LE MATEMATICHE
Vol. LV (2000) – Fasc. I, pp. 121–132

APPROXIMATION PROPERTIES OF CERTAIN MODIFIED SZASZ-MIRAKYAN OPERATORS

LUCYNA REMPULSKA - ZBIGNIEW WALCZAK

We introduce certain modified Szasz - Mirakyan operators in exponential weighted spaces of functions of one variable. We give theorems on the degree of approximation and the Voronovskaya type theorem.

1. Introduction.

1.1. Let $q > 0$ be a fixed number,

$$\nu_q(x) := e^{-qx}, \quad x \in R_0 := [0, +\infty), \tag{1}$$

and let C_q be the space of all real-valued functions f continuous on R_0 for which $\nu_q f$ is uniformly continuous and bounded on R_0 and the norm is defined by the formula

$$\|f\|_q \equiv \|f(\cdot)\|_q := \sup_{x\in R_0} \nu_q(x)|f(x)|. \tag{2}$$

C_q is called exponential weighted space ([1]).

Entrato in Redazione il 16 novembre 2000.

AMS Subject classification: 41 A 36

In the paper [1] were examined approximation properties of the Szasz-Mirakyan operators

(3) $$S_n(f;x) := \sum_{k=0}^{\infty} \varphi_k(nx) f(k/n), \quad x \in R_0, n \in N := \{1, 2, \ldots\},$$

for functions $f \in C_q$, where

(4) $$\varphi_k(t) := e^{-t}\frac{t^k}{k!}, \quad t \in R_0, \ \ k \in N_0 := N \cup \{0\}.$$

In [1] was proved that S_n is a positive linear operator from the space C_q into C_r provided that $r > q > 0$ and $n > n_0$, where n_0 is a fixed natural number such that $n_0 > q/\ln(r/q)$. For example: the function $1/v_q$, $q > 0$, belongs to C_q, but $S_n(1/v_q; \cdot) \notin C_q$ for $n \in N$.

1.2. Denote by B_q, $q > 0$, the space of all real-valued functions f defined on R_0 for which $v_q f$ is bounded function on R_0 and the norm is given by (2). Hence

(5) $$C_q \subset B_q \subset B_r, \quad \text{for } r > q > 0.$$

In this paper we modify the operator S_n given in (3). We introduce the operator $S_n[f;q]$, $q > 0$, $n \in N$, which (see Lemma 1 and (5)) is a positive linear operator from the space C_q into C_q.

Definition. *Let $q > 0$ be a fixed number. For functions $f \in B_q$ and $n \in N$ we define operators*

(6) $$S_n[f;q](x) \equiv S_n(f;q;x) := \sum_{k=0}^{\infty} \varphi_k(nx) f(k/(n+q)), \quad x \in R_0,$$

where $\varphi_k(\cdot)$ is given in (4).

By elementary calculations we get from (6):

(7) $$S_n(1;q;x) = 1,$$

(8) $$S_n(t-x;q;x) = -qx/(n+q),$$

(9) $$S_n\Big((t-x)^2;q;x\Big) = \Big(q^2x^2 + nx\Big)/(n+q)^2,$$

(10) $S_n\Big((t-x)^4;q;x\Big) = \Big(q^4x^4 + 6q^2nx^3 + (3n-4q)nx^2 + nx\Big)/(n+q)^4,$

for all $x \in R_0$, $n \in N$ and for every fixed $q > 0$. Moreover we have

$$S_n\Big(e^{qt};q;x\Big) = e^{q_n x}, \tag{11}$$

$$S_n\Big(te^{qt};q;x\Big) = \frac{nx}{n+q}e^{q/(n+q)}e^{q_n x},$$

$$S_n\Big(t^2e^{qt};q;x\Big) = \frac{nx}{(n+q)^2}e^{q/(n+q)}\Big\{nxe^{q/(n+q)} + 1\Big\}e^{q_n x},$$

for $x \in R_0$ and $n \in N$, where

$$q_n := n\Big(e^{q/(n+q)} - 1\Big). \tag{12}$$

Next properties of $S_n[f;q]$ we shall give in Section 2. Main theorems will be given in Section 3.

2. Lemmas.

Applying (11)–(12) and (2), we shall prove two main lemmas.

Lemma 1. *Let $q > 0$ be a fixed number. Then*

$$\Big\|S_n\Big[1/v_q;q\Big]\Big\|_q \le 1, \quad n \in N. \tag{13}$$

Moreover

$$\Big\|S_n\Big[f;q\Big]\Big\|_q \le \|f\|_q, \tag{14}$$

for every $f \in B_q$ and $n \in N$.

The formulas (1)–(6) *and the inequality* (14) *show that $S_n[f;q]$, $n \in N$, is a positive linear from the space B_q into C_q.*

Proof. For $n \in N$ and $q > 0$ we have

$$0 < e^{q/(n+q)} - 1 < \sum_{k=1}^{\infty} \left(\frac{q}{n+q} \right)^k = \frac{q}{n},$$

which by (12) implies $0 < q_n < q$ for all $n \in N$. From this and by (11) and (1) we get

$$\nu_q(x) S_n\Big(1/\nu_q(t); q; x\Big) = e^{(q_n - q)x} \leq 1 \quad \text{for } x \in R_0, n \in N,$$

and by (2) follows (13).

For $f \in B_q$ we get from (6) and by (2) and (13)

$$\|S_n[f : q]\|_q \leq \|f\|_q \Big\| S_n\Big[1/\nu_q; q\Big]\Big\|_q \leq \|f\|_q, \quad n \in N.$$

Thus the proof is completed. □

Lemma 2. *For every fixed $q > 0$ and for all $x \in R_0$ and $n \in N$ we have*

$$\nu_q(x) S_n\left(\frac{(t-x)^2}{\nu_q(t)}; q; x\right) \leq \frac{4e^2 q^2 x^2}{(n+q)^2} + \frac{3x}{n+q}. \tag{15}$$

Proof. From (11)–(12) it follows that

$$S_n\Big((t-x)^2 e^{qt}; q; x\Big) = \left\{ x^2 \left(\frac{n}{n+q} e^{q/(n+q)} - 1 \right)^2 + \frac{nx}{(n+q)^2} e^{q/(n+q)} \right\} e^{q_n x},$$

for $x \in R_0$ and $n \in N$. By the inequality $e^t - 1 \leq te^t$ for $t \geq 0$, we get

$$\left(\frac{n}{n+q} e^{q/(n+q)} - 1 \right)^2 \leq 2 \left\{ \left(\frac{n}{n+q} - 1 \right)^2 e^{2q/(n+q)} + \Big(e^{q/(n+q)} - 1 \Big)^2 \right\} \leq$$

$$\leq \frac{4q^2}{(n+q)^2} e^{2q/(n+q)} < \frac{4e^2 q^2}{(n+q)^2} \quad \text{for } n \in N.$$

From the above and by (13) we easily obtain (15). □

From (8)–(10) we obtain

Lemma 3. *Assuming that $q > 0$ is a fixed number, we have*

$$\lim_{n\to\infty} n S_n(t-x; q; x) = -qx, \quad \lim_{n\to\infty} n S_n((t-x)^2; q; x) = x,$$

$$\lim_{n\to\infty} n^2 S_n((t-x)^4; q; x) = 3x^2,$$

for every $x \in R_0$.

3. Theorems.

3.1. First we shall give two theorems on point-convergence of the sequence $\big(S_n(f;q;\cdot)\big)_1^\infty$.

Theorem 1. *Suppose that $f \in B_q$ with a fixed $q > 0$ and let $x_0 \in R_0$ be a point of continuity of f. Then*

$$\lim_{n\to\infty} S_n(f;q;x_0) = f(x_0). \tag{16}$$

Proof. From (6) we get

$$S_n(f;q;0) = f(0), \quad n \in N. \tag{17}$$

If $x_0 > 0$, then by (6) and (7) we have

$$S_n(f;q;x_0) - f(x_0) = \sum_{k=0}^{\infty} \varphi_k(nx_0)\big(f\big(k/(n+q)\big) - f(x_0)\big), \quad n \in N.$$

Choose $\varepsilon > 0$. By our assumptions there exists $\delta = \delta(\varepsilon; x_0) > 0$ such that

$$|f(k/(n+q)) - f(x_0| < \varepsilon/2 \quad \text{if} \quad |k/(n+q) - x_0| < \delta.$$

Denoting by $Z_1 = \{k \in N_0 : |k/(n+q) - x_0| < \delta\}$, $Z_2 = \{k \in N_0 : |k/(n+q) - x_0| \geq \delta\}$ we can write

$$\nu_q(x_0)|S_n(f;q;x_0) - f(x_0)| \leq \left(\sum_{k\in Z_1} + \sum_{k\in Z_2}\right)\nu_q(x_0)\varphi_k(nx_0)|f(k/(n+q)) -$$

$$- f(x_0)| := \sum\nolimits_1 + \sum\nolimits_2$$

and

$$\sum\nolimits_1 < \frac{\varepsilon}{2}\sum_{k=0}^{\infty}\varphi_k(nx_0) = \frac{\varepsilon}{2}, \quad n \in N.$$

If $k \in Z_2$, then $1 \leq \delta^{-2}(k/(n+q) - x_0)^2$. Moreover for $f \in B_q$ we have

$$|f(k/(n+q)) - f(x_0)| \leq \|f\|_q\Big(e^{qk/(n+q)} + e^{qx_0}\Big), \quad k \in N_0, n \in N.$$

Hence

$$\sum\nolimits_2 \leq \|f\|_q \delta^{-2} \nu_q(x_0) \sum_{k\in Z_2} \varphi_k(nx_0\Big(e^{qk/(n+q)} + e^{qx_0}\Big)(k/(n+q) - x_0)^2 \leq$$

$$\leq \|f\|_q \delta^{-2}\Big\{e^{qx_0} S_n\Big(e^{qt}(t-x_0)^2; q; x_0\Big) + S_n\Big((t-x_0)^2; q; x_0\Big)\Big\}.$$

Applying (9) and (15), we get

$$\sum\nolimits_2 \leq \|f\|_q \delta^{-2}\left(\frac{(4e^2+1)q^2x_0^2}{(n+q)^2} + \frac{4x_0}{n+q}\right), \quad n \in N.$$

It is obvious that for fixed positive x_0, δ and $\|f\|_q$ there exists $n_0 \in N$ such that

$$\sum\nolimits_2 < \frac{\varepsilon}{2} \text{ for all } n > n_0.$$

Consequently,

(18) $$e^{-qx_0}|S_n(f; q; x_0) - f(x_0)| < \varepsilon \text{ for } n > n_0.$$

From (17) and (18) follows (16). □

Analogously as Theorem 1 we obtain

Theorem 2. *Suppose that $f \in B_{2q}$ with a fixed $q > 0$ and let $x_0 \in R_0$ be a point of continuity of f. Then assertion (16) is satisfied.*

Theorems 1 and 2 imply

Corollary 1. *If $f \in C_q$ or $f \in C_{2q}$ with a fixed $q > 0$, then*

(19) $$\lim_{n\to\infty} S_n(f; q; x) = f(x), \quad x \in R_0.$$

3.2. Now we shall given two theorems on the degree of approximation. Let $\omega_1(f; C_q; \cdot)$ and $\omega_2(f; C_q; \cdot)$ be the modulus of continuity and the modulus of smoothness of $f \in C_q$, $q > 0$, i.e.,

$$\omega_1(f; C_q; t) := \sup_{0\leq h\leq t} \|\Delta_h f(\cdot)\|_q, \quad \omega_2(f; C_q; t) := \sup_{0\leq h\leq t} \left\|\Delta_h^2 f(\cdot)\right\|_q,$$

for $t \geq 0$, where

$$\Delta_h f(x) := f(x+h) - f(x), \quad \Delta_h^2 f(x) := f(x) - 2f(x+h) + f(x+2h)$$

for $x, h \in R_0$. Let for fixed $m \in N$ and $q > 0$

$$C_q^m = \Big\{ f \in C_q : f^{(k)} \in C_q, k = 1, 2, \ldots, m \Big\},$$

and let

(20) $$\psi(x) := (1 + x^2)^{-1}, \quad x \in R_0,$$

(21) $$\lambda_{n,q} := \left(\frac{1+q}{n+q} \right)^{1/2}, \quad n \in N, q > 0.$$

Theorem 3. *Suppose that $f \in C_q^2$ with a fixed $q > 0$. Then*

(22) $$\|\{S_n[f; q] - f\}\Psi\|_q \le \frac{q}{n+q} \left\| f' \right\|_q + \Big(4e^2 + 1\Big) \left\| f'' \right\|_q \lambda_{n,q}^2$$

for all $n \in N$.

Proof. Let $x \in R_0$ be a fixed point. Then for $f \in C_q^2$ and $t \in R_0$ we can write

$$f(t) = f(x) + f'(x)(t - x) + \int_x^t \int_x^s f''(u)\, du ds,$$

which implies

$$f(t) = f(x) + f'(x)(t - x) + \int_x^t (t - u) f''(u) du.$$

From this and by (7) we deduce that

$$S_n(f(t); q; x) = f(x) + f'(x) S_n(t - x; q; x) + S_n\left(\int_x^t (t - u) f''(u)\, du; q; x \right), \quad n \in N.$$

But by (1) and (2),

$$\left| \int_x^t (t - u) f''(u)\, du \right| \le \left\| f'' \right\|_q \left(\frac{1}{\nu_q(t)} + \frac{1}{\nu_q(x)} \right) (t - x)^2.$$

From the above and by (9) and (15) it follows that

$$\nu_q(x) |S_n(f(t); q; x) - f(x)| \le \left\| f' \right\|_q |S_n(t - x; q; x)| +$$

$$+ \left\| f'' \right\|_q \left\{ \nu_q(x) S_n\left(\frac{(t-x)^2}{\nu_q(t)}; q; x \right) + S_n\Big((t - x)^2; q; x\Big) \right\} \le$$

$$\le \left\| f' \right\|_q \frac{qx}{n+q} + \left\| f'' \right\|_q \left\{ \frac{\Big(4e^2 + 1\Big) q^2 x^2}{(n+q)^2} + \frac{4x}{n+q} \right\}$$

for $n \in N$, which by (2) and (20) and (21) yields (22). □

Theorem 4. *Assume that $f \in C_q$ with a fixed $q > 0$. Then*

$$\|\{S_n[f;q] - f\}\Psi\|_q \le \frac{M_1}{\sqrt{n+q}}\omega_1(f;C_q;\lambda_{n,q}) + \tag{23}$$
$$+ (11 + 36e^2)\omega_2(f;C_q;\lambda_{n,q})$$

for all $n \in N$, where $M_1 = \left(5q/\sqrt{1+q}\right)\exp\sqrt{q+q^2}$ and $\lambda_{n,q}$ is defined by (21).

Proof. As in [1] we shall use the Stieklov function f_h of $f \in C_q$:

$$f_h(x) := \frac{4}{h^2}\int_0^{h/2}\int_0^{h/2}\Big[2f(x+s+t) - f(x+2(s+t))\Big]dsdt, \quad x \in R_0,\ h > 0.$$

From this we get

$$f_h'(x) = \frac{1}{h^2}\int_0^{h/2}\Big[8\Delta_{h/2}f(x+s) - 2\Delta_h f(x+2s)\Big]ds,$$

$$f_h''(x) = h^{-2}\Big[8\Delta_{h/2}^2 f(x) - \Delta_h^2 f(x)\Big],$$

and consequently $f_h \in C_q^2$ if $f \in C_q$. Moreover, for $h > 0$, we have

$$\|f_h - f\|_q \le \omega_2(f;C_q;h), \tag{24}$$

$$\left\|f_h'\right\|_q \le 5e^{qh}h^{-1}\omega_1(f;C_q;h), \tag{25}$$

$$\left\|f_h''\right\|_q \le 9h^{-2}\omega_2(f;C_q;h), \tag{26}$$

Hence we can write

$$\|\{S_n[f;q] - f\}\Psi\|_q \le \|\{S_n[f - f_h;q] - f\}\Psi\|_q + \tag{27}$$
$$+ \|\{S_n[f_h;q] - f_h\}\Psi\|_q + \|\{f_h - f\}\Psi\|_q$$

for $n \in N$, $h > 0$. By (20), (14) and (24), we get

$$\|\{S_n[f - f_h;q]\}\Psi\|_q \le \|f - f_h\|_q \le \omega_2(f;C_q;h), \tag{28}$$

for $n \in N$ and $h > 0$. Applying Theorem 3 and (25) and (26), we get

$$\|\{S_n[f_h; q] - f_h\}\Psi\|_q \le \frac{q}{n+q}\left\|f_h'\right\|_q + \left(4e^2+1\right)\left\|f_h''\right\|_q \lambda_{n,q}^2 \le \tag{29}$$

$$\le \frac{5qe^{qh}}{h(n+q)}\omega_1(f; C_q h) + 9(4e^2+1)h^{-2}\lambda_{n,q}^2\omega_2(f; C_q h)$$

for $n \in N$ and $h > 0$. Collecting (24), (28) and (29), we obtain from (27)

$$\|\{S_n[f; q] - f\}\Psi\|_q \le \frac{5qe^{qh}}{h(n+q)}\omega_1\Big(f; C_q; h\Big) +$$

$$+\Big\{2 + 9\Big(4e^2+1\Big)h^{-2}\lambda_{n,q}^2\Big\}\omega_2\Big(f; C_q; h\Big)$$

for all $n \in N$ and $h > 0$. Now setting $h = \lambda_{n,q}$, for fixed $n \in N$ and $q > 0$, we obtain (23). □

Theorem 4 implies the following

Corollary 2. *If $f \in C_q$, $q > 0$, then*

$$\lim_{n\to\infty}\{S_n(f; q; x) - f(x)\} = 0$$

uniformly on every interval $\big[x_1, x_2\big]$, $x_2 > x_1 \ge 0$.

Corollary 3. *If $f \in C_q$ with a fixed $q > 0$ and if $\omega_2(f; C_q; t) = O(t^\alpha)$ for a fixed $0 < \alpha \le 2$, then there exists positive constant $M_2(q, \alpha)$, depending only on q and α, such that*

$$\|\{S_n[f; q] - f\}\Psi\|_q \le M_2(q, \alpha)\cdot(n+q)^{-\alpha/2}$$

for all $n \in N$.

3.3. Applying Lemma 3 and Theorem 2, we shall prove the Voronovskaya type theorem.

Theorem 5. *Let $f \in C_q^2$ with a fixed $q > 0$. Then*

$$\lim_{n\to\infty} n\{S_n(f; q; x) - f(x)\} = -qxf'(x) + \frac{x}{2}f''(x) \tag{30}$$

for every $x \in R_0$.

Proof. By (17) follows (30) for $x = 0$. Choosing $x > 0$, we have by the Taylor formula for $f \in C_q^2$:

$$f(t) = f(x) + f'(x)(t-x) + \frac{1}{2}f''(x)(t-x)^2 + \varepsilon_1(t;x)(t-x)^2,\ t \in R_0,$$

where $\varepsilon_1(t) \equiv \varepsilon_1(t;x)$ is a function belonging to C_q and $\varepsilon_1(x) = 0$. From this and by (7) we get

$$S_n(f;q;x) = f(x) + f'(x)S_n(t-x;q;x) +$$
$$+\frac{1}{2}f''(x)S_n((t-x)^2;q;x) + S_n\Big(\varepsilon_1(t)(t-x)^2;q;x\Big),$$

which by Lemma 3 yields

$$\lim_{n\to\infty} n\{S_n(f;q;x) - f(x)\} = -qxf'(x) + \frac{x}{2}f''(x) + \tag{31}$$
$$+\lim_{n\to\infty} nS_n\Big(\varepsilon_1(t)(t-x)^2;q;x\Big).$$

By the Hölder inequality we have

$$\left|S_n\Big(\varepsilon_1(t)(t-x)^2;q;x\Big)\right| \le \Big\{S_n\Big(\varepsilon_1^2(t);q;x\Big)\Big\}^{1/2}\Big\{S_n\Big((t-x)^4;q;x\Big)\Big\}^{1/2},$$

for $n \in N$. Since $\varepsilon_1^2 \in C_{2q}$, we get by Theorem 2

$$\lim_{n\to\infty} S_n\Big(\varepsilon_1^2(t);q;x\Big) = \varepsilon_1^2(x) = 0.$$

From this and by Lemma 3 we deduce that

$$\lim_{n\to\infty} nS_n\Big(\varepsilon_1(t)(t-x)^2;q;x\Big) = 0$$

and from (31) follows (30). □

3.4. Now we shall give some properties of derivatives $(S_n[f;q])^{(r)}$, $r \in N$.

Theorem 6. *Suppose that $f \in B_q$ with a fixed $q > 0$. Then $S_n[f;q] \in C_q^\infty$ for every fixed $r \in N$ and $n \in N$*

$$\left\|(S_n[f;q])^{(r)}\right\|_q \le n^r \left\|\Delta^r_{1/(n+q)} f(\cdot)\right\|_q, \tag{32}$$

where

$$\Delta_h^r f(x) := \sum_{k=0}^{r} \binom{r}{k}(-1)^{r-k} f(x+kh). \tag{33}$$

Proof. From (6) we derive the formula

$$\frac{d}{dx} S_n(f(t); q; x) = -nS_n(f(t); q; x) + nS_n(f(t + 1/(n+q)); q; x) =$$

$$= nS_n(\Delta_{1/(n+q)} f(t); q; x)$$

and next for every $r \in N$ we have

$$\frac{d^r}{dx^r} S_n(f(t); q; x) = n^r S_n\Big(\Delta^r_{1/(n+q)} f(t); q; x\Big), \quad x \in R_0, n \in N, \tag{34}$$

where $\Delta^r_h f(\cdot)$ is defined by (33). Applying Lemma 1, we immediately obtain (32) from (34). □

Corollary 4. *Assuming as in Theorem 6, we obtain from* (32) *and* (33) *and* (2)

$$\left\|(S_n[f; q])^{(r)}\right\|_q \le \left(1 + e^{q/(n+q)}\right)^r n^r \|f\|_q,$$

for $n, r \in N$.

Formulas (7) and (34) imply the following

Corollary 5. *Let* $f \in C_q$, $q > 0$. *Then*

(a) *if* f *is a increasing (decreasing) function on* R_0*, then* $S_n(f; q; \cdot)$, $n \in N$*, is also increasing (decreasing) on* R_0*;*

(b) *if* f *is a convex (concave) function on* R_0*, then* $S_n(f; q; \cdot)$, $n \in N$*, is also convex (concave) on* R_0.

Finally we shall prove analogy of Theorem 1 for first derivate.

Theorem 7. *Suppose that* $f \in B_q$, $q > 0$*, and for given* $x_0 > 0$ *there exists* $f'(x_0)$*. Then*

$$\lim_{n\to\infty} (S_n[f; q])'(x_0) = f'(x_0). \tag{35}$$

Proof. By assumptions on f we have

$$f(t) = f(x_0) + f'(x_0)(t - x_0) + \varepsilon_2(t; x_0)(t - x_0), \tag{36}$$

for $t \in R_0$, where ε_2 is function continuous at x_0 and $\varepsilon_2 \in B_q$.

From (6) it follows that

$$(S_n[f;q])'(x) = -nS_n(f(t);q;x) + \frac{n+q}{x} S_n(tf(t);q;x) = \tag{37}$$

$$= qS_n(f(t);q;x) + \frac{n+q}{x} S_n((t-x)f(t);q;x),$$

for $x > 0$ and $n \in N$. By (36) and (37) we get

$$(S_n[f;q])'(x_0) = f(x_0)\left\{\frac{n+q}{x_0} S_n(t-x_0;q;x_0) + q\right\} + \tag{38}$$

$$+ f'(x_0)\left\{\frac{n+q}{x_0} S_n((t-x_0)^2;q;x_0) + qS_n(t-x_0;q;x_0)\right\} +$$

$$+ qS_n(\varepsilon_2(t)(t-x_0);q;x_0 + \frac{n+q}{x_0} S_n(\varepsilon_2(t)(t-x_0)^2;q;x_0).$$

Properties of ε_2 and Theorem 2 imply that

$$\lim_{n\to\infty} S_n(\varepsilon_2(t)(t-x_0);q;x_0) = 0. \tag{39}$$

Arguing analogously as in the proof of Theorem 5, we get

$$\lim_{n\to\infty} nS_n(\varepsilon_2(t)(t-x_0)^2;q;x_0) = 0. \tag{40}$$

Applying (8), (9), (39) and (40), we obtain (35) from (38). □

REFERENCES

[1] M. Becker - D. Kucharski - R. J. Nessel, *Global approximation theorems for the Szasz-Mirakjan operators in exponential weight spaces, in Linear Spaces and Approximation,* (Proc. Conf. Oberwolfach, 1977) Birkhäuser Verlag, Basel ISNM, 40 (1978), pp. 319–333.

[2] R.A. De Vore - G. G. Lorentz, *Constructive Approximation,* Springer-Verlag, Berlin, 1993.

Institute of Mathematics,
Poznań University of Technology,
Piotrowo 3A
60-965 Poznań (POLAND)