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ON ALMOST COMMUTATIVE UNITAL COMPLEX NORMED Q-ALGEBRAS

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We show that a unital complex normed Q-algebra $(A, \|.\|)$ in which the spectral radius satisfies:

$$\rho_A(x) = \inf\{p(x) : p \in Eun(A),\ p \leq \|.\|\},$$

where $Eun(A)$ denotes the set of all algebra-norms p on A equivalent to the given algebra-norm $\|.\|$ such that $p(e) = 1$, is commutative modulo its Jacobson radical. The same conclusion is obtained if $(A, \|.\|)$ satisfies:

$$\rho_{\widehat{A}}(xy) \leq \rho_{\widehat{A}}(x)\, \|y\| \text{ for every } x \in \widehat{A}, y \in A,$$

where $\widehat{A}$ is the completion of $(A, \|.\|)$.

1. Introduction

According to definition, the spectral radius is an algebraic notion. In the framework of Banach algebras, it is linked to the norm. Indeed, if $(A, \|.\|)$ is a unital Banach algebra, then by [3, Theorem 8, p. 23], one has:

$$\rho_A(x) = \inf\left\{\|x^n\|^{\frac{1}{n}} : n \in \mathbb{N}^*\right\}. \tag{1}$$

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In the non necessarily complete case, condition (1) also provides a characterization of Q-algebras [7]. For any unital normed algebra A, another connection between the spectral radius and the norm [3, Proposition 8, p. 11, Corollary 2, p. 18] is the following: for every $x \in A$, one has:

$$\rho_A(x) = \inf\{p\,(x) : p \in Eun\,(A)\}, \tag{2}$$

where $Eun\,(A)$ denotes the set of all algebra-norms p on A, equivalent to the given algebra-norm $\|.\|$, such that $p(e) = 1$. In the norm-unital commutative case, an important improvement of the above result, is that the infimum, in equality (2), is taken from the elements of $Eun\,(A)$ dominated by the norm $\|.\|$ that is, for every $x \in A$, we have:

$$\rho_A(x) = \inf\{p(x) : p \in Eun\,(A) \text{ and } p \leq \|.\|\} \tag{3}$$

(see Corollary 2.3).Thus, in a norm-unital commutative context, the equality is unchanged if we replace the norm $\|.\|$ by another norm of $Eun\,(A)$. Hence the question on the validity of (3) in the non commutative case. Moreover, if $(A, \|.\|)$ is an almost commutative Q-algebra, then by [9, Theorem 2.2.14, p. 219], $A/Rad(A)$ is a commutative Q-algebra. Hence, by [9, Corollary 3.1.6, p. 311],

$$\rho_A(xy) \leq \rho_A(x)\rho_A(y) \text{ for every } x, y \in A.$$

As $\rho_A(y) \leq \|y\|$ for every $y \in A$, one has:

$$\rho_A(xy) \leq \rho_A(x)\,\|y\| \text{ for every } x, y \in A.$$

Here, we consider the following property:

$$\rho_{\widehat{A}}(xy) \leq \rho_{\widehat{A}}(x)\,\|y\| \text{ for every } x \in \widehat{A}, y \in A, \tag{4}$$

where $\left(\widehat{A}, \widehat{\|.\|}\right)$ is the completion of $(A, \|.\|)$.

The aim of this note is to examine conditions (3) and (4) in the non commutative framework. We show that a unital normed Q-algebra satisfying (3) is almost commutative (see Theorem 4.1). We get the same conclusion when condition (3) is replaced with condition (4) (see Theorem 4.6). Such algebras are then commutative in the semi-simple case. We end this note with an example of a unital Q-algebra not satisfying (3) and which is almost commutative. Thus condition (3) gives rise to a class of unital normed Q-algebras strictly between that of unital commutative normed Q-algebras and that of unital almost commutative normed Q-algebras.

2. Definitions and preliminaries

In this paper, all the algebras considered are complex algebras. An algebra A is said to be unital if it contains a unit element e for the multiplication i.e. $ex = xe$ for every $x \in A$. It is said to be normed if it has a vector space norm $\|.\|$ such that

$$\|xy\| \leq \|x\| \, \|y\| \text{ for every } x, y \in A.$$

If, in addition $\|e\| = 1$, then the algebra is called a norm-unital normed algebra. Let A be a unital complex algebra, and let $G(A)$ stand for the group of all invertible elements of A. The spectrum of an element x of A, denoted by $Sp_A(x)$, is defined by:

$$Sp_A(x) = \{\lambda \in \mathbb{C} : x - \lambda e \notin G(A)\}.$$

The spectral radius $\rho_A(x)$ of x is given by:

$$\rho_A(x) = \sup\{|\lambda| : \lambda \in Sp_A(x)\}.$$

A unital normed algebra $(A, \|.\|)$ is called a Q-algebra if the group $G(A)$ of invertible elements is open. In this case, one has:

$$\rho_A(x) = \rho_{\widehat{A}}(x) \text{ for every } x \in A,$$

where $\widehat{A}$ is the completion of $(A, \|.\|)$ (in fact, this last equality characterizes the Q-property among normed algebras). The Jacobson radical of an algebra A, denoted $Rad(A)$, is the intersection of the kernels of all irreducible representations of A [3, p. 124] that is also:

$$Rad(A) = \{x \in A : \rho_A(xy) = 0 \text{ for every } y \in A\}.$$

An algebra A is said to be semi-simple if $Rad(A) = \{0\}$. It is said to be almost commutative if the algebra $A/Rad(A)$ is commutative. Recall the following equality which will be useful later:

$$\rho_A(axa^{-1}) = \rho_A(x) \text{ for every } x \in A \text{ and } a \in G(A).$$

3. Condition (3) in the commutative case

Let's start by treating a finite dimensional example which shed more light on the condition (3). Consider the algebra $\mathcal{M}_n(\mathbb{C})$ $(n \geq 2)$, of $n \times n$ complex matrices, provided with the norm-unital algebra-norm given by:

$$\left\|(a_{i,j})\right\| = \max\left\{ \sum_{1 \leq j \leq n} |a_{ij}| : 1 \leq i \leq n \right\}.$$

For each invertible element P of $\mathcal{M}_n(\mathbb{C})$, we denote by $\|.\|_P$ the norm-unital algebra-norm defined, on $\mathcal{M}_n(\mathbb{C})$, by:

$$\|M\|_P = \left\|PMP^{-1}\right\| \text{ for every } M \in \mathcal{M}_n(\mathbb{C}).$$

We denote by $\mathcal{T}_n$ the subalgebra of $\mathcal{M}_n(\mathbb{C})$ consisting of all upper triangular matrices.

Example 3.1. The algebra $(\mathcal{T}_n, \|.\|)$ has the property (3). Indeed, let $\alpha_1, ..., \alpha_n \in \mathbb{C}$, not zero and P the diagonal matrix whose diagonal is formed by $\alpha_1, ..., \alpha_n$. A simple calculation shows that if

$$|\alpha_1| \leq ... \leq |\alpha_n|,$$

then $\|.\|_P \leq \|.\|$. Moreover, for every $M \in \mathcal{T}_n$, one has:

$$\rho_{\mathcal{T}_n}(M) = \inf\{\|M\|_P : |\alpha_1| \leq ... \leq |\alpha_n|\}.$$

Notice that the algebra $\mathcal{T}_n$ is not commutative, but that $\mathcal{T}_n/Rad(\mathcal{T}_n)$ is commutative.

Proposition 3.2. *Let $(A, \|.\|)$ be a norm-unital normed algebra and $x_0 \in A$, which commutes with any element of A, such that $\rho_A(x_0) < 1$. Then there exists $\|.\|' \in Eun(A)$ such that $\|.\|' \leq \|.\|$ and $\|x_0\|' \leq 1$.*

Proof. Let $\widehat{A}$ denote the completion of the algebra A. Then one has

$$\rho_{\widehat{A}}(x_0) \leq \rho_A(x_0) < 1.$$

So, by [4, Corollary 1.1.18], the sequence $(\left\|x_0^i\right\|)_{i\geq 0}$ converges to zero. For every $x \in A$, put:

$$\|x\|' = \inf\left\{\sum_{0\leq i\leq n} \|a_i\| : n \geq 0,\ x = \sum_{0\leq i\leq n} a_i x_0^i,, \text{ where } a_i \in A\right\}.$$

It is easy to see that $\|x\|'$ is an algebra norm on A such that $\|x_0\|' \leq 1$ and

$$\|.\|' \leq \|.\| \leq k\|.\|', \text{ where } k = \max\left\{\left\|x_0^i\right\| : i \geq 0\right\}.$$

□

As a consequence, we obtain the following:

Corollary 3.3. *Let $(A, \|.\|)$ be a norm-unital commutative Q-algebra. Then, for every $x \in A$, one has:*

$$\rho_A(x) = \inf\{\|x\|' : \|.\|' \in Eun(A), \|.\|' \leq \|.\|\}.$$

Proof. Let $x \in A$ and $\varepsilon > 0$. It suffices to apply Proposition 3.2 to the element $x_0 = \frac{x}{\varepsilon + \rho_A(x)}$. □

4. Main Results

The following result establishes that condition (3) is quite close to commutativity. In fact, it is equivalent to commutativity in the semi-simple case. More precisely, we have:

Theorem 4.1. *Let $(A, \|.\|)$ be a unital normed Q-algebra and $(\widehat{A}, \widehat{\|.\|})$ its completion. We suppose that $(A, \|.\|)$ satisfies (3). Then:*

- ***i)*** *$\widehat{A}/Rad(\widehat{A})$ is commutative,*
- ***ii)*** *$A/Rad(A)$ is commutative.*

For the proof, we need the following lemma:

Lemma 4.2. *Let $(A, \|.\|)$ be a unital normed Q-algebra and $(\widehat{A}, \widehat{\|.\|})$ its completion. We suppose that $(A, \|.\|)$ satisfies (3). Then*

- ***i)*** *If $x \in \widehat{A}$ and if $(a_n)_n$ is a sequence of $(A, \|.\|)$ such that $a_n \longrightarrow x$, then we have:*
$$\rho_{\widehat{A}}(x) \leq \sup_n(\rho_A(a_n)).$$
- ***ii)*** *$Rad(A) = Rad(\widehat{A}) \cap A$.*

Proof. **i)** Let $\varepsilon > 0$ and n_0 such that $\widehat{\|x - a_{n_0}\|} \leq \varepsilon$. Then there exists $\|.\|_\varepsilon \in Eun(A)$ such that $\|.\|_\varepsilon \leq \|.\|$ and

$$\|a_{n_0}\|_\varepsilon \leq \rho_A(a_{n_0}) + \varepsilon.$$

Denote by $\widehat{\|.\|_\varepsilon}$ the completed norm of $\|.\|_\varepsilon$. Then, one has:

$$\begin{aligned}
\rho_{\widehat{A}}(x) &\leq \widehat{\|x\|_\varepsilon} = \widehat{\|x - a_{n_0} + a_{n_0}\|_\varepsilon} \\
&\leq \widehat{\|x - a_{n_0}\|_\varepsilon} + \widehat{\|a_{n_0}\|_\varepsilon} \\
&\leq \widehat{\|x - a_{n_0}\|} + \|a_{n_0}\|_\varepsilon \\
&\leq \rho_A(a_{n_0}) + 2\varepsilon \\
&\leq \sup_n(\rho_A(a_n)) + 2\varepsilon.
\end{aligned}$$

As ε is arbitrary, we obtain

$$\rho_{\widehat{A}}(x) \leq \sup_n(\rho_A(a_n)).$$

ii) Let $a \in Rad(A)$, $x \in \widehat{A}$, and $(a_n)_n$ be a sequence of A such that $a_n \longrightarrow x$. By **i),** we have:

$$\rho_{\widehat{A}}(ax) \leq \sup_n(\rho_A(aa_n)) = 0.$$

It follows that $\rho_{\widehat{A}}(ax) = 0$ and so $a \in Rad(\widehat{A})$ [3, Proposition 1, p. 126]. Thus $Rad(A) \subset Rad(\widehat{A}) \cap A$. The reverse inclusion follows from the Q-property of A [4, Corollary 3.6.46]. $\square$

Proof of Theorem 4.1. **i)** Let $x, y \in A$, $\lambda \in \mathbb{C}$ and $(a_n)_n$ be a sequence of invertible elements of A such that $a_n \longrightarrow e^{\lambda y}$. By assertion **i)** of Lemma 4.2, we have:

$$\rho_{\widehat{A}}(x - e^{\lambda y} x e^{-\lambda y}) \leq \sup_n(\rho_A(x - a_n x a_n^{-1})).$$

On the other hand, for every n and $\varepsilon > 0$, there exists $\|.\|_n \in Eun(A)$ such that $\|.\|_n \leq \|.\|$ and

$$\begin{array}{rcl} \left\|a_n x a_n^{-1}\right\|_n & \leq & \rho_A(a_n x a_n^{-1}) + \varepsilon \\ & \leq & \rho_A(x) + \varepsilon. \end{array}$$

We then have

$$\begin{array}{rcl} \rho_A(x - a_n x a_n^{-1}) & \leq & \left\|x - a_n x a_n^{-1}\right\|_n \\ & \leq & \|x\|_n + \left\|a_n x a_n^{-1}\right\|_n \\ & \leq & \|x\| + \rho_A(x) + \varepsilon. \end{array}$$

It follows that

$$\rho_{\widehat{A}}(x - e^{\lambda y} x e^{-\lambda y}) \leq \|x\| + \rho_A(x).$$

Now, inspired by an idea of [10], we consider the function f defined by:

$$f(\lambda) = \rho_{\widehat{A}}\left(\frac{x - e^{\lambda y} x e^{-\lambda y}}{\lambda}\right) \text{ if } \lambda \neq 0 \text{ and } f(0) = \rho_{\widehat{A}}(xy - yx).$$

By [10], the function f is subharmonic. Moreover,

$$f(\lambda) \leq \frac{\|x\| + \rho_A(x)}{|\lambda|} \longrightarrow 0 \text{ when } \lambda \longrightarrow \infty.$$

According to Liouville's theorem for subharmonic functions, the function f is identically zero. Then

$$\rho_{\widehat{A}}(xy - yx) = 0.$$

But since A is a Q-algebra, we have:

$$\rho_A(xy - yx) = 0.$$

From the assertion **i)** of Lemma 4.2, we deduce that:

$$\rho_{\widehat{A}}(xy - yx) = 0, \text{ for every } x, y \in \widehat{A}.$$

And by [8] (see also [1, Corollaires 7 and 8, pp. 45-46] or [2, Theorem 5.2.1]), $\widehat{A}/Rad(\widehat{A})$ is commutative.

ii) Follows from assertions **ii)** of Lemma 4.2 and **i)** of Theorem 4.1. □

As an immediate consequence of Theorem 4.1 and Corollary 3.3, we have the following:

Corollary 4.3. *In order for a semi-simple norm-unital Q-algebra (or one whose completion is semi-simple) to satisfy (3), it is necessary and sufficient that it be commutative.*

We also have the following result establishing the preservation of property (3) by going to completion. It is a property which is not obvious without Theorem 4.1.

Corollary 4.4. *For a unital Q-algebra to satisfy (3), it is necessary and sufficient that its completion satisfies it.*

Remark 4.5. Examining closely the proof of the theorem (take $x \in A$, $y = 0$ and $(a_n) = (e)$, we obtain $\|x\|_n \leq \rho_A(x) + \varepsilon$), we see that in a unital normed Q-algebra $(A, \|.\|)$, the condition (3) is equivalent to:

$$\rho_A(x) = \inf\{\|x\|' : \|.\|' \in Eun(A),\ \|.\|' \leq \|.\|\}\ \forall x \in G(A).$$

Let $(A, \|.\|)$ be a unital Q-algebra such that $(\widehat{A}, \widehat{\|.\|})$ is almost commutative. Applying [7, Corollary 2] to $\widehat{A}$ and taking into account the fact that all Banach algebras are Q-algebras, then one has:

$$\rho_{\widehat{A}}(xy) \leq \rho_{\widehat{A}}(x)\,\|y\| \text{ for every } x \in \widehat{A}, y \in A.$$

One would be tempted to look for unital normed Q-algebras that are not necessarily almost commutative and verify this last property. In fact, as we will see, this property implies almost commutativity of unital normed Q-algebras as the following result shows:

Theorem 4.6. *Let $(A, \|.\|)$ be a unital normed Q-algebra which satisfies (4). Then A is almost commutative.*

Proof. Let $x \in A$, $y \in \widehat{A}$ and λ be a complex number such that $|\lambda| > \rho_{\widehat{A}}(y) + \|x\|$. Then $\lambda - y$ is invertible in $\widehat{A}$. Taking into account equality:

$$\lambda - (x+y) = (\lambda - y)\left[e - (\lambda - y)^{-1} x\right]$$

and the fact that

$$\rho_{\widehat{A}}\left[(\lambda - y)^{-1} x\right] \leq \frac{\|x\|}{|\lambda| - \rho_{\widehat{A}}(y)} < 1,$$

one has $\lambda - (x+y)$ is invertible in $\widehat{A}$. So

$$\rho_{\widehat{A}}(x+y) \leq \rho_{\widehat{A}}(y) + \|x\| \text{ for every } x \in A, y \in \widehat{A}$$

Now, for $a, b \in A$, we consider the function g defined by:

$$g(\lambda) = \rho_{\widehat{A}}\left(\frac{a - e^{\lambda b} a e^{-\lambda b}}{\lambda}\right) \text{ if } \lambda \neq 0 \text{ and } g(0) = \rho_{\widehat{A}}(ab - ba).$$

The same arguments as for the function f, considered in Theorem 4.1, show that g is identically zero. So $\rho_{\widehat{A}}(ab - ba) = 0$. Finally, let x be any element of A. Since A is a unital normed Q-algebra and $(ab - ba)x \in A$, one has:

$$\rho_{\widehat{A}}[(ab - ba)x] = \rho_A[(ab - ba)x].$$

Then, by (4)

$$\rho_A[(ab - ba)x] \leq \rho_{\widehat{A}}(ab - ba)\|x\| = 0$$

and therefore $ab - ba \in Rad(A)$. So A is almost commutative.

□

Remark 4.7. 1) If in (4), we restrict to the elements of a unital normed algebra A, i.e.

$$\rho_A(xy) \leq \rho_A(x)\|y\| \text{ for every } x, y \in A, \tag{5}$$

then we have $\rho_A(a) < +\infty$ for every $a \in A$ (indeed, take $x = e$ and $y = a$ in (5)). In the case of an algebra A not necessarily unital, this last property is not generally valid as shown in the following example: Let $\mathbb{C}[X]$ denote the unital commutative complex algebra of all polynomials in the indetermite X, with complex coefficients, endowed with the algebra norm

$$\|p(X)\| := \max\{|p(\lambda)| : \lambda \in [0,1]\}.$$

Consider the ideal A of $\mathbb{C}[X]$ consisting of all polynomials vanishing at 0, i.e. $A = X\mathbb{C}[X]$. It is clear that A is a non-unital complex normed algebra, and that $\rho_A(p(X)) = +\infty$ for all nonzero $p(X) \in A$, and so the inequality (5) is trivially verified.

2) Another property which implies almost commutativity for a unital normed Q-algebra A is the following:

$$B_x \subset G(\widehat{A}) \text{ for every } x \in G(\widehat{A}), \tag{6}$$

where $B_x = A \bigcap B\left(x, \frac{1}{\rho_{\widehat{A}}(x^{-1})}\right)$ and $B\left(x, \frac{1}{\rho_{\widehat{A}}(x^{-1})}\right)$ is the open ball of $\widehat{A}$ with center x and radius $\frac{1}{\rho_{\widehat{A}}(x^{-1})}$. Indeed, let $a \in G(\widehat{A})$, $b \in A$ and $\lambda \in \mathbb{C}$ such that

$$|\lambda| > \rho_{\widehat{A}}(a) + \|b\|,$$

then $\lambda - a$ is invertible in $\widehat{A}$ and

$$\rho_{\widehat{A}}((\lambda - a)^{-1})\,\|b\| < 1.$$

Thus $\lambda - (a+b) \in B_{\lambda - a} \subset G(\widehat{A})$. As in the proof of Theorem 4.1, one has

$$\rho_{\widehat{A}}(a+b) \le \rho_{\widehat{A}}(a) + \widehat{\|b\|} \text{ for every } a \in \widehat{A},\ b \in A.$$

Finally, by Theorem 4.6, the algebra $(A, \|.\|)$ is almost commutative.

Let $(A, \|.\|)$ be a unital normed Q-algebra. If the spectral radius function $\rho_{\widehat{A}}$ is continuous on $\widehat{A}$, then property (5) implies property (4). Thus, one has the following:

Proposition 4.8. *Let $(A, \|.\|)$ be a unital normed Q-algebra which satisfies (5) such that the spectral radius function $\rho_{\widehat{A}}$ is continuous on $\widehat{A}$. Then A is almost commutative.*

Here is an example of a Q-algebra which is commutative modulo its radical but not satisfying (3).

Example 4.9. Let $(A, \|.\|)$ be a normed algebra in which every element is nilpotent and whose completion $\widehat{A}$ is semi-simple (an example of such an algebra is given in [5]). For every $x \in A$, one has $\rho_{\widehat{A}}(x) \le \rho_A(x)$ and so A cannot be commutative. The unitized $A^1 = A + \mathbb{C}e$ of A is commutative modulo its radical.

Indeed, $A \subset Rad(A^1)$ due to the fact that A is a left ideal of A^1 in which every element is nilpotent. Moreover A is of codimension 1 in A^1, so $Rad(A^1) = A$. This implies that $A^1/Rad(A^1)$, which is isomorphic to $\mathbb{C}$, is commutative. On the other hand, we have

$$\widehat{A^1} = \widehat{A + \mathbb{C}e} = \widehat{A} + \mathbb{C}e = \left(\widehat{A}\right)^1,$$

$Rad\left(\widehat{A}\right)^1 = Rad(\widehat{A})$ and $Rad(\widehat{A}) = \{0\}$. This implies that $Rad(\widehat{A^1}) = \{0\}$. Therefore $\widehat{A^1}/Rad(\widehat{A^1}) = \widehat{A^1}$ is not commutative. Finally, the algebra A^1 does not verify (3), otherwise, we would have according to **i)** of theorem 4.1, $\widehat{A^1}/Rad(\widehat{A^1})$ should be commutative, which is not the case.

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