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PLASTICITY AS A CONSEQUENCE OF THE INSTABILITY OF VISCOELASTIC FLOW. A THERMODYNAMIC DESCRIPTION

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The Onsager equations for the simplest viscoelastic fluid with proper material coefficients fitted to the principle of objectivity (corotational Jeffrey body) display plastic behavior, as well as, creep and solid like properties even in the first (linear in the thermodynamic sense) approximation.

1. Introduction.

A fluid with a traceless symmetric second order tensor dynamic state variable has been studied with the standard methods of non-equilibrium thermodynamics (De Groot and Mazur 1962). Volume preserving motions were regarded only.The balance equations for the linear momentum, the angular momentum, the mass and the internal energy are the well known ones

$$\operatorname{grad} p + \varrho \frac{d\mathbf{v}}{\mathrm{dt}} = \operatorname{Div} \mathrm{t} + \varrho \mathbf{f}$$

$$\mathrm{t}^{\mathrm{T}} = \mathbf{f}$$

$$\operatorname{div} \mathbf{v} = 0$$

$$\varrho \frac{du}{dt} + \operatorname{div} \mathbf{J}_{\mathrm{q}} = \mathrm{t} : \hat{\mathrm{d}}$$

with p scalar pressure, ϱ density, $\mathbf{v}$ velocity, t the deviatoric part of Cauchy's stress tensor, $\mathbf{f}$ body force, u internal energy, $\mathbf{J}_{\mathrm{q}}$ heat flow, $\hat{\mathrm{d}}$ the symmetric part of the velocity gradient (and the corotational time derivative of the deformation with respect to the present configuration.)

2. Entropy.

With reference to the Morse lemma (Morse 1925) the entropy is

$$s = s^{e} \left(u - \frac{1}{2\varrho} \xi : \xi \right)$$

where $s^{e}(u)$ is the equilibrium entropy function and ξ is a traceless symmetric second order tensor as a dynamic state variable. The reciprocal of the derivative of this function is the temperature. The general form of the entropy balance

$$\varrho \frac{ds}{dt} + \operatorname{div} \mathbf{J}_{s} = \sigma_{s}$$

with adopting the simplest approximation of the entropy flow ($\mathbf{J}_{s} = \mathbf{T}^{-1} \mathbf{J}_{\mathrm{q}}$) gives

$$T \sigma_{s} = \mathrm{t} : \hat{\mathrm{d}} - \xi : \frac{\mathrm{d}\xi}{\mathrm{dt}}$$

for the energy dissipation in an isotherm body. The tensor $\hat{\xi}$ is the corotational time derivative of the tensor ξ

$$\hat{\xi} = \frac{d\xi}{dt} + \xi \omega - \omega \xi$$

The tensor ω is the skew-symmetric part of the velocity gradient. The material time derivative $\frac{d\xi}{dt}$ is not a suitable measure for the rate of the processes as it may be the consequence of the rotation of the frame, too. The Onsager equations are

$$\mathbf{t} = L_{11}\,\hat{\mathbf{d}} - L_{12}\,\xi$$

$$\hat{\xi} = L_{21}\,\hat{\mathbf{d}} - L_{22}\,\xi$$

with the Casimir reciprocal relation

$$L_{21} = -L_{12}$$

as the dynamic state variable ξ is supposed not changing under time inversion. The conductivity coefficients are material constants in the linear theory. The model the equations constitute is the (corotational) Jeffrey body, free of the contradictions of the Maxwell body. The Maxwell body is a limit case when $L_{11} = 0$. This latter approximation is somehow like to the reversible process. Eliminating ξ and introducing new notations for the material coefficients

$$\tau_d = \frac{L_{11}}{L_{11}L_{22} + L_{12}^2}, \quad \tau_t = \frac{1}{L_{22}}, \quad 2\eta = L_{11} + \frac{L_{12}^2}{L_{22}}$$

we obtain

$$\mathbf{t} + \tau_t\,\hat{\mathbf{t}} = 2\eta(\hat{\mathbf{d}} + \tau_d\,\hat{\hat{\mathbf{d}}})$$

with $\eta > 0$ and $\tau_t > \tau_d > 0$.

3. Simple shear flow.

One can check out the body is a simple one so the simple shear flow exhibits the viscometric functions. Let the field of the velocity be

$$v_1 = \chi x_2, \qquad v_2 = v_3 = 0$$

in a Cartesian frame. The equations give

$$[\mathbf{t}] = \begin{pmatrix} \sigma & \tau & 0 \\ \tau & -\sigma & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

with

$$\tau = \eta\chi\,\frac{1 + \tau_t\tau_d\chi^2}{1 + \tau_t^2\chi^2}, \qquad \sigma = \frac{\eta(\tau_t - \tau_d)\chi^2}{1 + \tau_t^2\chi^2}$$

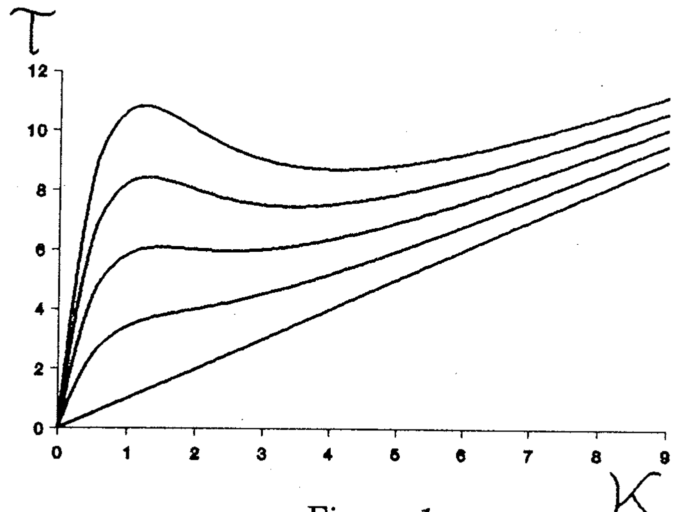

Figure 1

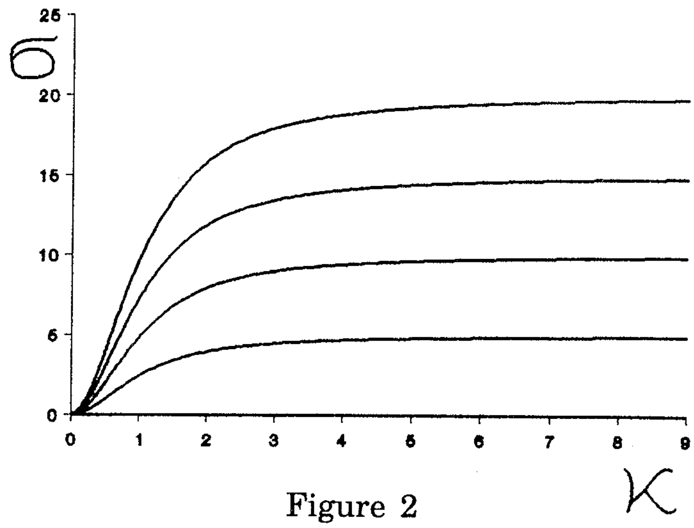

Figure 2

for the stress (Verhas 1984, 1987)

If the relation $\frac{\tau_t}{\tau_d} > 9$ holds an unstable decreasing section appears. Plastic behavior has been displayed together with hysteresis.

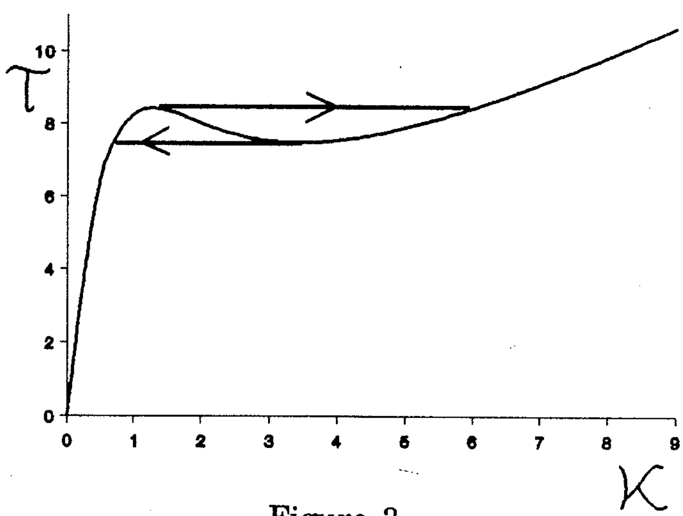

Figure 3

4. Elongational flow.

For elongational flow the equations give

$$\mathbf{t} = 2\eta\hat{\mathbf{d}}$$

as a stationary solution. The question emerges whether the flow is stable even if the load is high. Rotation may start as in the case of shear flow. The tensorial constitutive equation consists of 5 scalar equations. If the body rotates the spin tensor ω has 3 more unknown components. The supposition that the line elements parallel to the traction do not rotate gives 2 more equations while the free choice of the direction x_2 perpendicular to the traction so that $\omega_2 = 0$ supply the missing equation. The equations produce new solutions. If $\sigma > \dfrac{2\eta}{\tau_t - \tau_d}$, elongational flow becomes unstable and a faster flow pattern enters.

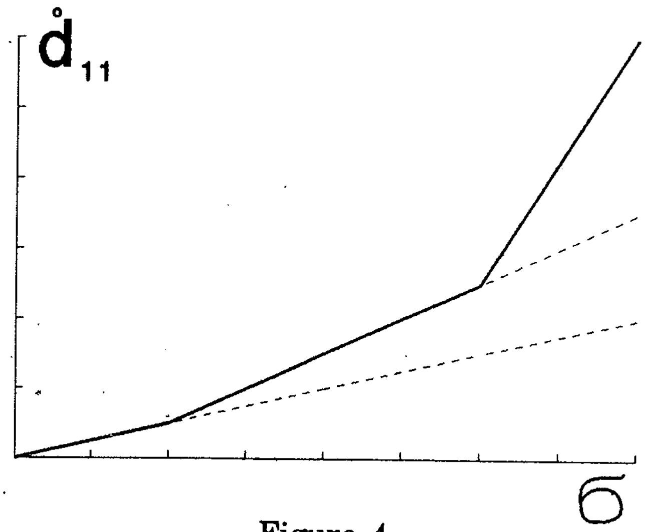

Figure 4

At $\sigma = \dfrac{6\eta}{\tau_t - \tau_d}$, a second bifurcation occurs, the previous solution gets unstable and the angular velocity starts to rotate around the traction. The flow pattern is more rapid. The effect on the rate of stretching is shown in figure 4. (The detailed calculations can be found in Verhas 1984, 1987.)

In case of uniaxial compression, it is supposed that the plane elements perpendicular to the compression do not rotate. The equations are the same as during uniaxial traction.

5. Creep and elastic behavior.

If the viscosity is high enough, the elongational flow (or any rotation free motion) is interpreted as creep. If we introduce

$$\hat{\mathbf{d}}_* = \hat{\mathbf{d}} - \frac{1}{2\eta}\mathbf{t}$$

we obtain

$$(\tau_t - \tau_d)\,\hat{\mathbf{t}} = 2\eta(\hat{\mathbf{d}}_* + \tau_d\,\hat{\hat{\mathbf{d}}}_*)$$

If the creep is negligible $\hat{\mathbf{d}}_*$ is the deformation rate. If the deformation is small with respect to any steady configuration really $\hat{\mathbf{d}}_*$ approximates a corotational derivative and integrating the equation we get

$$(\tau_t - \tau_d)\,\mathbf{t} = 2\eta(\mathbf{d}_* + \tau_d\,\hat{\mathbf{d}}_*)$$

the rheologycal equation for a Kelvin body. The viscoelastic motion before or after a plastic flow is exhibited.

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