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AN INFINITE PLATE WITH A CURVILINEAR HOLE IN S-PLANE

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Cauchy integral method has been applied to derive exact and closed expressions for Goursat's functions for the first and second fundamental problems for the infinite plate weakened by a hole having arbitrary shape.

The plate considered are conformally mapped on the area of the right half-plane. The work of many previous authors are considered as special cases of this work and the interesting cases when the the shape of the hole is an ellipse, a crescent, a triangle, or a cut having the shape of a circular are include as special cases.

1. Introduction.

The boundary value problems value for isotropic homogeneous perforated infinite plates have been discussed by several authors [1], [4], [8].

It is know that [8], the first and second fundamental problems in the plane theory of elasticity are equivalent to find two analytic functions $\phi_1(z)$ and $\psi_1(z)$ of one complex argument $z = x + iy$, satisfying the boundary condition

$$k\phi_1(t) - t\overline{\phi_1'} - \overline{\psi_1}(t) = f(t), \tag{1.1}$$

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Where $k = -1$, $f(t)$ is a given function of stesses, for the first fundamental problem;
While $k = \chi = \frac{\lambda+3\mu}{\lambda+\mu} > 1$, $f = 2\mu g(t)$ is a given function of the displacement for the second fundamental problem; λ and μ are called the constant's of Lame; χ is called Muskhelishvili's constant and t denoting the affix of a point on the boundary L.

Muskhelishvili [8] solved the problem of the stretched of an infinite plate weakened by an elliptic hole using the mapping function $z = c(\zeta + m\zeta^{-1})$. This transformation conformally maps the infinite domain bounded internally by an ellipse onto the domain outside the unit circle $|\xi| = 1$ in the ξ - plane.

In [1] El-Sirafy used the complex variable methods and rational mapping functions to obtain the Goursat functions for a stretched infinite plate weakened by an inner curvilinear hole using the transformation

$$\frac{z}{c} = \frac{(s+1)^2 + m(s-1)^2}{(s-1)(s+1)^2 - n(s-1)^2} \quad (c > 0,\ s = \sigma + i\tau;\ |n| < 1) \tag{1.2}$$

This transformation maps the perforated infinite plate onto the area of the right half-plane, Re $s \geq 0$.

The same author in [2] considered the case of stretched infinite plates weakened by hypotrochoidal holes with four or fives round corners, the Goursat functions $\phi(z)$ and $\psi(z)$ are obtained in a closed form.

Abdou and Hassan [5] obtained the two Goursat's functions for the stretched infinite plate weakened by a hole whose the edge is free from stresses, using the two rational mapping functions.

$$\frac{z}{c} = w(s) = \frac{(s+1)^3 + m(s-1)^3}{(s-1)(s+1)^2 - n(s+1)(s-1)^2}$$

and

$$\frac{z}{c} = w(s) = \frac{(s+1)^3 + m(s-1)^3}{(s-1)[(s+1)^2 - n(s-1)^2]^2} \quad (|n|, c > 0, s = \sigma + i\tau) \tag{1.3}$$

Here, m, n are real parameters subject to the conditions that $w'(s)$ does not vanish on the right half-plane (i.e Re $s \geq 0$) and $w\ (\infty)$ is bounded.

In this paper Cauchy integral methods and rational mapping functions.

$$\frac{z}{c} = w(s) = \frac{(s+1)^3 + m(s+1)(s-1)^2}{(s-1)(s+1)^2 - n(s+1)(s-1)^2},\ (c > 0, |n| < 1; s = \sigma + i\tau) \tag{1.4}$$

where $m, n,$ are real parameters subject to the conditions that $w(\infty)$ is bounded and $w'(s)$ does not vanish on the right half-plane (i.eRe $s \geq> 0$), are used to obtain exact and closed expressions for Goursat functions for the first and second fundamental problems of an infinite plate weakened by a curvilinear hole conformally mapped on the domain onto the right half-plane by (1.4)
If we let in (1.4) $s = \frac{\xi+1}{\xi-1}$, we have the transformation mapping

$$\frac{z}{c} = w(\xi) = \frac{\xi + m\xi^{-1}}{1 - n\xi^{-1}}$$

In terms of $z = cw(\xi)$, $c > 0$, $w'(\xi)$ does not vanish or become infinite for $|\xi| > 1$, the infinite region outside a closed contour conformally mapped outside the unit circle γ. (see [7])

Some applications of the first and second fundamental problems on these domain are investigated, the interesting cases of an infinite plate weakened by an elliptic hole, a crescent like hole or a cut having the shape of a circular arc, and the hypotrochoidal hole with three rounded corner are considered as special cases, and the functions $\phi(z)$ and $\phi(z)$ are obtained in a closed form.

2. Basic equations.

Consider a region of an elastic media of an infinite plate denoted by S and bounded by a single contour L, with a curvilinear hole C where the origin lies inside the hole.

If $x\hat{x}$, $y\hat{y}$, $x\hat{y}$ represent the components of stress, while u, v the components of displacement and in the absence of body forces, we have the formulae of Kolsov-Muskhelishvili [8] in the following form.

$$x\hat{x} + y\hat{y} = 4\,\mathrm{Re}\big\{\phi_1'(z)\big\} \tag{2.1}$$

$$y\hat{y} - x\hat{x} + 2ix\hat{y} = 2[z\phi_1''(z) + \psi_1'(z)] \tag{2.2}$$

and

$$2\mu\,(u + iv) = k\,\phi_1(z) - \overline{z\,\phi_1'(z)} - \psi_1(z). \tag{2.3}$$

In terms of conformal mapping function

$$z = cw(\xi),\ c > 0,\ w'(\xi) \neq 0 \ \text{ or } \ \infty \ \text{ for } \ |\xi| > 1,$$

The infinite region outside a closed contour conformally mapped on the region outside the unit circle Υ. The complex potentials $\phi(z)$ and $\psi(z)$ can be written in the form.

$$\phi_1(t) = -\frac{X + iY}{2\pi(1+\chi)} \ln t + \Gamma\, t + \phi_0\ (t) \tag{2.4}$$

$$\psi_1(t) = \frac{\chi(X - iY)}{2\pi(1+\chi)} \ln t + \Gamma^* t + \psi_0\ (t). \tag{2.5}$$

Where $X,\ Y$ are the components of the resultant vector of all external forces acting on L; Γ, Γ^* are constants and $\phi_0(t)$, $\psi_0(t)$ are holomorphic functions at infinity.
Using (2.4), (2.5) in (2.3), we obtain

$$k\phi_0(t) - \overline{t\phi_0'(t)} - \overline{\psi_0(t)} = f\ (t), \tag{2.6}$$

where $k = -1$, $f(t) = -f(t)$ for the first displacement problem; while $k = \chi$, $f(t) = 2\mu g(t)$ for the second fundamental problem.

3. Method of solution.

The expressions $\frac{\overline{w(i\tau)}}{w'(i\tau)}$ will be assumed in the form

$$\frac{\overline{w(i\tau)}}{w'(i\tau)} = \overline{\alpha(i\tau)} + \beta(i\tau), \tag{3.1}$$

where

$$\alpha(i\tau) = \frac{k^*}{a + i\tau}\ ,\quad a = \frac{1+n}{1-n}$$

$$k^* = 4na^2(n^3 + nm)J_0^{-1}\ ,\ J_0 = (1 - 2n^2 - mn^2) \tag{3.2}$$

and

$$\beta(s) = \frac{1}{s-a}\left[\frac{H(s)}{E(s)} + k^*\right],$$

where

$$\begin{aligned} H(s) &= (1-n)(s^2-1)(s+a)^2[m(s-1)(s+1)^2 + (s-1)^3] \\ E(s) &= 2[-(s+1)^4 + 2n(s+1)^3(s-1) + m(s+1)^2(s-1)^2] \end{aligned} \tag{3.3}$$

$\beta(s)$ is a regular function within the right half-plane except at infinity.
The boundary condition (2.6) takes the form

$$k\phi(i\tau) - \alpha(\tau)\phi'(i\tau) - \psi(i\tau) = f_*(\tau) \tag{3.4}$$

where

$$\begin{aligned}
&\phi(s) = \phi_0(w(s))\\
&f_*(\tau) = f(w(i\tau)) - \gamma_0 + w(i\tau)\big(\overline{\Gamma} - k\Gamma\big) + \overline{w(i\tau)}\,\overline{\Gamma}^*\\
&\qquad\qquad - \frac{X - iY}{2\pi(1+\chi)\overline{w'(i\tau)}}\,(w(i\tau) - \overline{w'(i\tau)})\\
&\psi(s) = \psi_0(w(s)) + \beta(s)\phi'(s) + \overline{\gamma_0} - \frac{X+iY}{2\pi(1+\chi)},\\
&\gamma_0 = c(\frac{1+m}{1-n})(\overline{\Gamma} - k\Gamma + \overline{\Gamma}^*),
\end{aligned} \tag{3.5}$$

and assume that $\phi(\infty) = \psi(\infty) = 0$.
Multiplying both sides of (2.4) by $\frac{1}{2\pi(s-i\tau)}$, and integrating with respect to τ from $-\infty$ to ∞, we have

$$k\phi(s) - \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha(i\tau)\overline{\phi'(i\tau)}}{s-i\tau}\,d\tau = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{f_*(\tau)}{s-i\tau}\,d\tau \tag{3.6}$$

and by using (3.1) in (3.6), we obtain:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\alpha(\tau)\overline{\phi'(i\tau)}}{s-i\tau}\,d\tau = \frac{ck^*b}{s+a}\,, \tag{3.7}$$

$$\begin{aligned}
\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{f_*(i\tau)\,d\tau}{s-i\tau} &= A_1(s) - \frac{2c\overline{\Gamma^*}}{1+s} + \frac{2c(k^*\Gamma - \overline{\Gamma})}{(1-n)^2}\left[\frac{m+n^2}{(s+a)}\right] +\\
&\quad + \frac{(1+n)(n^3+mn)(X-iY)}{(1+\chi)(1-n)(1+mn^2)(s+a)}
\end{aligned} \tag{3.8}$$

and

$$A_1(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{f(w(i\tau))}{s-i\tau}\,d\tau \tag{3.9}$$

Where b is a complex constant to be determined.
Substituting from (3.7)–(3.9) in (3.6), we get:

$$k\phi(s) = \frac{ck^*b}{s+a} + A_1(s) + \frac{\gamma_1(1+n)(X-iY)}{\pi\tau_2(1+\chi)(1-n)(s+a)} - \frac{2c\overline{\Gamma}^*}{1+s} + + \frac{2c(k\Gamma-\overline{\Gamma})}{(1-n)^2}\left(\frac{m+n^2}{a+s}\right) \tag{3.10}$$

where $\gamma_1 = (n^3+nm)$; $\gamma_2 = 1+mn^2$.
Differentiating (3.10) and inserting $\phi(i\tau)$ in (3.7), the complex constant b can be determined in the form:

$$b = \frac{2a^2}{c(16a^4k^2 - K^{*2})}\left\{\left[8a^2k\overline{A_1'(a)} - 2k^*A_1'(0)\right] + + c(1-n)^2(4a^2k\Gamma - k^*\overline{\Gamma^*}) - c\left[(4a^2k^2+k^*)\overline{\Gamma} - (4a^2+K^*)k\Gamma\right.\right. (\frac{m+n^2}{(1+n)^2}) - \frac{(1+n)\gamma_1}{c\gamma_2(1-n)(1+k)}\left[\frac{x}{4a^2k+k^*} + \frac{iY}{4a^2k-k^*}\right] \tag{3.11}$$

Inserting (3.11) in (3.10) the function $\phi(s)$ becomes.

$$k\phi(s) = A_1(s) + \frac{k\gamma_1 J_0 a(XJ_1 - iYJ_2)}{\pi\gamma_2(1+\chi)(s+a)} + \frac{2c\overline{\Gamma^*}}{(1+s)} + + \frac{2(m+n^2)h_1}{s+a} + \frac{2n\gamma_1 J_1 J_2}{(1-n)^2(s+a)}[h_2+h_3+h_4] \tag{3.12}$$

where

$$J_1 = (kJ_0 + n\gamma_1)^{-1}, \quad J_2 = (J_0k - n\gamma_1)^{-1},$$

$$\begin{aligned} h_1 &= c\frac{k\Gamma-\overline{\Gamma}}{(1-n)^2};\ h_2 = 2(1+n)^2\left[kJ_0\overline{A_1'(a)} - n\gamma_1 A_1'(a)\right], \\ h_3 &= c(1n^2)^2\left[kJ_0\Gamma^* - n\gamma_1\overline{\Gamma}^*\right], \\ h_4 &= c(m+n^2)\left[(J_0+n\gamma_1)k\Gamma - (k^2J_0+n\gamma_1)\overline{\Gamma}\right]. \end{aligned} \tag{3.13}$$

From the boundary condition (2.4), we can determine $\psi(s)$ in the form

$$\psi(s) = A_2(s) + \frac{kB_1(S)}{k^*} + \frac{2c(\Gamma-k\overline{\Gamma})}{1+S} - \frac{cK(1-n)^2(s+a+2)\overline{\Gamma}^*}{2k^*(1+s)^2} + \tag{3.14}$$

$$+\left(\frac{s+3a}{(s+a)^2}\right)\left\{\frac{2n\gamma_1 h_1(m+n^2)}{kJ_0}+\frac{n(1+n)\gamma_1^2(J_1X-iJ_2Y)}{\pi\gamma_2(1+\chi)(1-n)}+\right.$$

$$\left.+\frac{2n^2\gamma_1^2 J_1J_2}{(1-n)^2J_0k}(h_2+h_3+h_4)\right\}+\frac{2c(m+n^2)\Gamma^*}{(s+a)(1-n)^2}$$

where

$$B_1(s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\overline{A_1(i\tau)}\,d\tau}{(i\tau-a)(s-i\tau)} \tag{3.15}$$

And

$$A_2(s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\overline{fw(i\tau)}}{s-i\tau}\,d\tau$$

4. Special cases.

Now, we are in a position to consider several interesting cases:

(i) let in (3.12) and (3.13)

$$X=Y=f=0\ ,\ \Gamma'=-\frac{p}{2}e^{-2i0}\ ,\ \Gamma=\frac{p}{4}\ ,\ k=-1$$

we have

$$\phi(s)=\frac{cP}{(1-n)^2}\left[\frac{(m+n^2)J_3}{s+a}-\frac{(1-n)^2e^{2i0}}{1+s}\right], \tag{4.1}$$

$$\psi(s)=cp\left[\frac{1}{1+s}-\frac{(m+n)^2e^{-2i0}}{(1-n)^2(s+a)}\right]+\ cPk_1\cdot \tag{4.2}$$

$$\cdot\left[\frac{(m+n)^2(s+3a)J_3}{4(1+n)(s+a)^2}-\frac{(s+a+2)e^{2i0}}{(1+a)^2(1+s)^2}\right],$$

where

$$\begin{aligned} J_3&=\frac{(m-2)(n^2-1)+n^2(n^2-1)\cos 20}{n^4-1+2n^2(m+1)}\\ K_1&=\frac{4n^2a^2(m+n^2)}{1-(m+2)n^2}\end{aligned} \tag{4.3}$$

and p is a uniform tensile stress.

The previous results of (4.1)–(4.3) agree with (1.7) and (1.8) of El-Sirafy [1].

(ii) Let in (4.1) $s = \frac{\xi+1}{\xi-1}$; and in (3.12), (3.13) let

$$X = Y = f = 0\,;\ \Gamma^* = -\frac{p}{2}e^{-2i0}\ ,\ \Gamma = \frac{p}{4}\ ,$$

then excluding the constant term, we have the mapping function $\frac{z}{c} = \frac{\xi+m\xi^{-1}}{1-n\xi^{-1}}$, and the corresponding two Goursat functions take the form

$$\phi(\xi) = \frac{1}{2}cp\left[e^{2i0}\xi^{-1} + (m+n^2)(\frac{1}{2} - J_4)(\xi-n)^{-1}\right], \tag{4.4}$$

$$\psi(\xi) = \frac{-cp}{4} + \frac{cpw}{2w'(\xi)}\left[-\frac{1}{2}e^{2i0}\xi^{-2} + (m+n^2)(\frac{1}{2} - J_4)(\xi-n)^{-2}\right] - \tag{4.5}$$

$$-\frac{cpn^2k_2\xi}{2(1-n\xi)}\left[e^{2i0} + (m+n^2)(\frac{1}{2} - J_4)(1-n^2)^{-2} - \frac{1}{2n^2}\right]$$

where

$$\begin{aligned} J_4 &= \frac{(m+2)n^2 - 1 + n^2(n^2-1)^2\cos 20}{n^4 - 1 + 2n^2(m+1)} + in^2\sin 20 \\ K_2 &= \frac{(m+n^2)(1-n^2)^2}{1-(m+2)n^2} \end{aligned} \tag{4.6}$$

The formulae (4.4)–(4.6) agree with the formulae (4.1)–(4.2) of [3] on noting the difference on notation (see Fig. 1).

(iii) Let

$$X = Y = f = 0\ ,\ \Gamma = \frac{P}{4}\ ,\ \Gamma^* = \frac{P}{2}e^{-2i0},$$

the Goursat functions become

$$\phi(s) = \frac{cp}{(1-n)^2}\left[\frac{J_1^* + iJ_2^*}{(s+a)} - \frac{(1-n)^2e^{2i0}}{s+a}\right]$$

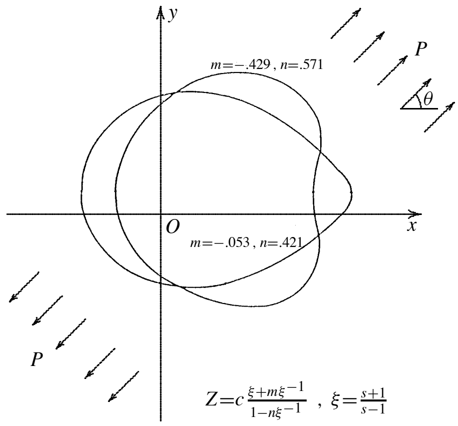

Figure 1

$$\psi(s) = cp\left[\frac{1}{1+s} - \frac{n^2e^{2i0}}{(1-n)^2(s+a)}\right] + \frac{cpk}{4}\left[\frac{(s+3a)(J_1^* + iJ_2^*)}{(1+n)^2(s+a)^2}\right] -$$
$$- \frac{(1-n)^2(s+2+a)e^{2i0}}{(1+s)^2}$$

where

$$J_1^* = \frac{n^2J_0^* - n^4(1-n^2)^2\cos 0}{J_0^* - n^4}$$
(4.8)
$$J_2^* = \frac{n^4(1-n^2)^2 sin20}{J_0^* + n^4}\ , \quad J_0^* = 1 - 2n^2$$

The previous results of (4.7)–(4.8) agree with (2.12)–(2.14) of Abdou [6], on noting the difference on notation.

(iv) Let $m = -n^2$ the hole is bounded by the circle $|z - nc| = c$ and the functions $\phi(s)$ and $\psi(s)$ become.

(4.9) $$k\phi(s) = A_1(s) - \frac{2c\overline{\Gamma}^*}{1+s}$$

and

$$\psi(s) = A_2(s) - \frac{2c(\Gamma - k\overline{\Gamma})}{1+s}$$

(v) For $m = -1$, the hole degenerates into a circle cut and the two complex functions $\phi(s)$ and $\psi(s)$ become

$$k\phi(s) = A_1(s) - \frac{2c\overline{\Gamma}^*}{1+s} - \frac{2c(k\Gamma - \overline{\Gamma})(1+n)}{(1-n)(s+a)} + \tag{4.11}$$

$$+ \frac{nk(1+n)}{\pi(1+\chi)(s+a)}\left[\frac{X}{n^2-k} + \frac{iY}{n^2+k}\right] +$$

$$+ \frac{2n^2}{(1-n)^2(n^4-k^2)(s+a)}\Big\{2(1+n)^2(kA_1'(a) + n^2A_1'(a)) +$$

$$+ c(1-n^2)^2(k\Gamma^* + n^2\overline{\Gamma}^*) + c(n^2-1)((n^2-k^2)\overline{\Gamma} + (1+n^2)k\Gamma)\Big\}$$

$$\psi(s) = A_2(s) - \frac{4(1+n)^2n^2}{k(1-n)^2}B_1(s) + \frac{2c(\Gamma - k\overline{\Gamma})}{(1+s)} + \frac{2c(n+1)\Gamma^*}{(n-1)(s+a)} + \tag{4.12}$$

$$+ \frac{2cn^2(s+a+2)(1+n)^2\overline{\Gamma}^*}{k(1+s)^2} + \frac{s+3a}{(s+a)^2}\left\{\frac{n^3(n+1)}{\pi(1+\chi)(n-1)}\left(\frac{x}{n^2-k} + \right.\right.$$

$$\left. + \frac{iY}{n^2+k}\right) + \frac{2n^2c(n+1)(k\Gamma - \overline{\Gamma})}{k(1-n)} + \frac{2n^4\left[c(1-n^2)^2(k\Gamma^* + n^2\overline{\Gamma}^*\right]}{k(1-n)^2(k^2-n^4)} +$$

$$\left. + 2(1=n)^2(k\overline{A_1'(a)} + n^2A_1'(a)) + c(n^2-1)(\overline{\Gamma} - k\Gamma) + k(\Gamma - k\overline{\Gamma})\right\}$$

(vi) Let in (4.11)–(4.12)

$$k = -1\ ,\ \Gamma = \frac{p}{4}\ ,\ \Gamma^* = -\frac{1}{2}Pe^{-2i0}\ ,\ X = Y = f(w(s)) = 0$$

and $s = \frac{\xi+1}{\xi-1}$ the results will agree with (3.1)–(3-2) of [3], on noting the different notation (see Fig. 2).

(vii) For $m = 0, s = \frac{\xi+1}{\xi-1}$, we have the mapping function $z = \frac{c\xi}{1-n\xi^{-1}}$ (see Fig. 3), where the inner edge of the infinite plate is the inverse of an elliptic limacon.

Also if

$$m = 0\ ,\ k = -1\ ,\ \Gamma = \frac{p}{4}\ ,\ \Gamma^* = -\frac{1}{2}pe^{-2i0}\ ,\ X = Y = f = 0$$

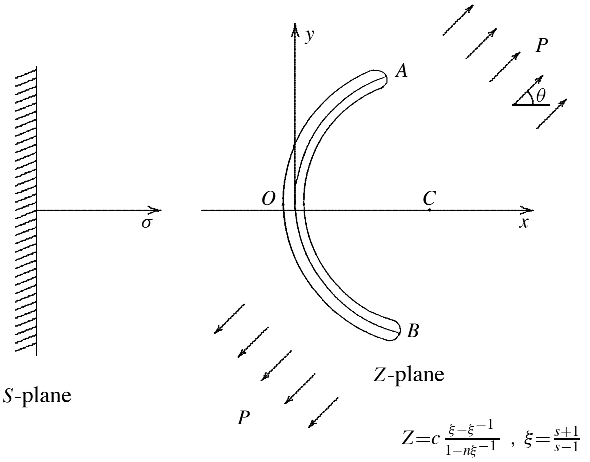

Figure 2

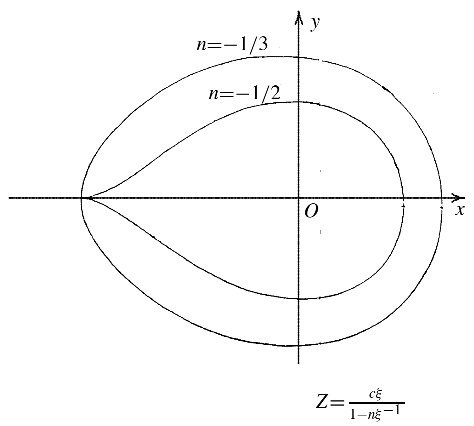

Figure 3

the Goursat functions become

$$\phi(s) = \frac{cP}{(1-n)^2}\left[\frac{J_5}{s+a} - \frac{(1-n)^2 e^{2i0}}{1+s}\right], \tag{4.13}$$

$$\psi(s) = cP\left[\frac{1}{1+s} - \frac{n^2 e^{2i0}}{(1-n)^2(s+a)}\right] + \tag{4.14}$$

$$+\frac{cpk_2}{4}\left[\frac{J_5(s+3a)}{(1+n)^2(s+a)^2} - \frac{(1-n)^2(s+a+2)e^{2i0}}{(1+s)^2}\right]$$

where

$$J_5 = \frac{n^2(1-2n^2) - n^4(1-n^2)cos20}{1-2n^2-n^4} + in^4 \ \sin 20 \quad , \quad k_2 = \frac{4a^2n^4}{1-2n^2}$$

The previous results agree with (1).

(vii) For $n = 0$, the hole takes a triangle form, and the two complex functions can be directly determined from (3.12)–(3.14).

5. Examples.

Now, we are in a position to consider some examples of the first and second fundamental problems.

(i) For

$$K = -1 \ , \ \Gamma = \frac{P}{4} \ , \ \Gamma^* = -\frac{1}{2}Pe^{-2i0} \ , \ X = \psi = f = 0$$

the Goursat functions for our tranformation take the form

$$\phi(s) = \frac{cP}{(1-n)^2}\left[\frac{(1-n)(1+s)(J_6+iJ_7)}{(1-n)(s+a)(1+s)} - \frac{(1-n)^2e^{2i0}}{1+s}\right] \tag{5.1}$$

$$\psi(s) = cp\left[\frac{1}{(1+s)} - \frac{(m+n^2)e^{-2i0}}{(1-n)^2(s+a)}\right] - \tag{5.2}$$

$$-\frac{(1-n)^2(s+a+2)e^{2i0}}{(1+s)^2} + \frac{cpk^*}{4}\left[\frac{(J_6+iJ_7)(s+3a)}{(1+n)^2(s+a)^2}\right]$$

where

$$J_6 = \frac{(m+n^2)J_0 - n\gamma_1(1-n^2)^2\cos 20}{(J_0 - n\gamma_1)}$$

and

$$J_7 = \frac{n\gamma_1((1-n^2)^2 \sin 20}{(J_0 + n\gamma_1)}$$

in the previous example, we have an infinite plate stretched at infinity by the application of a uniform tensile stress of intesity P, making an angle 0 with the xaxis. The plate is weakened by a curvilinear hole C which is free from stress. This result agree with [1].

(ii) For

$$k = -1\ ,\ X = Y = \Gamma = \Gamma^* = 0$$

and $f = Pt$, complex functions take the form

$$\phi(s) = \frac{2cP}{(1-n)^2(s+a)}(m+n^2)\left[1 + \frac{n\gamma_1}{J_0 - n\gamma_1}\right], \tag{5.3}$$

$$\psi(s) = \frac{-2cp\gamma_1^2(J_0 + n\gamma_1 - 2n^4 + 4n^2 - 2)(s+3a)}{J_0(J_0 - n\gamma_1)(1-n)^2(s+a)^2} +$$

$$+ \frac{2cp}{1+s}\left[1 - \ell J_0^{-1}\gamma_1(1+n)^2(1 + \frac{2}{(1-n)(1+s)}\right].$$

The previous results give the solution of the first fundamental problem for an isotropic infinite plate with a curvilinear hole when there is no external forces and the edge of the hole is subject to a uniform pressure p. If $P = -iT$, we have case when the edge of the hole is subject to a uniform tangential stress T.

(iii) If $\Gamma = \Gamma^* = f = 0$ and $k = \chi$, then the two complex functions are transformed to

$$\phi(s) = \frac{J_0 J_1 J_2 \gamma_1(1+n)[X(\chi J_0 + n\gamma_1) - iY(\chi J_0 + n\gamma_1)}{\pi\gamma_2(1+\chi)(1-n)(s+a)} \tag{5.5}$$

$$\psi(s) = \frac{n(1+n)\gamma_1^2(s+3a)}{\pi\gamma_2(1+\chi)(1-n)(s+a)^2}\left[\frac{X}{\chi J_0 + n\gamma_1} - \frac{iY}{\chi J_0 + n\gamma_1}\right]. \tag{5.6}$$

Therefore, we have the solution of the second fundamental Problem when a force (X, Y) acts on the center of the curvilinear kernel.

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