9]):

$$\begin{cases} \dfrac{dr}{dR} = \dfrac{4}{3} + \dfrac{5}{3}y - \dfrac{2}{3}y^2, \quad y = \dfrac{r}{R}, \\ \dfrac{df}{dR} = \dfrac{f}{R} - y, \end{cases}. \tag{2.13}$$

The first integrals of this system can be taken in the form:

$$\frac{y+1}{R^2(2-y)} = C_1, \qquad \frac{f}{R} + \frac{1}{2}\ln\Big\{\frac{27}{(y+1)(2-y)^2}\Big\} = C_2. \tag{2.14}$$

Therefore, the general solution to (2.11$'$) can be written as:

$$f = \frac{R}{2}\ln\Big\{\frac{(r+R)(2R-r)^2}{27R^3}\Big\} + RF_0\Big(\frac{r+R}{R^2(2R-r)}\Big), \tag{2.15}$$

where F_0 is an arbitrary function in a single variable.

In order to satisfy (2.12),we take the constant $(1/2)\ln(27/4)$ as the function F_0. Finally, we have

$$f = \frac{R}{2}\ln\Big\{\frac{(r+R)(2R-r)^2}{4R^3}\Big\}. \tag{2.16}$$

The function (2.16) satisfies all the desired conditions. Additionally, we find the derivatives f_R, f_r:

$$\begin{cases} f_R = \dfrac{1}{2}\ln\Big\{\dfrac{(r+R)(2R-r)^2}{4R^3}\Big\} + \dfrac{3r^2}{2(r+R)(2R-r)}, \\ f_r = -\dfrac{3Rr}{2(r+R)(2R-r)}. \end{cases} \tag{2.17}$$

So, the system (1.10) admits existence of the additional entropy conservation law (1.12) which we write as follows:

$$(R\Lambda)_\tau + (Ru\Lambda)_s = G. \tag{2.18}$$

Here

$$\Lambda = S + \frac{1}{2}\ln\left\{\frac{\mathcal{P}\omega^2}{4P^3}\right\}, \qquad \omega = 2P - \Sigma\ (\neq 0), \tag{2.19}$$

$$G = \frac{(Ru)^2}{\mathcal{P}\tau_P} + \frac{3R\Sigma^2}{2\mathcal{P}\omega\tau_\sigma} - \frac{R^2(2P+\Sigma)}{\mathcal{P}\omega}\frac{\widetilde{E} - 3/2}{\tau_W}. \tag{2.20}$$

Now the formulae (2.8), (2.9) for the Lagrange multipliers q_i, $i = \overline{1,4}$ of the generating function L look as follows:

$$\begin{cases} q_1 = \dfrac{1}{2}\ln\left\{\dfrac{\mathcal{P}\omega^2}{4R^5}\right\} + \dfrac{Ru^2 - 5\mathcal{P}}{2\mathcal{P}}, \\ q_2 = -\dfrac{Ru}{\mathcal{P}}, \\ q_3 = \dfrac{R(2P+\Sigma)}{\mathcal{P}\omega}, \\ q_4 = -\dfrac{3R\Sigma}{2\mathcal{P}\omega}, \end{cases} \tag{2.8$'$}$$

$$L = -R. \tag{2.9$'$}$$

In a view of (1.1), the relation (2.18) can be rewritten as:

$$\Lambda_\tau + u\Lambda_s = \frac{G}{R}. \tag{2.18$'$}$$

Consequently, in a view of (2.18′), the system (1.10) can be rewritten as a *symmetric t-hyperbolic system*:

$$\begin{cases} \dfrac{d\Lambda}{d\tau} = \dfrac{G}{R}, \\ R\frac{du}{d\tau} + \mathcal{P}_s = RF_2, \\ \dfrac{1}{\lambda}\dfrac{dP}{d\tau} + u_s = \dfrac{1}{\lambda}F_3, \\ \dfrac{1}{\mu}\dfrac{d\Sigma}{d\tau} + u_s = \dfrac{1}{\mu}F_4, \end{cases} \tag{2.21}$$

provided that

$$\begin{cases} \lambda = \dfrac{5P + 2\Sigma}{3} > 0, \\ \mu = \dfrac{4P + 7\Sigma}{3} > 0, \text{ i.e. } \Sigma > -\dfrac{4}{7}P. \end{cases} \tag{2.22}$$

Here $\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + u\frac{\partial}{\partial s}$.

Remark 2.1. The additional entropy conservation law (2.18′) is a consequence of equations in (1.10). Therefore, we can replace any one of these equations by the relation (2.18′). In some cases, such a substitution leads to a symmetric system immediately. In our case, substituting the first equation in (1.10) for (2.18′), we come to the *symmetric t-hyperbolic (by Friedrichs)* system (2.21).

Concluding this section, we note that the additional entropy law for the system (1.10) is not unique. Indeed, taking a smooth function $h = h(\Lambda)$, we have

$$(Rh)_\tau + (Ruh)_s = h'G. \tag{2.18''}$$

Such an ambiguity can be used while numerically studying the system (1.10) (see, e.g., [8]).

3. Symmetrization of the system (1.10).

In the previous section we have proposed a variant of symmetric form of the system (1.10) (see (2.21)). To obtain another variant, we rewrite the system (1.10) in a slightly different form. Summing up the third and forth equations in (1.10), finally we obtain:

$$\frac{d\mathcal{P}}{d\tau} + 3\mathcal{P}u_s = F_3 + F_4. \tag{3.1}$$

Next, we multiply the third equation by 2 and subtract the forth equation:

$$\frac{d\omega}{d\tau} + \omega u_s = 2F_3 - F_4 \tag{3.2}$$

and, using (1.9), rewrite the expression for Λ (see (2.19)) in the following form:

$$\Lambda = \frac{1}{2}\ln\left\{\frac{\mathcal{P}\omega^2}{4}\right\} - \frac{5}{2}\ln R. \tag{3.3}$$

In a view of (3.1), (3.2), (3.3), we derive another symmetric form of the system (1.10)

$$\begin{cases} \dfrac{d\Lambda}{d\tau} = \dfrac{G}{R}, \\ \dfrac{dX}{d\tau} = 2\dfrac{F_3 + F_4}{3\mathcal{P}} - 2\dfrac{2F_3 - F_4}{\omega}, \\ Re^{-\pi}\dfrac{du}{d\tau} + \pi_s = Re^{-\pi}F_2, \\ \dfrac{1}{3}\dfrac{d\pi}{d\tau} + u_s = \dfrac{F_3 + F_4}{3\mathcal{P}}. \end{cases} \tag{3.4}$$

Here

$$\pi = \ln\mathcal{P}, \qquad X = \frac{2}{3}\pi - \ln\omega^2.$$

While obtaining (3.4), we have assumed that inequalities (1.11) are fulfilled. We have also assumed that $\Sigma \neq 2P$ $(\omega \neq 0)$ (see (2.19))

If (1.11) are valid the system (3.4) is symmetric t-hyperbolic (by Friedrichs).

Finally, one more variant of symmetric form of the system (1.1)–(1.4) is connected with the obtained in section 2 canonical variables q_i, $i = \overline{1,4}$ and the generating function L. We need one more generating function M:

$$M = \sum_{i=1}^{4} q_i J^{(i)} - \Phi = -Ru = uL. \tag{3.5}$$

Further symmetrization formalism is detailed in [3, 4, 5]. It follows from (2.9), (3.5) that

$$R = L_{q_1}, J^{(1)} = L_{q_2}, e = L_{q_3}, \sigma = L_{q_4},$$

$$J^{(1)} = M_{q_1}, J^{(2)} = M_{q_2}, J^{(3)} = M_{q_3}, J^{(4)} = M_{q_4}.$$

Next, the system (1.1)–(1.4) can be rewritten as follows:

$$(L_{q_i})_\tau + (M_{q_i})_s = \mathcal{F}^{(i)}, \quad i = \overline{1,4} \quad (\mathcal{F}^{(1)} = 0). \tag{3.6}$$

We obtain from (3.6):

$$A_0 q_\tau + A_1 q_s = \mathcal{F}, \tag{3.6$'$}$$

where

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}, \qquad \mathcal{F} = \begin{pmatrix} 0 \\ \mathcal{F}^{(2)} \\ \mathcal{F}^{(3)} \\ \mathcal{F}^{(4)} \end{pmatrix},$$

$A_0 = (L_{q_i q_j})$, $A_1 = (M_{q_i q_j})$, $i, j = \overline{1,4}$ are symmetric matrices. After cumbersome calculations we find the matrices A_0, A_1:

$$A_0 = -\begin{pmatrix} R & Ru & R\widetilde{E} & \sigma \\ R & uRu^2 + \mathcal{P} & \frac{Ru^3}{2} + \frac{3}{2}\lambda u & \frac{2}{3}Ru^3 + \mu u \\ R\widetilde{E} & \frac{Ru^3}{2} + \frac{3}{2}\lambda u & a^0_{33} & a^0_{34} \\ \sigma & \frac{2}{3}Ru^3 + \mu u & a^0_{43} & a^0_{44} \end{pmatrix},$$

where

$$a^0_{33} = \frac{Ru^4}{4} + \frac{3}{2}\lambda u^2 + \frac{3(5P^2+\Sigma^2)}{4R}$$

$$a^0_{34} = a^0_{43} = \frac{Ru^4}{3} + \frac{14P+11\Sigma}{6}u^2 + \frac{\Sigma(7P+\Sigma)}{2R}$$

$$a^0_{44} = \frac{4Ru^4}{9} + \frac{16P+28\Sigma}{9}u^2 + \frac{2(2P^2+2P\Sigma+3\Sigma^2)}{3R};$$

$$A_1 = uA_0 - \mathcal{P}\widetilde{A}_1,$$

$$\widetilde{A}_1 = \begin{pmatrix} 0 & 1 & u & \frac{3}{4}u \\ 1 & 2u & \frac{3}{2}u^2 + \frac{3\lambda}{2R} & 2u^2 + \frac{\mu}{R} \\ u & \frac{3}{2}u^2 + \frac{3\lambda}{2R} & u^3 + \frac{3\lambda}{R}u & \frac{4}{3}u^3 + \frac{14P+11\Sigma}{3R} \\ \frac{4}{3}u & 2u^2 + \frac{\mu}{R} & \frac{4}{3}u^3 + \frac{14P+11\Sigma}{3R}u & \frac{16}{9}u^3 + \frac{5u\mu}{3R} \end{pmatrix},$$

and prove the matrix $B_0 = -A_0$ is positive definite provided that $\mathcal{P} > 0$. Taking into account this fact, finally we rewrite the system (1.1)–(1.4) as a symmetric t-hyperbolic system (by Friedrichs):

$$B_0 q_\tau + B_1 q_s = -\mathcal{F}. \tag{3.6$''$}$$

Here $B_1 = uB_0 + \mathcal{P}\widetilde{A}_1$.

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