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CONFIGURATIONS IN TRIPLE SYSTEMS: AVOIDANCE AND DECOMPOSITIONS

ALEX ROSA

A *triple system* $TS\,(V,\lambda)$ is a pair $(V,\mathbf{B})$ where V is a v-set of elements, and $\mathbf{B}$ is a collection of 3-subsets of V called *triples* or *lines* such that every 2-subset of V is contained in exactly λ triples. The number v is the *order* of the triple system and λ its *index*. If in the above definition of the triple system, "*in exactly* λ" is replaced by "*in at most* λ", we have a partial triple system $PTS(v,\lambda)$. A configuration $\mathbf{C}$ is just (an isomorphism class of) a partial triple system $(V,\mathbf{C})$, usually with a small or fixed number of lines.

Given a configuration $\mathbf{C}$, a $TS(v,\lambda)$ [or a $PTS\,(v,\lambda)$] $(V,\mathbf{B})$ contains $\mathbf{C}$, if there exists a $PTS\,(U',\mathbf{C}')$ with $U\subseteq V$, $\mathbf{C}'\subseteq\mathbf{B}$ and $\mathbf{C}\cong\mathbf{C}'$. Otherwise, $(V,\mathbf{B})$ avoids $\mathbf{C}$. The avoidance set for $\mathbf{C}$ and λ is the set $\Omega(\mathbf{C},\lambda)=\{v: v\varepsilon B(\lambda)$ and $\exists TS\,(v,\lambda)$ which avoids $\mathbf{C}\}$ where $B(\lambda)$ is the set of *admissible orders* of v for given λ.

For instance, if $\mathbf{C}$ is the Pasch configuration, it has been conjectured that $\Omega(\mathbf{C},1)=B(1)\setminus\{7,13\}$ although this remains far from proved. The avoidance sets for all configurations of up to three lines have been determined almost completely.

Another set of questions from among the many that one can ask about configurations in triple systems concerns *decompositions*. For a configuration $\mathbf{C}$ with k lines, let $A(\mathbf{C})=\{v: k\,\text{divides}\,\lambda v\,(v-1)/6\}$.

Does there exist a $TS\,(v,\lambda)$ whose set of triples can be partitioned into copies of $\mathbf{C}$? What is the spectrum of $\mathbf{C}$, $S(\mathbf{C})=\{v:\exists TS\,(v,\lambda)$ whose set of triples can be partitioned into copies of $\mathbf{C}\}$?

Can the set of triples of every $TS\,(v, \lambda)$ with $v \varepsilon B(\lambda), v \geq v_0$, be partitioned into copies of $\mathbf{C}$?

One may even go one step further and ask for *simultaneous* decompositions of triple systems: given a set Σ of configurations, for which values of v and λ does there exist a $TS\,(v, \lambda)$ whose set of triples can be partitioned into copies of $\mathbf{C}$ for all configurations $\mathbf{C}$ in Σ?

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