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HÖLDER CONTINUITY FOR SECOND ORDER NON VARIATIONAL PARABOLIC SYSTEMS

MARIA STELLA FANCIULLO

Let Q be the cylinder $\Omega \times (-T, 0)$, with $T > 0$, we prove that if $u \in W^2(Q, \mathbb{R}^N)$ (N integer ≥ 1) is a solution in Q of the system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = 0,$$

where $X = (x, t)$, $a(X, \xi)$ is a vector of $\mathbb{R}^N$, measurable in X, continuous in ξ and satisfying the conditions $a(X, 0) = 0$ and (A), then u and Du are Hölder continuous in Q, if $n \leq 4$ and $n = 2$, respectively.

We obtain similar results for the solutions in Q of the systems

$$a(X, H(u)) - \frac{\partial u}{\partial t} = f(X)$$

and

$$a(X, H(u)) - \frac{\partial u}{\partial t} = b(X, u, Du),$$

where $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$ and $b(X, u, p)$ is a vector of $\mathbb{R}^N$ with linear growth.

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1. Introduction.

Let Ω be an open bounded set of $\mathbb{R}^n$, $n \geq 2$, of generic point $x = (x_1, x_2, \dots, x_n)$, let T be a real positive number, we denote the cylinder $\Omega \times (-T, 0)$ by Q and the point $(x, t) \in \mathbb{R}^n_x \times \mathbb{R}_t$ by X.

If $u(X) : Q \to \mathbb{R}^N$, N integer ≥ 1, we set

$$D_i u = \frac{\partial u}{\partial x_i},$$

$$Du = (D_1 u, D_2 u, \dots, D_n u),$$

$$H(u) = \{D_i D_j u\} = \{D_{ij} u\}, \qquad i, j = 1, 2, \dots, n;$$

Du is an element of $\mathbb{R}^{nN}$ and $H(u)$ is an element of $\mathbb{R}^{n^2 N}$.

We denote by

$$W^p(Q, \mathbb{R}^k) = \left\{u : u \in L^p(-T, 0, H^{2,p}(\Omega, \mathbb{R}^k)), \frac{\partial u}{\partial t} \in L^p(Q, \mathbb{R}^k)\right\}$$

and

$$W^p_0(Q, \mathbb{R}^k) = \{u \in W^p(Q, \mathbb{R}^k) : u \in L^p(-T, 0, H^{1,p}_0(\Omega, \mathbb{R}^k)), u(x, -T) = 0\},$$

where $p \in [1, +\infty[$, k is an integer ≥ 1, $H^{2,p}(\Omega, \mathbb{R}^k)$ and $H^{1,p}_0(\Omega, \mathbb{R}^k)$ are the usual Sobolev spaces.

We consider the system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = 0, \tag{1.1}$$

where $a(X, \xi)$ is a vector in $\mathbb{R}^N$, measurable in X, continuous in ξ and satisfying the conditions

$$a(X, 0) = 0; \tag{1.2}$$

(A) *there exist three positive constants α, γ and δ, with $\gamma + \delta < 1$, such that, $\forall \tau, \xi \in \mathbb{R}^{n^2 N}$ and for a.e. $X \in Q$, it results*

$$\Big\| \sum_{i=1}^n \tau_{ii} - \alpha[a(X, \tau + \xi) - a(X, \xi)] \Big\|^2 \leq \gamma \|\tau\|^2 + \delta \Big\| \sum_{i=1}^n \tau_{ii} \Big\|^2;$$

(B) *there exists a bounded non-negative function* $\omega(t)$, *defined for* $t > 0$, *which is non-decreasing and goes to zero as* $t \to 0^+$, *such that,* $\forall X, Y \in Q$ *and* $\forall \xi \in \mathbb{R}^{n^2N}$

$$\|a(X,\xi) - a(Y,\xi)\|^2 \le \omega(d(X,Y))\|\xi\|^2,$$

where $d(X,Y) = \max\left\{\|x-y\|, |t-\tau|^{\frac{1}{2}}\right\}$, $X = (x,t)$ *and* $Y = (y,\tau)$.

A solution of the system (1.1) is a function $u \in W^2(Q, \mathbb{R}^N)$ which satisfies (1.1) for a.e. $X \in Q$.

In [3] and [6] there are Hölder continuity results in Q for the solutions of the basic system

$$a(H(u)) - \frac{\partial u}{\partial t} = 0;$$

these results are been obtained by S. Campanato in [3], using the Sobolev imbedding Theorem and, with different technique (using fundamental estimates for $H(Du)$, $\frac{\partial (Du)}{\partial t}$, $H(u)$ and $\frac{\partial u}{\partial t}$) by M. Marino and A. Maugeri in [6].

Furthermore, in [6] M. Marino and A. Maugeri obtained similar results for the following systems

$$a(H(u)) - \frac{\partial u}{\partial t} = f(X)$$

and

$$a(H(u)) - \frac{\partial u}{\partial t} = b(X, u, Du),$$

with $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$ and $b(X,u,p)$ vector in $\mathbb{R}^N$ with linear growth, i.e.

$$\|b(X,u,p)\| \le c(1 + \|u\| + \|p\|)$$

for a.e. $X \in Q$, $\forall u \in \mathbb{R}^N$ and $\forall p \in \mathbb{R}^{nN}$.

In this paper, for system (1.1), thanks to the hypothesis (B), we obtain $L^{2,2+(n+2)(1-\frac{2}{q})-\varepsilon}_{\text{loc}}$ regularity results for $H(u)$ and $\frac{\partial u}{\partial t}$ and then we prove that u and Du are Hölder continuous in Q, if $n \le 4$ and $n = 2$, respectively.

In Section 3 we obtain similar results for the following system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = f(X) \tag{1.3}$$

with $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$. At last in Section 4 we study the Hölder continuity in Q for system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = b(X, u, Du), \tag{1.4}$$

where $b(X,u,p)$ is a vector of $\mathbb{R}^N$ with linear growth.

2. Hölder continuity for systems of type (1.1).

Let $u \in W^2(Q, \mathbb{R}^N)$ be a solution in Q of the following parabolic system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = 0.$$

In this section we shall give some Hölder continuity results in Q for u and Du.

Theorem 2.1. *If the vector* $a(X, \xi)$ *satisfies the hypothesis* (1.2), (A) *and* (B), *then,* $\forall \varepsilon > 0$ *an* $\forall q \in (2, \min(\tilde{q}, n+2))$ (1), *it results:*

$$H(u) \in L^{2,2+(n+2)(1-\frac{2}{q})-\varepsilon}_{\mathrm{loc}}(Q, \mathbb{R}^{n^2 N})$$

and

$$\frac{\partial u}{\partial t} \in L^{2,2+(n+2)(1-\frac{2}{q})-\varepsilon}_{\mathrm{loc}}(Q, \mathbb{R}^N). \tag{2.1}$$

Proof. Fixed $Q(X^0, \sigma) = Q(\sigma) \subset\subset Q$ (2), with $0 < \sigma < 1$, let w be the solution of the Cauchy-Dirichlet problem (the existence and uniqueness are ensured by Theorem 1.2 in [6]):

$$\begin{cases} w \in W^2_0(Q(\sigma), \mathbb{R}^N) \\ a(X^0, H(u) + H(w)) - \dfrac{\partial w}{\partial t} = \dfrac{\partial u}{\partial t} \qquad \text{in} \quad Q(\sigma). \end{cases}$$

For this solution, (1.8) of [6] ensures the following inequality

$$\int_{Q(\sigma)} \left(\|H(w)\|^2 + \left\| \frac{\partial w}{\partial t} \right\|^2 \right) dX \leq \tag{2.2}$$

$$\leq c \int_{Q(\sigma)} \left\| \frac{\partial u}{\partial t} - a(X^0, H(u)) \right\|^2 dX.$$

Set $v = u + w$ in $Q(\sigma)$, it results: $v \in W^2(Q(\sigma), \mathbb{R}^N)$ and

$$a(X^0, H(v)) - \frac{\partial v}{\partial t} = 0 \qquad \text{in} \quad Q(\sigma).$$

(1) $\tilde{q}$ is the constant (> 2) that occurs in (2.19) of [6].

(2) If $X^0 = (x^0, t^0) \in Q$ and if $\rho > 0$, the symbol $Q(X^0, \rho)$ denotes the cylinder $B(x^0, \rho) \times (t^0 - \rho^2, t^0)$.

Thanks to Theorem 3.2 in [6], for v the following inequality holds

$$\int_{Q(\tau\sigma)} \left(\|H(v)\|^2 + \left\| \frac{\partial v}{\partial t} \right\|^2 \right) dX \leq \tag{2.3}$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(v)\|^2 + \left\| \frac{\partial v}{\partial t} \right\|^2 \right) dX,$$

$\forall \tau \in (0, 1)$ and $\forall q \in (2, \min(\tilde{q}, n+2))$.

Now, thanks to (2.2) and (2.3), since $u = v - w$ and is solution of system (1.1) and thanks to the hypothesis (B), it follows, $\forall \tau \in (0, 1)$ and $\forall q \in (2, \min(\tilde{q}, n+2))$

$$\int_{Q(\tau\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX \leq \tag{2.4}$$
$$\leq 2\int_{Q(\tau\sigma)} \left(\|H(v)\|^2 + \left\| \frac{\partial v}{\partial t} \right\|^2 \right) dX + 2\int_{Q(\sigma)} \left(\|H(w)\|^2 + \left\| \frac{\partial w}{\partial t} \right\|^2 \right) dX \leq$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(v)\|^2 + \left\| \frac{\partial v}{\partial t} \right\|^2 \right) dX +$$
$$+ 2\int_{Q(\sigma)} \left(\|H(w)\|^2 + \left\| \frac{\partial w}{\partial t} \right\|^2 \right) dX \leq$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX +$$
$$+ c\int_{Q(\sigma)} \left(\|H(w)\|^2 + \left\| \frac{\partial w}{\partial t} \right\|^2 \right) dX \leq$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX +$$
$$+ c\int_{Q(\sigma)} \left\| \frac{\partial u}{\partial t} - a(X^0, H(u)) \right\|^2 dX \leq$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX +$$
$$+ c\int_{Q(\sigma)} \|a(X, H(u)) - a(X^0, H(u))\|^2 dX \leq$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX + c\omega(\sigma) \int_{Q(\sigma)} \|H(u)\|^2 dX$$

and hence

$$\int_{Q(\tau\sigma)} \left(\|H(u)\|^2 + \left\|\frac{\partial u}{\partial t}\right\|^2\right) dX \leq \tag{2.5}$$
$$\leq c\left[\tau^{2+(n+2)(1-\frac{2}{q})} + \omega(\sigma)\right] \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\|\frac{\partial u}{\partial t}\right\|^2\right) dX.$$

From (2.5) and Lemma 1.III in [1] it deduces that $\forall \varepsilon > 0$ there exists $\sigma_\varepsilon \in (0, 1)$ such that $\forall \tau \in (0, 1)$ and $\forall \sigma \in (0, \sigma_\varepsilon]$

$$\int_{Q(\tau\sigma)} \left(\|H(u)\|^2 + \left\|\frac{\partial u}{\partial t}\right\|^2\right) dX \leq \tag{2.6}$$
$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})-\varepsilon} \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\|\frac{\partial u}{\partial t}\right\|^2\right) dX,$$

$\forall q \in (2, \min(\tilde{q}, n+2))$.

From (2.6) the thesis follows. □

Thanks to Theorem 2.1 and Lemma 2.II in [4] it deduces that

$$Du \in \mathcal{L}^{2,4+(n+2)(1-\frac{2}{q})-\varepsilon}_{\text{loc}}(Q, \mathbb{R}^{nN}) \tag{2.7}$$

$\forall \varepsilon > 0$ and $\forall q \in (2, \min(\tilde{q}, n+2))$.

If $n < 2\tilde{q} - 2$ (and in particular if $n = 2$), there exist $\varepsilon > 0$ and $q \in (2, \min(\tilde{q}, n+2))$ such that $4 + (n+2)\left(1 - \frac{2}{q}\right) - \varepsilon > n+2$ and (2.7) holds (with this choice of ε and q), and then

Du is Hölder continuous in Q.

On the other hand, from Lemma 2.I in [4] and from conditions (2.1) and (2.7), it follows:

$$u \in \mathcal{L}^{2,6+(n+2)(1-\frac{2}{q})-\varepsilon}_{\text{loc}}(Q, \mathbb{R}^{N}),$$

$\forall \varepsilon > 0$ and $\forall q \in (2, \min(\tilde{q}, n+2))$, from which we have that, if $n < 3\tilde{q} - 2$ (and in particular if $n \leq 4$), the vector

u is Hölder continuous in Q.

3. Hölder continuity for system of type (1.3)..

Let $f : Q \to \mathbb{R}^N$ be a function of class $L^2(Q, \mathbb{R}^N)$ and let $u \in W^2(Q, \mathbb{R}^N)$ be a solution in Q of the parabolic system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = f(X), \tag{3.1}$$

with $a(X, \xi) : Q \times \mathbb{R}^{n^2 N} \to \mathbb{R}^N$ measurable in X, continuous in ξ and satisfying conditions (1.2), (A) and (B).

Lemma 3.1. *For every cylinder* $Q(X^0, \sigma) = Q(\sigma) \subset Q$, *with* $\sigma < 1$, $\forall \tau \in (0, 1)$ *and* $\forall q \in (2, \min(\tilde{q}, n+2))$ [1], *one has:*

$$\int_{Q(\tau\sigma)} \Big(\|H(u)\|^2 + \Big\|\frac{\partial u}{\partial t}\Big\|^2 \Big) dX \leq \tag{3.2}$$

$$\leq c\Big[\tau^{2+(n+2)(1-\frac{2}{q})} + \omega(\sigma)\Big] \int_{Q(\sigma)} \Big(\|H(u)\|^2 + \Big\|\frac{\partial u}{\partial t}\Big\|^2 \Big) dX +$$

$$+ c \int_{Q(\sigma)} \|f\|^2 \, dX,$$

where the constant c does not depend on σ *and* τ.

Proof. Fixed $Q(X^0, \sigma) = Q(\sigma) \subset Q$, with $\sigma < 1$, let us denote, as in the proof of Theorem 2.1, by w the solution of the Cauchy-Dirichlet problem

$$\begin{cases} w \in W^2_0(Q(X^0, \sigma), \mathbb{R}^N) \\ a(X^0, H(u) + H(w)) - \dfrac{\partial w}{\partial t} = \dfrac{\partial u}{\partial t} \qquad \text{in} \quad Q(\sigma). \end{cases}$$

Set $v = w + u$ in $Q(\sigma)$, it results: $v \in W^2(Q(\sigma), \mathbb{R}^N)$ and

$$a(X^0, H(v)) - \frac{\partial v}{\partial t} = 0 \qquad \text{in} \quad Q(\sigma).$$

For w and v the estimates (2.2) and (2.3) hold, respectively. Making use of these estimates and proceeding as in the proof of Theorem 2.1, one gets, $\forall \tau \in (0, 1)$ and $\forall q \in (2, \min(\tilde{q}, n+2))$

$$\int_{Q(\tau\sigma)} \Big(\|H(u)\|^2 + \Big\|\frac{\partial u}{\partial t}\Big\|^2 \Big) dX \leq \tag{3.3}$$

$$\leq c\tau^{2+(n+2)(1-\frac{2}{q})} \int_{Q(\sigma)} \Big(\|H(u)\|^2 + \Big\|\frac{\partial u}{\partial t}\Big\|^2 \Big) dX +$$

$$+ c \int_{Q(\sigma)} \Big\| \frac{\partial u}{\partial t} - a(X^0, H(u)) \Big\|^2 dX.$$

Now let us estimate the last integral in the right hand side of the (3.3).

Since u is solution of system (3.1), one has:

$$\frac{\partial u}{\partial t} - a(X^0, H(u)) = a(X, H(u)) - a(X^0, H(u)) - f(X) \quad \text{in} \quad Q(\sigma),$$

and then, thanks to hypothesis (B)

$$\int_{Q(\sigma)} \left\| \frac{\partial u}{\partial t} - a(X^0, H(u)) \right\|^2 dX \leq \tag{3.4}$$

$$\leq 2 \int_{Q(\sigma)} \|a(X, H(u)) - a(X^0, H(u))\|^2 \, dX + 2 \int_{Q(\sigma)} \|f(X)\|^2 \, dX \leq$$

$$\leq c\omega(\sigma) \int_{Q(\sigma)} \|H(u)\|^2 dX + 2 \int_{Q(\sigma)} \|f(X)\|^2 \, dX.$$

(3.2) is a trivial consequence of inequalities (3.3) and (3.4). □

The next theorem follows easily from Lemma 3.1:

Theorem 3.1. *If* $u \in W^2(Q, \mathbb{R}^N)$ *is a solution of system* (3.1)*, if the vector* $a(X, \xi)$ *satisfies conditions* (1.2)*, (A) and (B) and if* $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$*,* $0 < \mu < \tilde{\lambda} = \min \left\{ 2 + (n+2)(1 - \frac{2}{\tilde{q}}), n+2 \right\}$*, then*

$$Du \in \mathcal{L}^{2,\mu+2}_{\text{loc}}(Q, \mathbb{R}^{nN}) \tag{3.5}$$

and

$$u \in \mathcal{L}^{2,\mu+4}_{\text{loc}}(Q, \mathbb{R}^N). \tag{3.6}$$

Proof. The proof is similar to that of Theorem 4.1 in [6]. Fixed $Q(\sigma) \subset Q$, with $\sigma < 1$, $\forall \tau \in (0,1)$ and for every $q \in (2, \min(\tilde{q}, n+2))$, in virtue of Lemma 3.1 and from the hypothesis $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$, one has:

$$\int_{Q(\tau\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX \leq \tag{3.7}$$

$$\leq c \left[\tau^{2+(n+2)(1-\frac{2}{q})} + \omega(\sigma) \right] \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\| \frac{\partial u}{\partial t} \right\|^2 \right) dX + c\sigma^{\mu} \|f\|^2_{\mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)}.$$

From (3.7) the thesis follows proceeding as in the proof of Theorem 4.1 in [6] and using Lemma 2.VII in [2], instead of Lemma 1.I in [1]. □

From (3.5) it follows, as in [6], that, if $n < 2\tilde{q} - 2$ (and in particular if $n = 2$) and if $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$, with $\mu \in (n, \tilde{\lambda})$,

$$Du \in \mathcal{L}^{2,\mu+2}_{\text{loc}}(Q, \mathbb{R}^{nN}),$$

with $\mu + 2 > n + 2$, and then

Du is Hölder continuous in Q.

If $n < 3\tilde{q} - 2$ (and in particular if $n \leq 4$) and if $f \in \mathcal{L}^{2,\mu}(Q, \mathbb{R}^N)$, with $\mu \in (n-2, \tilde{\lambda})$, for (3.6) one has

$$u \in \mathcal{L}^{2,\mu+4}_{\text{loc}}(Q, \mathbb{R}^N),$$

with $\mu + 4 > n + 2$, and then

u is Hölder continuous in Q.

4. Hölder continuity for systems of type (1.4).

Let $u \in W^2(Q, \mathbb{R}^N)$ be a solution in Q of the system

$$a(X, H(u)) - \frac{\partial u}{\partial t} = b(X, u, Du), \tag{4.1}$$

where $a(X, \xi)$ is a vector in $\mathbb{R}^N$, measurable in X, continuous in ξ and satisfying conditions (1.2), (A) and (B).

Let $b(X, u, p)$ be a vector in $\mathbb{R}^N$, measurable in X,continuous in (u, p) and satisfying the condition

$$\|b(X, u, p)\| \leq c(1 + \|u\| + \|p\|) \tag{4.2}$$

for a.e. $X \in Q$, $\forall u \in \mathbb{R}^N$ and $\forall p \in \mathbb{R}^{nN}$.

Theorem 3.1 in [5] ensures that there exists $\overline{q} \in \left(2, \frac{2(n+2)}{n}\right]$ such that, $\forall q \in (2, \overline{q})$,

$$u \in W^q_{\text{loc}}(Q, \mathbb{R}^N),$$

hence, thanks to Lemma 2.II in [4], one has, $\forall\, Q(\sigma) \subset\subset Q$ and $\forall q \in (2, \overline{q})$:

$$\int_{Q(\sigma)} \|Du - (Du)_{Q(\sigma)}\|^2 \, dX \leq c\sigma^2 \int_{Q(\sigma)} \left(\|H(u)\|^2 + \left\|\frac{\partial u}{\partial t}\right\|^2\right) dX \leq$$
$$\leq c\sigma^{2+(n+2)(1-\frac{2}{q})} \left[\int_{Q(\sigma)} \left(\|H(u)\|^q + \left\|\frac{\partial u}{\partial t}\right\|^q\right) dX\right]^{\frac{2}{q}},$$

from which

$$Du \in \mathcal{L}^{2,2+(n+2)(1-\frac{2}{q})}_{\mathrm{loc}}(Q, \mathbb{R}^{nN}),$$

$\forall q \in (2, \overline{q})$.

On the other hand, making use of Lemma 2.1 in [5] (with $p = 2$), it deduces

$$u \in \mathcal{L}^{2,4+(n+2)(1-\frac{2}{q})}_{\mathrm{loc}}(Q, \mathbb{R}^{N}),$$

$\forall q \in (2, \overline{q})$.

Then u and $D_i u$, $i = 1, 2, \dots, n$, belong to the space $\mathcal{L}^{2,\mu}(Q^*, \mathbb{R}^N)$, $\forall \mu \in \left(0, 2 + (n+2)\left(1 - \frac{2}{\overline{q}}\right)\right)$ and $\forall Q^* \subset\subset Q$, from which, for condition (4.2), it follows that the vector $f(X) = b(X, u, Du) \in \mathcal{L}^{2,\mu}(Q^*, \mathbb{R}^N)$, $\forall \mu \in \left(0, 2 + (n+2)\left(1 - \frac{2}{\overline{q}}\right)\right)$ and $\forall Q^* \subset\subset Q$.

For Theorem 3.1 one has

$$Du \in \mathcal{L}^{2,\mu+2}_{\mathrm{loc}}(Q, \mathbb{R}^{nN}),$$

$$u \in \mathcal{L}^{2,\mu+4}_{\mathrm{loc}}(Q, \mathbb{R}^{N}),$$

$\forall \mu \in (0, \lambda^*)$, where $\lambda^* = \min\left\{2 + (n+2)\left(1 - \frac{2}{q^*}\right), n+2\right\}$, $q^* = min(\overline{q}, \tilde{q})$.

Now reasoning exactly as in Section 5 of [6] it follows that

Du is Hölder continuous in Q if $n < 2q^* - 2$

and

u is Hölder continuous in Q if $n < 3q^* - 2$.

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Dipartimento di Matematica,
Università di Catania,
Viale Andrea Doria 6,
95125 Catania