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INVESTIGATION ON STABILITY OF ELECTROHYDRODYNAMIC SHOCK WAVES

A.M. BLOKHIN - I.Z. MERAZHOV - YU.L. TRAKHININ

Well-posedness of a linear mixed problem on stability of electrohydrodynamic shock waves is investigated in the paper. Stability of shock waves for a hydrodynamic model of movement of a continuum with a volume electric charge is proved.

1. Introduction.

The renewed interest to investigation of movements of a continuum with a volume charge is caused, on one hand, by a great number of various possible practical applications (see survey [3]). On the other hand the problem of derivation and justification of basic equations of electrohydrodynamics (EHD) is not solved as yet with a satisfactory result in distinguish, for example, from magnetohydrodynamics (MHD) (deducing and justification of MHD equations are discussed, for example, in [3], [10]).

In the present paper we discuss the variant of EHD equations suggested in [3] from the viewpoint of theory of partial equations. We preferred this variant to the one from [10] because in this case the considered system possesses certain mathematical advantages from the viewpoint of theory of differential equations (what is important, for example, for justification of numerical methods applied to solution of EHD problems). From this point of view it is necessary to consider the problem on stability of shock waves in EHD also. While investigating this

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problem we apply the method from [1]. It is known that this approach is based on the technique of dissipative energy integrals for the mixed linear problem on stability of shock electrohydrodynamical waves.

The paper consists of an introduction (the first section) and six sections. In the second section we give EHD equations in the single-liquid approximation (see [3]). In the third section we discuss problems on obtaining a linearized EHD system. Relations on a strong discontinuity for EHD equations are given in the fourth section. Here the main problem on stability of strong discontinuity in EHD is formulated. In the fifth section we discuss conditions on a stationary discontinuity. The main result of this paper is given in the sixth section. Exactly, with the help of technique of dissipative energy integrals, we prove the well-posedness of the main problem in the case when the stationary discontinuity is a shock wave. Concluding remarks can be found in the seventh section.

2. EHD equations in a single-liquid approximation.

Movement equations of a continuum with a volume charge in a single-liquid approximation can be written in the divergent form as it follows from [3], [10]:

$$\rho_t + \mathrm{div}(\rho \boldsymbol{u}) = 0, \tag{2.1}$$

$$(\rho \boldsymbol{u})_t + \mathrm{div}\tilde{\boldsymbol{\Pi}} = 0, \tag{2.2}$$

$$(\rho e)_t + \mathrm{div}\boldsymbol{W} = (\boldsymbol{J}, \boldsymbol{E}). \tag{2.3}$$

Here ρ is the density of continuum, $\boldsymbol{u} = (u_1, u_2, u_3)^*$ is the velocity of continuum (asterisk stands for transposition), $\tilde{\boldsymbol{\Pi}}$ is the impulse flux density tensor with the components

$$\tilde{\boldsymbol{\Pi}}_{ik} = \rho u_i u_k + p\delta_{ik} - P_{ik} \quad (i, k = 1, 2, 3),$$

components P_{ik} of the Maxwell stress tensor P are of the form [6]:

$$P_{ik} = \frac{1}{4\pi}(E_i E_k - \frac{|\boldsymbol{E}|^2}{2}\delta_{ik});$$

p is the pressure, $\boldsymbol{E} = (E_1, E_2, E_3)^*$ is the electric field strength, $|\boldsymbol{E}|^2 = (\boldsymbol{E}, \boldsymbol{E})$, $e = e_0 + \frac{1}{2}|\boldsymbol{u}|^2$, $|\boldsymbol{u}|^2 = (\boldsymbol{u}, \boldsymbol{u})$, e_0 is the internal energy, $\boldsymbol{W} = (W_1, W_2, W_3)^* = \rho \boldsymbol{u}(e + pV)$, $V(= \frac{1}{\rho})$ is the specific volume, $\boldsymbol{J}$ is the electric current density. The thermodynamical variables are connected by the Gibbs relation

$$T\,ds = de_0 + p\,dV \tag{2.4}$$

(s is the entropy, T is the temperature).

To system (2.1) – (2.3) it is necessary to add the charge conservation law ($q > 0$)

$$q_t + \operatorname{div}\boldsymbol{J} = 0, \tag{2.5}$$

and stationary laws of conservation (Maxwell equations for electric field in the electrohydrodynamic approximation)

$$\operatorname{div}\boldsymbol{E} = 4\pi q, \tag{2.6}$$

$$\operatorname{rot}\boldsymbol{E} = 0. \tag{2.7}$$

Maxwell equations (2.6), (2.7) can be reduced to the single Poisson equation for the scalar electric potential φ ($\boldsymbol{E} = -\nabla\varphi$):

$$\Delta\varphi = -4\pi q. \tag{2.8}$$

The electric current density $\boldsymbol{J}$ is connected with the velocity $\boldsymbol{u}$ and the electric field strength $\boldsymbol{E}$ by the Ohm law

$$\boldsymbol{J} = q(\boldsymbol{u} + b\boldsymbol{E}) \tag{2.9}$$

($b > 0$ is the mobility constant [3], [10]). By virtue of (2.4), the following is true

$$p = -(e_0)_V = \rho^2(e_0)_\rho,\ T = (e_0)_s. \tag{2.10}$$

Then, with account of the state equation

$$e_0 = e_0(\rho, s)$$

and (2.9), (2.10), we can treat system (2.1) – (2.3), (2.5) – (2.7) as a system to determine components of the vectors $\boldsymbol{U} = (p, s, \boldsymbol{u}^*)^*$, $\boldsymbol{E}$ and the charge q.

Since, by virtue of (2.6), (2.7),

$$\operatorname{div}\boldsymbol{P} = q\boldsymbol{E}.$$

the vector equation (2.2) can be rewritten as follows:

$$(2.2') \qquad (\rho \boldsymbol{u})_t + \operatorname{div} \boldsymbol{\Pi} = q\boldsymbol{E}.$$

Here $\boldsymbol{\Pi}$ is the impulse flux density tensor with the components $\Pi_{ik} = \rho u_i u_k + p\delta_{ik}$. We note that system (2.1), (2.2′), (2.3) are the system of gas dynamics equations with right parts.

System (2.1), (2.2′), (2.3) can be rewritten in the nondivergent form [3]:

$$(2.11) \qquad \begin{aligned} &\frac{1}{\rho c^2}\frac{dp}{dt} + \operatorname{div} \boldsymbol{u} = b\frac{(e_0)_{\rho s} q}{c^2 T}|\boldsymbol{E}|^2, \\ &\frac{ds}{dt} = b\frac{q}{\rho T}|\boldsymbol{E}|^2, \\ &\rho\frac{d\boldsymbol{u}}{dt} + \nabla p = q\boldsymbol{E}, \end{aligned}$$

where $\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + (\boldsymbol{u}, \nabla)$; $c^2 = (\rho^2 (e_0)_\rho)_\rho$ is the squared sound velocity in gas [8]. It is seen that, in its turn, (2.11) can be written in the matrix form

$$(2.11') \qquad B^{(0)}\boldsymbol{U}_t + \sum_{k=1}^{3} B^{(k)}\boldsymbol{U}_{x_k} = \boldsymbol{F}.$$

Here $B^{(0)} = B^{(0)}(\boldsymbol{U}) = \operatorname{diag}(1/(\rho c^2),\ 1, \rho, \rho, \rho)$ is a diagonal matrix, $B^{(k)} = B^{(k)}(\boldsymbol{U})$ are symmetric matrices, $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{U}, \boldsymbol{E}, q)$ is the vector of right parts. We note that $B^{(0)} > 0$ under the natural assumption that thermodinamical variables satisfy inequalities $\rho > 0$, $(\rho^2 (e_0)_\rho)\rho > 0$. Thus, with respect to the vector $\boldsymbol{U}$, system (2.11′) is symmetric t-hyperbolic (by Friedrichs) [1], [2]. The matrices $B^{(k)} (k = 1, 2, 3)$, the vector of right parts $\boldsymbol{F}$ can be easily written down.

Remark 2.1. In the case of polytropic gas [1], [8]:

$$(2.12) \qquad \begin{aligned} &e_0 = \frac{pV}{\gamma - 1}, \quad T = \frac{pV}{c_V(\gamma - 1)}, \\ &(e_0)_{\rho s} = \frac{pV^2}{c_V}, \quad c^2 = \gamma p V \end{aligned}$$

($\gamma > 1$ is the isentropic exponent, $c_V > 0$ is the specific heat).

3. Linearization of EHD equations.

The EHD equation system from above has a constant solution

$$U = \widehat{U} = (\hat{p}, \hat{s}, \hat{\boldsymbol{u}})^*, \quad \hat{\boldsymbol{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)^*,$$

$$\boldsymbol{E} = \widehat{\boldsymbol{E}} = (\widehat{E}_1, \widehat{E}_2, \widehat{E}_3)^*, \quad q = 0.$$

where

$$\hat{p} = \hat{\rho}^2 (e_0)_\rho(\hat{\rho}, \hat{s}), \quad \widehat{T} = (e_0)_s(\hat{\rho}, \hat{s});$$

$\hat{\rho}, \hat{s}, \hat{u}_k, \widehat{E}_k (k = 1, 2, 3)$ are certain constants.

After linearization [8] of EHD equations system with respect to the constant solution (this means that we slightly perturb the constant solution), we obtain a linear system of differential equations for small perturbations of the constant solution:

$$\begin{aligned} Lp + \operatorname{div} \boldsymbol{u} &= \frac{\gamma - 1}{\gamma} q, \\ Ls &= \frac{\gamma - 1}{\gamma} q, \\ M^2 L\boldsymbol{u} + \nabla p &= \hat{\alpha} q, \\ Lq + (\hat{\omega}, \nabla) q &= 0, \\ \operatorname{div} \boldsymbol{E} &= 4\pi q, \\ \operatorname{rot} \boldsymbol{E} &= 0. \end{aligned} \tag{3.1}$$

Here small perturbations of the sought variables p, s, u_k, E_k $(k = 1, 2, 3)$, q are denoted by the same symbols. Besides, the gas is supposed to be polytropic (see Remark 2.1);

$$L = \tau + (\hat{\boldsymbol{w}}, \nabla), \quad \tau = \frac{\partial}{\partial t}, \quad \nabla = (\xi_1, \xi_2, \xi_3)^*,$$

$$\xi_k = \frac{\partial}{\partial x_k}\ (k = 1, 2, 3), \quad \hat{\boldsymbol{w}} = (\hat{w}_1, \hat{w}_2, \hat{w}_3)^* = \frac{\hat{\boldsymbol{u}}}{\hat{u}_1} = \left(1, \frac{\hat{u}_2}{\hat{u}_1}, \frac{\hat{u}_3}{\hat{u}_1}\right)^*,$$

$$\hat{u}_1 \neq 0, \quad M^2 = \frac{\hat{u}_1^2}{\hat{c}^2}, \quad \hat{c}^2 = \gamma \hat{p} \widehat{V}, \quad \widehat{V} = \frac{1}{\hat{\rho}},$$

$$\hat{\alpha} = \frac{\hat{\omega}}{\gamma |\hat{\omega}|^2}, \quad \hat{\omega} = (\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3)^* = \frac{b}{\hat{u}_1} \widehat{\boldsymbol{E}}.$$

System (3.1) is written in the dimensionless form, where the coordinates $x_k (k = 1, 2, 3)$, time t, the components of the vectors $\boldsymbol{U}$, $\boldsymbol{E}$, the charge q are related to the following characteristic parameters:

$$\hat{l},\ \frac{\hat{l}}{\hat{u}_1},\ \gamma\hat{p},\ \gamma c_V,\ \hat{u}_1,\ \frac{\hat{p}b}{\hat{u}_1|\hat{\omega}|^2},\ \frac{\hat{p}b}{\hat{l}\hat{u}_1|\hat{\omega}|^2};$$

$\hat{l}$ is the characteristic length, $|\widehat{\boldsymbol{E}}| \neq 0$.

We note that without two last equations system (3.1) can be written in the matrix form

$$\hat{A}^{(0)}\boldsymbol{V}_t + \sum_{k=1}^{3} \hat{A}^{(k)}\boldsymbol{V}_{x_k} + \hat{A}^{(4)}\boldsymbol{V} = 0, \tag{3.2}$$

where

$$\hat{A}^{(0)} = \operatorname{diag}(1, 1, M^2, M^2, M^2, 1), \quad \boldsymbol{V} = \begin{pmatrix} \boldsymbol{U} \\ q \end{pmatrix},$$

$$\hat{A}^{(1)} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & M^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & M^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & M^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\hat{\omega}_1 \end{pmatrix},$$

$$\hat{A}^{(2)} = \begin{pmatrix} \hat{w}_2 & 0 & 0 & 1 & 0 & 0 \\ 0 & \hat{w}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & M^2\hat{w}_2 & 0 & 0 & 0 \\ 1 & 0 & 0 & M^2\hat{w}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & M^2\hat{w}_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{w}_2+\hat{\omega}_2 \end{pmatrix},$$

$$\hat{A}^{(3)} = \begin{pmatrix} \hat{w}_3 & 0 & 0 & 0 & 1 & 0 \\ 0 & \hat{w}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & M^2\hat{w}_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & M^2\hat{w}_3 & 0 & 0 \\ 1 & 0 & 0 & 0 & M^2\hat{w}_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{w}_3+\hat{\omega}_3 \end{pmatrix},$$

$$\hat{A}^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{\gamma-1}{\gamma} \\ 0 & 0 & 0 & 0 & 0 & -\frac{\gamma-1}{\gamma} \\ 0 & 0 & 0 & 0 & 0 & \hat{\alpha}_1 \\ 0 & 0 & 0 & 0 & 0 & \hat{\alpha}_2 \\ 0 & 0 & 0 & 0 & 0 & \hat{\alpha}_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Eigen-values of the matrix $\hat{A}^{(1)}$ which will be used for formulation of mixed problems for system (3.2) are of the form:

$$\lambda_1 = 1, \quad \lambda_{2,3} = M^2, \quad \lambda_4 = 1 + \hat{\omega}_1,$$

(3.3)

$$\lambda_{5,6} = \frac{1 + M^2 \pm \sqrt{(1+M^2)^2 + 4(1-M^2)}}{2}.$$

4. Equations of strong discontinuity.

The problem on obtaining relations on surfaces of strong discontinuities [8], [9] for EHD equations was detailed in [3]. We will consider piecewise smooth solutions of system (2.1) – (2.3), (2.5) – (2.7) such that their smooth pieces are separated by a surface of strong discontinuity with the equation

$$\tilde{f}(t, \boldsymbol{x}) = f(t, \boldsymbol{x}') - x_1 = 0 \tag{4.1}$$

$$(\boldsymbol{x} = (x_1, \boldsymbol{x}'), \quad \boldsymbol{x}' = (x_2, x_3)).$$

Following [8], [9], we write down equations of electrohydrodynamical strong discontinuity

$$f_t[\rho] - [\rho u_1] + f_{x_2}[\rho u_2] + f_{x_3}[\rho u_3] = 0, \tag{4.2}$$

$$f_t[\rho u_i] - [\widetilde{\Pi}_{1i}] + f_{x_2}[\widetilde{\Pi}_{2i}] + f_{x_3}[\widetilde{\Pi}_{3i}] = 0 \quad (i = 1, 2, 3), \tag{4.3}$$

$$f_t[\rho e] - [W_1] + f_{x_2}[W_2] + f_{x_3}[W_3] = 0, \tag{4.4}$$

$$[J_N] = \frac{\partial \sigma}{\partial t}, \tag{4.5}$$

$$[E_N] = -4\pi\sigma, \tag{4.6}$$

$$[E_k] + f_{x_k}[E_1] = 0 \quad (k = 2, 3). \tag{4.7}$$

Here

$$J_N = (\boldsymbol{J}, \boldsymbol{N}), \quad E_N = (\boldsymbol{E}, \boldsymbol{N}), \quad \boldsymbol{N} = \frac{1}{|\nabla \tilde{f}|}\,(-1, f_{x_2}, f_{x_3})^*,$$

$$|\nabla \tilde{f}| = \sqrt{1 + f_{x_2}^2 + f_{x_3}^2};$$

$\boldsymbol{N}$ is the normal to surface (4.1). Besides, we used usual notation $[F] = F - F_\infty$ (F is a value of F on the right side (at $\tilde{f} \to -0$), F_∞ is a value of F on the left side (at $\tilde{f} \to +0$) of the discontinuity surface). Equations (4.2), (4.4) can also be written in the form:

$$[j] = 0, \quad [ej + pu_N] = 0,$$

where $j = \rho(u_N - D_N)$ is the mass flux through the discontinuity, $u_N = (\boldsymbol{u}, \boldsymbol{N})$, $D_N = -f_t/|\nabla \tilde{f}|$. While obtaining relations (4.4) – (4.6) we assumed the existence of the charge $\sigma = \sigma(t, \boldsymbol{x}')$ on surface (4.1) and neglected the surface electric current as it is recommended in [3], [9].

We linearize EHD equations and relations on strong discontinuity (4.2) – (4.7) with respect to the piecewise constant solution. At $x_1 < 0$:

$$\boldsymbol{U}(t\boldsymbol{x}) = \widehat{\boldsymbol{U}}_\infty = \begin{pmatrix} \hat{p}_\infty \\ \hat{s}_\infty \\ \hat{u}_{1\infty} \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{E}(t, \boldsymbol{x}) = \widehat{\boldsymbol{E}}_\infty = \begin{pmatrix} \widehat{E}_{1\infty} \\ 0 \\ 0 \end{pmatrix},$$

$$q(t, \boldsymbol{x}) = 0,$$

$$\hat{u}_{1\infty} \neq 0, \quad \hat{p}_\infty = \hat{\rho}_\infty^2 (e_0)_\rho(\hat{\rho}_\infty, \hat{s}_\infty), \quad \widehat{T}_\infty = (e_0)_s(\hat{\rho}_\infty, \hat{s}_\infty),$$

at $x_1 > 0$:

$$\boldsymbol{U}(t, \boldsymbol{x}) = \widehat{\boldsymbol{U}} = \begin{pmatrix} \hat{p} \\ \hat{s} \\ \hat{u}_1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{E}(t, \boldsymbol{x}) = \widehat{\boldsymbol{E}} = \begin{pmatrix} \hat{E}_1 \\ 0 \\ 0 \end{pmatrix},$$

$$q(t, \boldsymbol{x}) = 0,$$

$$\hat{u}_1 \neq 0,\ \hat{p} = \hat{\rho}^2 (e_0)_\rho (\hat{\rho}, \hat{s}), \quad \hat{T} = (e_0)_s (\hat{\rho}, \hat{s}),$$

at $x_1 = 0$ relations (4.2)-(4.7) are fulfilled (under condition that the front of discontinuity is immovable and is described by the equation $x_1 = 0$):

$$[\hat{j}] = 0,\ [\hat{\rho}\hat{u}_1^2 + \hat{p} - \frac{\hat{E}_1^2}{8\pi}] = 0,$$

(4.8)

$$[\hat{\rho}\hat{u}_1(\hat{e}_0 + \frac{1}{2}\hat{u}_1^2 + \hat{p}\hat{V})] = 0,\ [\hat{E}_1] = 4\pi\hat{\sigma},$$

where $\hat{e}_0 = e_0(\hat{\rho}, \hat{s})$, $\hat{j} = \hat{\rho}\hat{u}_1 (\neq 0)$; $\hat{\sigma}$ is the surface charge. After linearization we obtain the main mixed problem on stability of strong discontinuity in EHD.

Main Problem. *In the domain* $t > 0,\ \boldsymbol{x} \in R^3_+$ *we seek the solution to system* (3.1) *(at* $\hat{\boldsymbol{w}} = (1, 0, 0)^*$, $\hat{\omega} = (\hat{\omega}_1, 0, 0)^*$, $\hat{\omega}_1 = b\hat{E}_1/\hat{u}_1$, $L = \tau + \xi_1$*), and in the domain* $t > 0,\ \boldsymbol{x} \in R^3_-$ *the solution to the system*

$$L_\infty p + div\boldsymbol{u} = \frac{\gamma - 1}{\gamma} q\,,$$

$$L_\infty s = \frac{\gamma - 1}{\gamma} q,$$

(3.1′)

$$M_\infty^2 L_\infty \boldsymbol{u} + \nabla p = \hat{\alpha}_\infty q,$$

$$L_\infty q + \hat{\omega}_{1\infty}\xi_1 q = 0,$$

$$\mathrm{div}\boldsymbol{E} = 4\pi q,$$

$$\mathrm{rot}\boldsymbol{E} = 0,$$

the solutions must satisfy the following boundary conditions at $x_1 = 0$*:*

(4.9)

$$[u_1 + p - s] = (1 - k)F_t,$$

(4.10)

$$2u_1 - \frac{2}{k}u_{1\infty} + \left(\frac{1}{M^2} + 1\right)p - \frac{1}{k}\left(\frac{1}{M_\infty^2} + 1\right)p_\infty -$$

$$- \hat{a}E_1 + \hat{a}_\infty E_{1\infty} - s + \frac{1}{k}s_\infty = 0,$$

(4.11) $$u_k - \frac{1}{k}u_{k\infty} - \hat{a}E_k + \hat{a}_\infty E_{k\infty} = \hat{\mu}F_{x_k} \quad (k = 2, 3),$$

(4.12) $$\left(\frac{\gamma}{(\gamma-1)M^2} + \frac{1}{2}\right)p - \frac{1}{k}\left(\frac{\gamma}{(\gamma-1)M_\infty^2} + \frac{1}{2}\right)p_\infty +$$
$$+\left(\frac{1}{(\gamma-1)M^2} + \frac{3}{2}\right)u_1 - \frac{1}{k}\left(\frac{1}{(\gamma-1)M_\infty^2} + \frac{3}{2}\right)u_{1\infty} -$$
$$-\frac{1}{2}(s - \frac{1}{k}s_\infty) = \hat{\nu}F_t,$$

(4.13) $$(1+\hat{\omega}_1)q - \frac{\hat{d}}{k}(1+\hat{\omega}_{1_\infty})q_\infty = \Omega_t,$$

(4.14) $$E_1 - \hat{d}E_{1\infty} = 4\pi\Omega,$$

(4.15) $$E_k - \hat{d}E_{k\infty} = -\hat{\chi}F_{x_k} \quad (k = 2, 3),$$

at $t = 0$ the initial data:

$$\boldsymbol{U}|_{t=0} = \boldsymbol{U}_0(\boldsymbol{x}), \quad \boldsymbol{E}|_{t=0} = \boldsymbol{E}_0(\boldsymbol{x}),$$

(4.16) $$q|_{t=0} = q_0(\boldsymbol{x}), \quad \boldsymbol{x} \in R^3_\pm,$$
$$F|_{t=0} = F_0(\boldsymbol{x}'), \quad \Omega|_{t=0} = \Omega_0(\boldsymbol{x}'), \quad \boldsymbol{x}' \in R^2.$$

Here
$$R^3_\pm = \{\boldsymbol{x}|x^1 \gtrless 0, \boldsymbol{x}' \in R^2\}, \quad L_\infty = k\tau + \xi_1,$$
$$k = \frac{\hat{u}_1}{\hat{u}_{1\infty}}, \quad M_\infty^2 = \frac{\hat{u}_{1\infty}^2}{\hat{c}_\infty^2}, \quad \hat{c}_\infty^2 = \gamma\hat{p}_\infty\hat{V}_\infty, \quad \hat{V}_\infty = \frac{1}{\hat{\rho}_\infty},$$
$$\hat{\alpha}_\infty = \frac{\hat{\omega}_\infty}{\gamma\hat{\omega}_{1\infty}^2}, \quad \hat{\omega}_\infty = (\hat{\omega}_{1\infty}, 0, 0)^*, \quad \hat{\omega}_{1\infty} = \frac{b_\infty\hat{E}_{1\infty}}{\hat{u}_{1\infty}};$$

$b_\infty > 0$ is the mobility constant at $x_1 < 0$ (mobilities are assumed to be different on different sides of the discontinuity). System (3.1′) is written in the dimensionless form (the coordinates x_k ($k = 1, 2, 3$), time t, components of the vectors $\boldsymbol{U}$, $\boldsymbol{E}$, the charge q are related to the characteristic values:

$$\hat{l}, \quad \frac{\hat{l}}{\hat{u}_1}, \quad \gamma\hat{p}_\infty, \quad \gamma c_V, \quad \hat{u}_{1\infty}, \quad \frac{b_\infty\hat{p}_\infty}{\hat{u}_{1\infty}\hat{\omega}_{1\infty}^2}, \quad \frac{b_\infty\hat{p}_\infty}{\hat{l}\hat{u}_{1\infty}\hat{\omega}_{1\infty}^2}).$$

Boundary conditions (4.9) – (4.15) are derived after linearization of relations (4.2) – (4.7) and written in the dimensionless form, and

$$\hat{a} = \frac{1}{4\pi\gamma M^2\hat{\omega}_1}, \quad \hat{a}_\infty = \frac{1}{4\pi\gamma k M_\infty^2 \hat{\omega}_{1\infty}}, \quad \hat{\mu} = \frac{1-\bar{p}}{\gamma M^2} + \hat{e},$$

$$\bar{p} = \frac{\hat{p}_\infty}{\hat{p}}, \quad \hat{e} = \frac{[\hat{E}_1^2]}{8\pi\hat{\rho}\hat{u}_1^2}, \quad \hat{\nu} = \frac{1-\bar{p}}{\gamma(\gamma-1)M^2} + \frac{1}{2}(1-\frac{1}{k}),$$

$$\hat{d} = \bar{p}k\frac{b_\infty\hat{\omega}_1^2}{\hat{b}\omega_{1\infty}^2}, \quad \hat{\chi} = \frac{\hat{u}_1\hat{\omega}_1^2}{\hat{p}b}[\hat{E}_1], \quad F = \frac{\delta f(t,\boldsymbol{x}')}{\hat{l}}, \quad \Omega = \frac{\hat{u}_1\hat{\omega}_1^2}{\hat{p}b}\delta\sigma(t,\boldsymbol{x}');$$

$\delta f(t, \boldsymbol{x}')$ is a small perturbation of the discontinuity front, $\delta\sigma(t, \boldsymbol{x}')$ is a small perturbation of the surface charge. We also note that while solving mixed problem (3.1), (3.1′), (4.9) – (4.16) the function $F = F(t, \boldsymbol{x}')$, $\Omega = \Omega(t, \boldsymbol{x}')$ becomes known too. Two relations from conditions (4.9) – (4.16) are considered as equations to determine the functions F, Ω.

We note that system (3.1′) without two last equations can be written in the matrix form

$$\hat{A}_\infty^{(0)}\boldsymbol{V}_t + \sum_{k=1}^{3}\hat{A}_\infty^{(k)}\boldsymbol{V}_{x_k} + \boldsymbol{A}_\infty^{(4)}\boldsymbol{V} = 0, \tag{3.2′}$$

where the matrices $\hat{A}_\infty^{(\alpha)}$ ($\alpha = \overline{0,4}$) can be easily determined, the eigen-values of the matrix $\hat{A}_\infty^{(1)}$ are given below

$$\begin{aligned} &\lambda_1 = 1, \quad \lambda_{2,3} = M_\infty^2, \quad \lambda_4 = 1 + \hat{\omega}_{1\infty}, \\ &\lambda_{5,6} = \frac{1 + M_\infty^2 \pm \sqrt{(1+M_\infty^2)^2 + 4(1-M_\infty^2)}}{2}. \end{aligned} \tag{3.3′}$$

Our aim of the further investigation is the well-posedness of main mixed problem. There exist some variants of this problem in dependence on the specifics of discontinuity (4.8).

Remark 4.1. By virtue of (2.8) the last two relations in systems (3.1), (3.1′) are reduced to the single Poisson equation for a small perturbation of the potential φ:

$$\Delta\varphi = -4\pi q, \quad \boldsymbol{x} \in R_\pm^3, \quad t > 0. \tag{4.17}$$

Here φ is a dimensionless value related to $b\hat{p}\hat{l}/(\hat{u}_1\hat{\omega}_1^2)$ at $x_1 > 0$ and $b_\infty \hat{p}_\infty/(\hat{l}\hat{u}_{1\infty}\hat{\omega}_{1\infty}^2)$ at $x_1 < 0$. Boundary conditions (4.14), (4.15) are transformed into

$$\frac{\partial \varphi}{\partial x_1} - \hat{d}\frac{\partial \varphi_\infty}{\partial x_1} = -4\pi\Omega, \tag{4.18}$$

$$\varphi - \hat{d}\varphi_\infty = \hat{\chi}F. \tag{4.19}$$

5. Investigation of conditions (4.8) on stationary discontinuity.

Let the stationary discontinuity which meets conditions (4.8) be a shock wave, i.e., $\hat{u}_1, \hat{u}_{1\infty} \neq 0$, $[\hat{\rho}] \neq 0$ (detailed classification of strong discontinuities is given in [3]). The second and third relations from (4.8) will be written in the form

$$\begin{gathered} \bar{p} = -\gamma M^2 \bar{v} + \tilde{\Delta}, \\ 1 - \bar{p}\bar{v} + \frac{\gamma - 1}{2} M^2 (1 - \bar{v}^2) = 0, \end{gathered} \tag{5.1}$$

where

$$\bar{v} = \frac{\hat{V}_\infty}{\hat{V}} = \frac{1}{k}, \tilde{\Delta} = 1 + \gamma M^2 (1 - \hat{e}),$$

$$\hat{e} = \frac{[\hat{E}_1^2]}{8\pi\hat{\rho}\hat{u}_1^2} = \frac{\hat{\sigma}(\hat{E}_1 + \hat{E}_{1\infty})}{2\hat{\rho}\hat{u}_1^2} = \frac{\hat{\sigma}(\hat{E}_1 - 2\pi\hat{\sigma})}{\hat{\rho}\hat{u}_1^2}.$$

A quadratic equation on $\bar{v}$

$$\frac{\gamma + 1}{2} M^2 \bar{v}^2 - \tilde{\Delta}\bar{v} + 1 + \frac{\gamma - 1}{2} M^2 = 0 \tag{5.2}$$

follows from (5.1). Further we will suppose that $\hat{e}$ is a small parameter ($|\hat{e}| \ll 1$). Besides, the following conditions are fulfilled

$$\hat{s} > \hat{s}_\infty, \ \hat{p} > \hat{p}_\infty > 0, \ \hat{\rho} > \hat{\rho}_\infty > 0, \ \hat{u}_{1\infty} > \hat{u}_1 > 0. \tag{5.3}$$

At $\hat{e} = 0$ they are the condition of evolutionarity [6] of shock waves in usual gas dynamics [8], [9] (the evolutionarity of EHD shock waves in the general case, i.e., at nonzero $\hat{e}$, is considered in [3]). Inequalities (5.3), with account of (2.12) and first relation from (4.8), can be rewritten in the form:

$$\bar{p}\bar{v}^{\gamma} < 1, \quad 0 < \bar{p} < 1, \quad \bar{v} > 1. \tag{5.4}$$

Then from two roots of equation (5.2) we choose the root

$$\bar{v} = \frac{\gamma}{\gamma+1}\left\{\Delta + \sqrt{\Delta^2 - 2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}\right\} \tag{5.5}$$

$$\left(\Delta = l + 1 - \hat{e},\ l = \frac{1}{\gamma M^2}\right),$$

which at $\hat{e} = 0$ is less than unit (another root equals unit at $\hat{e} = 0$). Then from condition $\bar{v} > 0$ we derive $\Delta > 0$, i.e., $\hat{e} < 1 + l$. On the other hand, the natural requirement

$$\Delta > \sqrt{2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}$$

leads us to the inequality

$$\hat{e} < 1 + l - \sqrt{2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}, \tag{5.6}$$

which the parameter $\hat{e}$ must satisfy. From (5.1), (5.5) it follows that

$$\bar{p} = \frac{\gamma M^2}{\gamma+1}\left\{\Delta - \gamma\sqrt{\Delta^2 - 2\frac{\gamma+1}{\gamma}\left(l + \frac{\gamma-1}{2\gamma}\right)}\right\}, \tag{5.7}$$

and $\bar{p} > 0$ if the parameter $\hat{e}$ satisfies the inequality:

$$\hat{e} > 1 + l - \sqrt{2\frac{\gamma}{\gamma-1}\left(l + \frac{\gamma-1}{2\gamma}\right)}. \tag{5.8}$$

We note that at small $\hat{e}$ inequality (5.6) is fulfilled, and inequality (5.8) is equivalent to condition

$$M^2 > \frac{\gamma-1}{2\gamma}. \tag{5.8'}$$

It is easy to show that fulfilment of equalities (5.6), (5.8), in view of (5.5) and (5.7), implies fulfilment of the second and third conditions from (5.4) at $M < 1$. It is apparent that the first inequality from (5.4) holds true also because at $\hat{e} = 0$ it is justified by mutual location of the Hugoniot and Poisson adiabats [9]. We also note that (5.5), (5.7) imply $M_\infty^2 = M^2\bar{v}/\bar{p} > 1$.

6. Investigation of well-posedness of main problem.

Let conditions

$$1+\hat{\omega}_{1\infty}>0, \quad 1+\hat{\omega}_1>0, \quad M<1 \tag{6.1}$$

hold. Then $M_\infty > 1$ (see the fifth section) and all eigen-values (3.3′) of system (4.2′) are positive, i.e., this system does not require boundary conditions at $x_1 = 0$ [1]. In this paper we restrict ourself to a partial case $q(x) \equiv 0$ at $x_1 < 0$. Then

$$q(t,x)\equiv 0 \quad \text{at } x_1<0 \tag{6.2}$$

and, without loss of generality, we also suppose that

$$\boldsymbol{U}(t,x)\equiv 0 \quad \text{at } x_1<0. \tag{6.2'}$$

By virtue of (3.3), (6.1) system (3.2) requires five boundary conditions at $x_1 = 0$. From physical reasons we require $\Omega \equiv 0$ (otherwise the main problem is underdetermined with respect to the number of boundary conditions).

As a result, in account of (4.18), (4.19) (see remark 4.1), (6.2), (6.2′), boundary conditions for the main problem is formulated as follows:

$$\begin{aligned} &u_1+dp+d_0E_{1\infty}=0,\\ &u_k=\hat{\lambda}F_{x_k}+\hat{d}_0E_{k\infty} \quad (k=2,3),\\ &F_t=\mu p-\mu_0E_{1\infty},\\ &s=\nu p+\nu_1E_{1\infty},\\ &q=0; \end{aligned} \tag{6.3}$$

$$\begin{aligned} &\frac{\partial\varphi}{\partial x_1}-\hat{d}\frac{\partial\varphi_\infty}{\partial x_1}=0,\\ &\varphi-\hat{d}\varphi_\infty=\hat{\chi}F, \end{aligned} \tag{6.4}$$

where

$$d=\frac{(\gamma+1)(1-k)+2\hat{\nu}(\gamma-1)}{(2+M^2(\gamma-1))(1-k)+2\hat{\nu}(\gamma-1)M^2},$$

$$d_0=\frac{M^2(\gamma-1)(1-k-2\hat{\nu})\hat{d}_0}{(2+(\gamma-1)M^2)(1-k)+2\hat{\nu}(\gamma-1)M^2},$$

$$\hat{d}_0=\hat{a}\hat{d}-\hat{a}_\infty=\frac{\hat{p}_\infty}{\hat{\rho}\hat{u}_1^2\hat{\omega}_{1\infty}\hat{E}_{1\infty}}\hat{\sigma}, \quad \hat{\lambda}=\hat{\mu}-\hat{a}\hat{\chi},$$

$$\mu = \frac{1-\nu-d}{1-k}, \quad \nu = 1 + \frac{1}{M^2} - 2d, \quad \mu_0 = \frac{-2d_0}{1-k}, \quad \nu_1 = -3d_0,$$

and, by virtue of (5.5), (5.7) and smallness of the parameter $\hat{e}$, the following presentations take place:

$$d = d^{(0)} + O(\hat{e}), \quad \hat{d}_0 = O(\hat{e}), \quad d_0 = O(\hat{e}),$$

$$\hat{\lambda} = \hat{\lambda}^{(0)} + O(\hat{e}), \quad \mu = \mu^{(0)} + O(\hat{e}), \quad \mu_0 = O(\hat{e}),$$

$$\nu = \nu^{(0)} + O(\hat{e}), \quad \nu_1 = O(\hat{e}), \quad \hat{\chi} = O(\hat{e})$$

$$d^{(0)} = \frac{3-\gamma+(3\gamma-1)M^2}{2M^2(2+(\gamma-1)M^2)}, \quad \hat{\lambda}^{(0)} = \frac{2(1-M^2)}{(\gamma+1)M^2},$$

$$\mu^{(0)} = -\frac{\gamma+1}{4M^2}, \quad \nu^{(0)} = \frac{(\gamma-1)(1-M^2)^2}{M^2(2+(\gamma-1)M^2)}.$$

Remark 6.1. It is meant that smallness of the parameter $\hat{e}$ ($|\hat{e}| \ll 1$) is the consequence of smallness of the jump of the normal component of electric filed strength, i.e.,

$$\left| \frac{\hat{\sigma}}{\hat{u}_1\sqrt{\frac{\rho}{2\pi}}} \right| = \left| \frac{\hat{E}_1}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} - \frac{\hat{E}_{1\infty}}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} \right| \ll \frac{1}{\left| \frac{\hat{E}_1}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} + \frac{\hat{E}_{1\infty}}{\hat{u}_1\sqrt{8\pi\hat{\rho}}} \right|}$$

and besides $\operatorname{sgn}\hat{E}_1 = \operatorname{sgn}\hat{E}_{1\infty}$, the values $\dfrac{\hat{E}_1}{\hat{u}_1\sqrt{8\pi\hat{\rho}}}, \dfrac{\hat{E}_{1\infty}}{\hat{u}_1\sqrt{8\pi\hat{\rho}}}$ are not small. It implies that $\hat{d}_0 = O(\hat{e})$ and so on.

We note that if the functions $E_{k\infty}$ ($k = \overline{1,3}$) in boundary conditions (6.3) are considered to be known, then hyperbolic mixed problem (3.2), (6.3) with corresponding initial data from (4.16) is well-posed with respect to the number of boundary conditions (remind that one of boundary conditions is used for determination of the function F).

To determine the potential φ we obtain the problem of diffraction [4]: at $x_1 < 0$, $\boldsymbol{x}' \in R^2$, $t > 0$ the solution to the Laplace equation

$$\Delta\varphi = 0 \tag{6.5}$$

is sought, at $x_1 > 0$, $\boldsymbol{x}' \in R^2$, $t > 0$ the solution to the Poisson equation

$$\Delta\varphi = -4\pi q \tag{6.6}$$

is sought, at $x_1 = 0$, $\boldsymbol{x}' \in R^2$, $t > 0$ the solutions to equations (6.5), (6.6) satisfy boundary conditions (6.4).

We describe the procedure of derivation of the apriori estimation without loss of smoothness for the solution to problem (3.2), (6.3) – (6.6). For this purpose we construct an extended system [1]. The procedure of constructing will be divided into two steps.

The first step is construction of the following symmetric t-hyperbolic (by Friedrichs) system:

$$\hat{A}_p^{(0)}(\boldsymbol{V}_p)_t + \sum_{k=1}^{3} \hat{A}_p^{(k)}(\boldsymbol{V}_p)_{x_k} + \hat{a}_p^{(4)}\boldsymbol{V}_p = 0. \tag{6.7}$$

Here

$$\boldsymbol{V}_p = (\boldsymbol{V}^*, \tau\boldsymbol{V}^*, \xi_1\boldsymbol{V}^*, \xi_2\boldsymbol{V}^*, \xi_3\boldsymbol{V}^*, \tau^2\boldsymbol{V}^*, \tau\xi_1\boldsymbol{V}^*, \tau\xi_2\boldsymbol{V}^*,$$
$$\tau\xi_3\boldsymbol{V}^*, \xi_1^2\boldsymbol{V}^*, \xi_1\xi_2\boldsymbol{V}^*, \xi_1\xi_3\boldsymbol{V}^*, \xi_2^2\boldsymbol{V}^*, \xi_2\xi_3\boldsymbol{V}^*, \xi_3^2\boldsymbol{V}^*)^*;$$

$\hat{A}_p^{(\alpha)} = \mathrm{diag}(I_5 \times \hat{A}^{(\alpha)}, \varepsilon(I_{10} \times \hat{A}^{(\alpha)}))$ $(\alpha = \overline{0,4})$ are block diagonal matrices, $I_5 \times \hat{A}^{(\alpha)}$ is the Kronecker product of the matrices I_5 and $\hat{A}^{(\alpha)}$, I_5 is the unit matrix of order 5 and so on, ε is a positive constant.

After integration the differential presentation of the energy integral for symmetric system (6.7) over the domain R^3_+, we come to

$$\frac{d}{dt}I_0(t) - \iint_{R^2} (\hat{A}_p^{(1)}\boldsymbol{V}_p, \boldsymbol{V}_p)\Big|_{x_1=0} d\boldsymbol{x}' + \iiint_{R^3_+} ((\hat{A}_p^{(4)} + \hat{A}_p^{(4)*})\boldsymbol{V}_p, \boldsymbol{V}_p)d\boldsymbol{x} = 0, \tag{6.8}$$

where

$$I_0(t) = \iiint_{R^3_+} (\hat{A}_p^{(0)}\boldsymbol{V}_p, \boldsymbol{V}_p)d\boldsymbol{x},$$

$$(\hat{A}_p^{(0)}\boldsymbol{V}_p, \boldsymbol{V}_p) = (\hat{A}^{(0)}\boldsymbol{V}, \boldsymbol{V}) + (\hat{A}^{(0)}\boldsymbol{V}_t, \boldsymbol{V}_t) + \sum_{i=1}^{3}(\hat{A}^{(0)}\boldsymbol{V}_{x_i}, \boldsymbol{V}_{x_i}) +$$
$$+\varepsilon\{(\hat{A}^{(0)}\boldsymbol{V}_{tt}, \boldsymbol{V}_{tt}) + (\hat{A}^{(0)}\boldsymbol{V}_{tx_1}, \boldsymbol{V}_{tx_1}) + \cdots + (\hat{A}^{(0)}\boldsymbol{V}_{x_3x_3}, \boldsymbol{V}_{x_3x_3})\}.$$

While deducing (6.8) we suppose that $(\boldsymbol{V}_p, \boldsymbol{V}_p)^{1/2} = |\boldsymbol{V}_p| \to 0$ at $x_1 \to \infty$ or $|x_{2,3}| \to \infty$.

Using boundary conditions (6.3) and system (3.1) for estimation of the second and third terms in equality (6.8) we obtain the inequality:

$$\frac{d}{dt}I_0(t) - N_1 \iint_{R^2} \{p^2 + u_2^2 + u_3^2 + p_t^2 + p_{x_1}^2 + p_{x_2}^2 + p_{x_3}^2 + \tag{6.9}$$

$$+ \varepsilon(P + R)\}\Big|_{x_1=0} d\boldsymbol{x}' - N_{\hat{e}} \iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}' \leq N_2 I_0(t),$$

where $N_1, N_2 > 0$, $N_{\hat{e}} = O(\hat{e})$ are constants,

$$P = P_{tt}^2 + p_{tx_1}^2 + p_{tx_2}^2 + p_{tx_3}^2 + \sum_{i=1}^{3}\sum_{j=i}^{3} p_{x_i x_j}^2,$$

$$R = \sum_{i=2}^{3}\sum_{j=2}^{3}\sum_{k=j}^{3}(u_i)_{x_j x_k}^2,$$

$$\mathcal{E}\Big|_{x_1=0} = \Big|\boldsymbol{E}_\infty\Big|^2 + \Big|\frac{\partial \boldsymbol{E}_\infty}{\partial t}\Big|^2 + \Big|\frac{\partial \boldsymbol{E}_\infty}{\partial x_2}\Big|^2 + \Big|\frac{\partial \boldsymbol{E}_\infty}{\partial x_3}\Big|^2 + \varepsilon\left\{\Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial t^2}\Big|^2 + \right.$$

$$\left. + \Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial t \partial x_2}\Big|^2 + \Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial t \partial x_3}\Big|^2 + \Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial x_2^2}\Big|^2 + \Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial x_2 \partial x_3}\Big|^2 + \Big|\frac{\partial^2 \boldsymbol{E}_\infty}{\partial x_3^2}\Big|^2 \right\}.$$

To estimate the integral $\iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}'$ through the integral

$$\iint_{R^2} \left\{p^2 + \cdots + \varepsilon(P + R)\right\}\Big|_{x_1=0} d\boldsymbol{x}'$$

we apply the Fourier transform to problem (6.4) – (6.6). As a result we come to a mixed problem for ordinary differential equations:

$$\frac{d^2}{dx_1^2}\hat{\varphi} - \omega^2\hat{\varphi} = 0, \quad x_1 < 0, \tag{6.10}$$

$$\frac{d^2}{dx_1^2}\hat{\varphi} - \omega^2\hat{\varphi} = -4\pi\hat{q}, \quad x_1 > 0, \tag{6.11}$$

(6.12)
$$\left(\frac{d\hat{\varphi}}{dx_1} - \hat{d}\frac{d\hat{\varphi}_\infty}{dx_1}\right)\Big|_{x_1=0} = 0,$$
$$(\hat{\varphi} - \hat{d}\hat{\varphi}_\infty)\Big|_{x_1=0} = \hat{\chi}\hat{F}.$$

Here
$$\hat{\varphi} = \hat{\varphi}(t, x_1, \xi') = \iint_{R^2} e^{-2\pi i(\xi', \boldsymbol{x}')}\varphi(t, \boldsymbol{x})d\boldsymbol{x}',$$
$$\hat{q} = \hat{q}(t, x_1, \xi') = \iint_{R^2} e^{-2\pi i(\xi', \boldsymbol{x}')}q(t, \boldsymbol{x})d\boldsymbol{x}',$$
$$\hat{F} = \hat{F}(t, \xi') = \iint_{R^2} e^{-2\pi i(\xi', \boldsymbol{x}')}F(t, \boldsymbol{x}')d\boldsymbol{x}',$$
are the Fourier transform on functions $\varphi(t, \boldsymbol{x})$, $q(t, \boldsymbol{x})$, $F(t, \boldsymbol{x}')$,
$$\xi' = (\tilde{\xi}_2, \tilde{\xi}_3), \quad \omega^2 = 4\pi^2|\xi'|^2 = 4\pi^2(\tilde{\xi}_2^2 + \tilde{\xi}_3^2) < \infty.$$

From (6.10) it follows that

$$\hat{\varphi} = c_{1\infty}e^{\omega x_1}, \quad \hat{\varphi}' = c_{1\infty}\omega e^{\omega x_1}, \quad x_1 < 0 \tag{6.13}$$

(primes stand for differentiation with respect to x_1), $c_{1\infty}$ is a constant, $\omega = 2\pi|\xi'|$. If the function $\hat{\varphi}$ at $x_1 > 0$ satisfies equation (6.11), the vector $\boldsymbol{y} = (y_1, y_2)^*$, where $y_1 = \hat{\varphi}$, $y_2 = y_1' - \omega y_1$, satisfies the system

$$\boldsymbol{y}' = \begin{pmatrix} \omega & 1 \\ 0 & -\omega \end{pmatrix} \boldsymbol{y} + \boldsymbol{f}, \tag{6.14}$$

where
$$\boldsymbol{f} = \begin{pmatrix} 0 \\ -4\pi\hat{q} \end{pmatrix}.$$

As
$$\begin{pmatrix} \omega & 1 \\ 0 & -\omega \end{pmatrix} = \begin{pmatrix} 1 & -\tilde{m} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix}\begin{pmatrix} 1 & \tilde{m} \\ 0 & 1 \end{pmatrix}, \ \tilde{m} = \frac{1}{2\omega},$$
then

$$\boldsymbol{y} = \begin{pmatrix} -e^{-\omega x_1}\tilde{m} \\ e^{-\omega x_1} \end{pmatrix} c_2 + \tilde{\boldsymbol{y}}, \tag{6.15}$$

where
$$\tilde{\boldsymbol{y}} = (\tilde{y}_1, \tilde{y}_2)^* = \int_0^\infty G(x_1 - \tau)\boldsymbol{f}(t, \tau, \xi')d\tau,$$

c_2 is a constant,

$$G(x_1) = \begin{cases} -\begin{pmatrix} e^{\omega x_1} & \tilde{m}e^{\omega x_1} \\ 0 & 0 \end{pmatrix}, & x_1 < 0, \\ \begin{pmatrix} 0 & -\tilde{m}e^{-\omega x_1} \\ 0 & e^{-\omega x_1} \end{pmatrix}, & x_1 > 0 \end{cases}$$

is the Green matrix [1]. From (6.15) we derive

$$\begin{aligned} &\hat{\varphi} = -\tilde{m}e^{-\omega x_1}c_2 + \tilde{y}_1, \\ &\hat{\varphi}' = \frac{1}{2}c_2 e^{-\omega x_1} + \omega\tilde{y}_1 + \tilde{y}_2, \quad x_1 > 0. \end{aligned} \tag{6.16}$$

With account of (6.13), (6.16), the constants $c_{1\infty}$, c_2 are found from boundary conditions (6.12):

$$c_{1\infty} = \frac{2\pi}{\hat{d}\omega}\int_0^\infty e^{-\omega\tau}\hat{q}(t,\tau,\xi')d\tau - \frac{\hat{\chi}}{2\hat{d}}\hat{F}, c_2 = -\hat{\chi}\omega\hat{F}. \tag{6.17}$$

Then from (6.13), (6.17) it follows that

$$\hat{\varphi}'_\infty(t,\xi') = \frac{2\pi}{\hat{d}}\int_0^\infty e^{-\omega\tau}\hat{q}(t,\tau,\xi')d\tau - \frac{\omega\hat{\chi}}{2\hat{d}}\hat{F}. \tag{6.18}$$

In view of (6.18) and the Parseval equality, we estimate the integral of the first term from $\mathcal{E}\Big|_{x_1=0}$:

$$\begin{aligned} \iint_{R^2} E^2_{1\infty}d\boldsymbol{x}' = \iint_{R^2} |\hat{\varphi}'_\infty|^2 d\xi' \le \frac{\hat{\chi}^2}{2\hat{d}^2}\iint_{R^2}\omega^2|\hat{F}|^2 d\xi' + \\ + \frac{8\pi^2}{\hat{d}^2}\iint_{R^2}\Big|\int_0^\infty e^{-\omega\tau}\hat{q}d\tau\Big|^2 d\xi'. \end{aligned} \tag{6.19}$$

Using the Parseval equality, the second and third boundary conditions from (6.3), relations (6.13), we estimate the first addendum from the right part of (6.19):

$$\begin{aligned} &\iint_{R^2}\omega^2|\hat{F}|^2 d\xi' = \iint_{R^2}(F^2_{x_2} + F^2_{x_3})d\boldsymbol{x}' \le \\ &\le \frac{2}{\hat{\lambda}^2}\iint_{R^2}(u_2^2 + u_3^2)\Big|_{x_1=0}d\boldsymbol{x}' + \frac{2\hat{d}_0^2}{\hat{\lambda}^2}\iint_{R^2}(E^2_{2\infty} + E^2_{3\infty})d\boldsymbol{x}' = \\ &= \frac{2}{\hat{\lambda}^2}\iint_{R^2}(u_2^2 + u_3^2)\Big|_{x_1=0}d\boldsymbol{x}' + \frac{2\hat{d}_0^2}{\hat{\lambda}^2}\iint_{R^2}\Big|\hat{\varphi}'_\infty\Big|^2 d\xi'. \end{aligned} \tag{6.20}$$

Then from (6.19), (6.20) we have

$$\iint_{R^2} E_{1\infty}^2 dx' \leq K_1 \iint_{R^2} (u_2^2 + u_3^2)\Big|_{x_1=0} d\boldsymbol{x}' + \tag{6.21}$$
$$+ K_2 \iint_{R^2} \Big| \int e^{-\omega\tau} \hat{q} d\tau \Big|^2 d\xi',$$

where

$$K_1 = \frac{\hat{\chi}^2}{\hat{\lambda}^2 \hat{d}^2 - \hat{\chi}^2 \hat{d}_0^2}, \quad K_2 = \frac{8\pi^2 \hat{\lambda}^2}{\hat{\lambda}^2 \hat{d}^2 - \hat{\chi}^2 \hat{d}_0^2}$$

($K_1, K_2 > 0$ by smallness of $\hat{e}$).

From the last equation of system (3.2)

$$q_t + (1 + \hat{\omega}_1) q_{x_1} = 0$$

it follows that the function

$$\Phi = \Phi(t, \xi') = \int_0^\infty e^{-\omega\tau} \hat{q}(t, \tau, \xi') d\tau$$

satisfies the equation

$$\Phi_t + (1 + \hat{\omega}_1)\omega\Phi = 0 \tag{6.22}$$

($q \to 0$ under assumption that $x_1 \to \infty$). From (6.22) we obtain

$$\Phi = e^{-(1+\omega_1)\omega t} \int_0^\infty e^{-\omega\tau} \hat{q}_0(\tau, \xi') d\tau. \tag{6.23}$$

We suppose that the function $q_0(x)$ is finite with respect to x_1, with the carrier supp$q_0 = (x_0, x_1) \times R^2$ where $0 \leq x_0 < x_1 < \infty$. Then, by the Hölder inequality and (6.23), we have

$$|\Phi|^2 \leq \Big| \int_0^\infty e^{-\omega\tau} \hat{q}_0 d\tau \Big|^2 \leq \left(\int_0^\infty |\hat{q}_0| d\tau \right)^2 = \tag{6.24}$$
$$= \left(\int_{x_0}^{x_1} |\hat{q}_0| d\tau \right)^2 \leq C_0 \int_0^\infty |\hat{q}_0|^2 d\tau.$$

where $C_0 = x_1 - x_0$. Thus, from (6.21), (6.24), we finally derive the estimation:

$$\iint_{R^2} E_{1\infty}^2 d\boldsymbol{x}' \leq K_1 \iint_{R^2} (u_2^2 + u_3^2)\Big|_{x_1=0} dx' + C_0 K_2 \iiint_{R^3} q_0^2(\boldsymbol{x}) d\boldsymbol{x} \leq$$

$$\leq K_1 \iint_{R^2} (u_2^2 + u_3^2)\Big|_{x_1=0} d\boldsymbol{x}' + K_3 I_0(t),$$

where $K_3 > 0$ is a constant. By analogy with account of boundary conditions (6.3) and system (3.1) at $x_1 = 0$, one can estimate integrals of other terms from $\mathcal{E}\Big|_{x_1=0}$. Finally

$$\iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}' \leq N_3 \iint_{R^2} \left\{p^2 + \cdots + \varepsilon(P + R)\right\}\Big|_{x_1=0} dx' + N_4 I_0(t)$$

($N_3, N_4 > 0$ are constants). Using properties of the trace of a function from W_2^1 on the plane $x_1 = 0$ [7], from the last inequality we conclude that

$$\iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}' \leq \varepsilon N_3 \iint_{R^2} (P + R)\Big|_{x_1=0} d\boldsymbol{x}' + \tilde{N}_4 I_0(t) \tag{6.25}$$

($\tilde{N}_4 > 0$ is constant).

From boundary conditions (6.3) and system (3.1) at $x_1 = 0$ we deduce

$$(\xi_2^2 + \xi_3^2) u_k = (\beta_1 \tau + \beta_2 \xi_1) \xi_k p - d_0 \tau \xi_k E_{1\infty} \quad (k = 2, 3), \quad x_1 = 0,$$

where

$$\beta_1 = -1 - d, \quad \beta_2 = \beta^2 / M^2, \quad \beta = \sqrt{1 - M^2} \quad (M < 1).$$

Then, using the known inequality from [5]

$$\iint_{R^2} R\Big|_{x_1=0} d\boldsymbol{x}' \leq \text{const} \iint_{R^2} \sum_{k=2}^{3} (\xi_2^2 u_k + \xi_3^2 u_k)^2 \Big|_{x_1=0} d\boldsymbol{x}' \leq$$

$$\leq C_1 \iint_{R^2} \sum_{k=2}^{3} ((\beta_1 \tau + \beta_2 \xi_1) \xi_k p)\Big|_{x_1=0} d\boldsymbol{x}' + C_2 \iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}'$$

($C_{1,2} > 0$ are constants) and properties of the trace of a function from $W_2^1(R_+^3)$ on the plane $x_1 = 0$, in a view of (6.25), we reduce inequality (6.9) to the form

$$\frac{d}{dt} I_0(t) - \varepsilon \tilde{N}_1 \iint_{R^2} P\Big|_{x_1=0} d\boldsymbol{x}' \leq \tilde{N}_2 I_0(t), \tag{6.26}$$

where $\tilde{N}_1, \tilde{N}_2 > 0$ are constant. We note that while deducing inequality (6.26) we assume $\varepsilon < \frac{1}{N_3(1+C_2)}$. Therewith inequality (6.25) can be rewritten as follows:

$$(6.25') \qquad \iint_{R^2} \mathcal{E}\Big|_{x_1=0} d\boldsymbol{x}' \leq \varepsilon \tilde{N}_3 \iint_{R^2} P\Big|_{x_1=0} d\boldsymbol{x}' + \tilde{N}_4 I_0(t),$$

where

$$\tilde{N}_3 = \frac{N_3 C_1}{(1 - \varepsilon N_3(1 + C_2))} > 0, \quad \hat{N}_4 = \frac{\tilde{N}_4}{(1 - \varepsilon N_3(1 + C_2))} > 0.$$

We now come to the second, more complicated, step in construction of the extended system. First we note that the function p satisfies the wave equation

$$M^2 L^2 p - \Delta p = \mathcal{F}_1$$

$$(\mathcal{F}_1 = \frac{M^2(\gamma - 1)}{\gamma} Lq - \frac{1}{\gamma \hat{\omega}_1} \xi_1 q),$$

which is a consequence of system (3.1). This equation can be rewritten in the form

$$(6.27) \qquad \{(\tau')^2 - (\xi_1')^2 - \xi_2^2 - \xi_3^2\} p = \mathcal{F}_1.$$

where new differential operators are given by formulae

$$\tau = \frac{\beta}{M} \tau', \quad \xi_1 = \frac{1}{\beta} \xi_1' + \frac{M}{\beta} \tau'.$$

If the function p satisfies equation (6.27), then the vector

$$\boldsymbol{Y} = (\tau' p, \xi_1' p, \xi_2 p, \xi_3 p)^*$$

satisfies a symmetric system [1]

$$(6.28) \qquad (B\tau' + Q\xi_1' + R_2 \xi_2 + R_3 \xi_3)\boldsymbol{Y} = \mathcal{F}.$$

Here

$$B = B(m_1, l_2, l_3) = \begin{pmatrix} 1 & -m_1 & -l_2 & -l_3 \\ -m_1 & 1 & 0 & 0 \\ -l_2 & 0 & 1 & 0 \\ -l_3 & 0 & 0 & 1 \end{pmatrix},$$

$$Q = Q(m_1, l_2, l_3) = \begin{pmatrix} m_1 & -1 & 0 & 0 \\ -1 & m_1 & l_2 & l_3 \\ 0 & l_2 & -m_1 & 0 \\ 0 & l_3 & 0 & -m_1 \end{pmatrix},$$

$$R_2 = R_2(m_1, l_2, l_3) = \begin{pmatrix} l_2 & 0 & -l & 0 \\ 0 & -l_2 & m_1 & 0 \\ -1 & m_1 & l_2 & l_3 \\ 0 & 0 & l_3 & -l_2 \end{pmatrix},$$

$$R_3 = R_3(m_1, l_2, l_3) = \begin{pmatrix} l_3 & 0 & 0 & -l \\ 0 & -l_3 & 0 & m_1 \\ 0 & 0 & -l_3 & l_2 \\ -1 & m_1 & l_2 & l_3 \end{pmatrix},$$

$$\mathcal{F} = \mathcal{F}(m_1, l_2, l_3) = (\mathcal{F}_1, -m_1\mathcal{F}_1, -l_2\mathcal{F}_1, -l_3\mathcal{F}_1)^*,$$

where m_1, l_2, l_3 are constant and $B > 0$ if $m_1^2 + l_2^2 + l_3^2 < 1$.

We obtain a boundary condition at $x_1 = 0$ for system (6.28). As in gas dynamics [1], in a view of conditions (6.3), system (3.1) and equation (6.27) at $x_1 = 0$, the function p satisfies the conditions at $x_1 = 0$:

$$m(\tau')^2 p + n(\xi_1')^2 p - \frac{1}{M}\tau'\xi_1' p + \mathcal{F}_0 = 0, \tag{6.29}$$

where

$$m = \frac{M^2}{\beta^2}\mu\hat{\lambda} + d, \quad n = -\frac{M^2}{\beta^2}\mu\hat{\lambda},$$

$$\mathcal{F}_0 = -\frac{M^2}{\beta^2}\big\{d_0\tau^2 - \tau d_0\tau\xi_1 - \mu_0\hat{\lambda}(\xi_2^2 + \xi_2^2)\big\}E_{1\infty}.$$

Boundary condition (6.29) can be rewritten in the form:

$$(\tau' - a\xi_1')\hat{L}p + \mathcal{F}_0 = 0, \quad x_1 = 0,$$

where

$$\hat{L} = a_1\tau' + a_2\xi_1'.$$

Constants a, a_1, a_2 are found from the system

$$a_1 = m, \quad aa_2 = -n, \quad am - a_2 = \frac{1}{M}.$$

We solve the system and take a, for example, in the form

$$a = \frac{1/M + \sqrt{1/M^2 - 4mn}}{2m}.$$

We note that

$$\frac{1}{M^2} - 4mn = \frac{\beta^4(\gamma - 1)}{M^4(2 + (\gamma - 1)M^2)} + O(\hat{e}) > 0,$$

i.e., a is real. The vector

$$\boldsymbol{Y}_p = (\tau' \boldsymbol{Y}^*, \xi_1' \boldsymbol{Y}^*, \xi_2 \boldsymbol{Y}^*, \xi_3 \boldsymbol{Y}^*, \hat{L}\boldsymbol{Y}^*)^*$$

satisfies an extended system

$$\text{(6.30)} \qquad \left\{B_p \tau' + Q_p \xi_2', + R_{2p}\xi_2 + R_{3p}\xi_3\right\} \boldsymbol{Y}_p = \mathcal{F}_p$$

constructed from (6.28). Here B_p, Q_p, R_{2p}, R_{3p} are block diagonal matrices of order 20

$$B_p = \operatorname{diag}(\sigma_1 B_1, \sigma_2 B_2, \sigma_3 B_3, \sigma_4 B_4, \sigma_5 B_5), \quad B_i = B(m_{1i}, l_{2i}, l_{3i})$$

and so on, $\sigma_i > 0$, m_{1i}, l_{2i}, l_{3i} are constants, $m_{li}^2 + l_{2i}^2 + l_{3i}^2 < 1$. We take

$$l_{2i} = l_{3i} = 0, \quad i = \overline{1, 5}, \quad m_{11} = 0, \quad m_{12} = -\frac{1}{2},$$

$$m_{13} = m_{14} = b, \quad b = \frac{1}{2} \min\left\{\frac{ma}{n}, \frac{n}{ma}\right\},$$

$$m_{15} = \frac{2a}{1 + a^2}, \quad \sigma_1 = \frac{m}{n}\sigma_2,$$

$$\sigma_3 = \sigma_4 = \frac{mn}{1 + a^2}\sigma_5, \quad \sigma_2 = \left(\frac{n}{ma} - b\right)\sigma_4,$$

σ_5 is an arbitrary positive number. We note that

$$m = \frac{2\gamma M^2 + 1 - \gamma}{2 + (\gamma - 1)M^2} + O(\hat{e}), \quad n = \frac{1}{2M^2} + O(\hat{e}),$$

i.e. by virtue of (5.8′) $m, n > 0$. Thus, $a, b, \sigma_{1,2,3,4} > 0$, i.e., system (6.30) is symmetric t-hyperbolic (by Friedrichs). With account of choice of the constants σ_i, the estimation for the quadratic form is as follows:

$$(6.31) \qquad -(Q_p \boldsymbol{Y}_p \boldsymbol{Y}_p)\Big|_{x_1=0} \geq \Big(\sum_{i=2}^{3} \{k_{1i}(\hat{L}\xi_1 p)^2 + k_{2i}(\tau'\xi_i p)^2 + k_{3i}(\xi_1'\xi_i p)^2 + k_{4i}(\xi_2\xi_i p)^2\} + k_5(\xi_3^2 p)^2 + \left(\frac{2}{nM} - \frac{1}{2}\right)(\tau'\xi_1' p)^2 + \frac{1}{2}\sigma_2((\xi_1')^2 p)^2 - K_{\hat{e}}(\hat{L}\tau' p)^2 - K_4\mathcal{F}_0^2\Big)\Big|_{x_1=0} \geq$$

$$\geq N_3 P\Big|_{x_1=0} - \tilde{N}_{\hat{e}}\mathcal{E}\Big|_{x_1=0},$$

where k_{ij} $(i = \overline{1,4},\ j = 2,3)$, k_5, $K_{\hat{e}}$, K_4, N_3, $\tilde{N}_{\hat{e}}$ are positive constants, $\tilde{N}_{\hat{e}}$, $\tilde{K}_{\hat{e}} = O(\hat{e})$. The differential form for the integral of energy for system (6.30) is given below:

$$(6.32) \qquad (D_p\boldsymbol{Y}_p, \boldsymbol{Y}_p)_t + \beta(Q_p\boldsymbol{Y}_p, \boldsymbol{Y}_p)_{x_1} + (R_{2p}\boldsymbol{Y}_p, \boldsymbol{Y}_p)_{x_2} + (R_{3p}\boldsymbol{Y}_p, \boldsymbol{Y}_p)_{x_3} + 2(\boldsymbol{Y}_p, \mathcal{F}_p) = 0.$$

Here $D_p = \dfrac{M}{\beta}B_p - \dfrac{M^2}{\beta}Q_p > 0$. Integrating (6.32) over the domain R_+^3 under assumption that $|\boldsymbol{Y}_p| \to 0$ at $x_1 \to \infty$ or $|x_{2,3}| \to \infty$ and accounting (6.31), (6.25′), we obtain the inequality

$$(6.33) \qquad \frac{d}{dt}I_1(t) + N_4 \iint_{R^2} P\Big|_{x_1=0} d\boldsymbol{x}' \leq N_5(I_1(t) + I_0(t)),$$

where $N_5 > 0$ is a constant; $N_4 > 0$ because $N_3 > 0$ and $\tilde{N}_{\tilde{e}} = O(\hat{e})$;

$$I_1(t) = \iiint_{R^3} (D_p\boldsymbol{Y}_p, \boldsymbol{Y}_p)d\boldsymbol{x}.$$

Summing up inequalities (6.26) and (6.23) and choosing the constant ε ($\varepsilon < \min\{1/(N_3(1 + C_2)), N_4/\tilde{N}_1\}$) such that the quadratic form

$$(N_4 - \varepsilon\tilde{N}_1)P\Big|_{x_1=0}$$

is positive definite, we come to the inequality

$$\frac{d}{dt} I(t) \le N_6 I(t), \quad t > 0,$$

where $I(t) = I_0(t) + I_1(t)$, $N_6 > 0$ is a constant. The desired a priori estimation

$$I(t) \le e^{N_6 t} I(0), \quad t > 0 \tag{6.34}$$

follows from the last inequality. Then from (6.34) we derive the estimation

$$\| \boldsymbol{V}(t) \|_{W_2^2(R_+^3)} \le N_7, \quad 0 < t \le \tilde{T} < \infty, \tag{6.35}$$

where $N_7 < \infty$ is a positive constant depending on $\tilde{T}$. In its turn from (6.13), (6.16), (6.35) and the Parseval equality it follows that

$$\| \boldsymbol{Z}(t) \|_{W_2^2(R_+^3)} \le N_8, \quad 0 < t \le \tilde{T} < \infty, \tag{6.36}$$

$$\| \boldsymbol{E}(t) \|_{W_2^2(R_-^3)} \le N_9, \quad 0 < t \le \tilde{T} < \infty, \tag{6.37}$$

where $\boldsymbol{Z} = (\boldsymbol{V}^*, \boldsymbol{E}^*)^*$; $N_8, N_9 < \infty$ are positive constants depending on $\tilde{T}$.

We note that, by analogy with [1], we can obtain the estimation for the function $F(t, x')$:

$$\| F \|_{W_2^3((0,\tilde{T}) \times R^2)} \le N_{10} \tag{6.38}$$

($N_{10} < \infty$ is a positive constant depending on $\tilde{T}$).

A priori estimations (6.36) – (6.38) justify that under the assumptions on smallness of the jump of the normal component of the electric field strength on the discontinuity (see remark 6.1) and finiteness with respect to x_1 of the function $q_0(\boldsymbol{x})$ behind the discontinuity (at $x_1 > 0$), the main problem on stability of electrohydrodynamical shock waves from above is well-posed.

7. Concluding remarks.

Remark 7.1. The problem on stability of shock waves under assumptions (6.1) in the case when $q_0(x) \not\equiv 0$ generally does not differ from the considered regarding methods to obtain a priori estimations, and it will be the object of investigations of forthcoming publications.

Remark 7.2. Hold conditions

$$1 + \hat{\omega}_{1\infty} < 0, \quad 1 + \hat{\omega}_1 < 0, \quad M < 1, \tag{7.1}$$

the main problem is well-posed regarding the number of boundary conditions (under assumptions $\Omega \equiv 0$). In this case the described technique to obtain a priori estimations can also be applied.

Remark 7.3. If

$$1 + \hat{\omega}_{1\infty} > 0, \quad 1 + \hat{\omega}_1 < 0, \quad M < 1, \tag{7.2}$$

then the equations for $q(t, \boldsymbol{x})$, $\boldsymbol{x} \in R^3_\pm$ (the last two equations from (3.2) and (3.2′)) do not require boundary conditions at $x_1 = 0$ (their solutions are completely determined by the initial data $q_0(\boldsymbol{x})$, $\boldsymbol{x} \in R^3_\pm$). Then at $\Omega \not\equiv 0$ the main problem is also well-posed regarding the number of boundary conditions. From the second inequality (7.2) it follows that $\hat{E}_1 < 0$, and if $-1 < \hat{\omega}_{1\infty} < 0$ (i.e., $\hat{E}_{1\infty} < 0$), then condition of smallness of the coefficients $\hat{d}_0, d_0, \mu_0, \nu_1$ can be fulfilled (see Remark 6.1). Therewith the function $\Omega(t, x')$ is determined from boundary condition (4.13) via the initial data $q_0(\boldsymbol{x})$ at $x_1 = 0$, and further reasoning will be similar to the ones from cases (6.1) and (7.1).

Remark 7.4. Physically condition (6.1) means that at $q_0(\boldsymbol{x}) \equiv 0$, $x_1 < 0$, by the Ohm law (2.9), electric current flows from the left to the right. Hold conditions (7.2), electric current is directed towards the discontinuity and generates a surface charge on the shock wave.

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Institute of Mathematics,
Universitetskii pr 4,
630090 Novosibirsk (RUSSIA)