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ON SOME OF MY FAVOURITE PROBLEMS IN GRAPH THEORY AND BLOCK DESIGNS

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I published many papers on these subjects and can not entirely avoid repetitions but I hope that some of the problems I will discuss are both new and interesting. First of all I refer to a few of my papers which state unsolved problems.

1. P. Erdös, *On the combinatorial problems I would most like to see solved,* Combinatorica 1 (1981), 25-42.

2. P. Erdös, *Problems and results in graph theory and combinatorial analysis,* Graph theory and related topics (A. Bondy and U.S.R. Murty Editors) Academic Press, 1978, 153-163.

3. P. Erdös, *On some problems in graph theory, combinatorial analysis and combinatorial number theory,* Graph theory and Combinatorics. Proc. cambridge Combinatorial conference (ed. B. Bollabás), 1-17.

1. This is an old conjecture of Faber, Lovász and myself which goes back to 1972: Let G_i, $1 \leq i \leq n$ be n edge disjoint complete graphs of size n. Is it true that the chromatic number of

$$G = \bigcup_{i=1}^{n} G_i$$

is n? G has of course $n\binom{n}{2}$ edges but the number of vertices of G is not fixed. I offer 500 dollars for a proof or disproof of this deceptively simple conjecture. A well known theorem of De Bruijn and myself easily implies that G does not contain a complete graph $K(n+1)$ of size $n+1$. Hindman proved our conjecture for $n \leq 10$, Chang and Lowler recently proved that the chromatic number of G is $\leq \frac{3n}{2} - 2$ and Seymour has various interesting results about the independence number of G, but as far as I know the conjecture is still completely open.

Recently I thought of the following slightly more general conjecture. Let G_i, $1 \leq i \leq n$ be n edge disjoint complete graphs of arbitrary size and assume that $G = \bigcup_{i=1}^{n} G_i$ has chromatic number r. Is it then true that one can put together the complete graphs G_i in an edge disjoint way so as to obtain $G' = \bigcup_{i=1}^{n} G_i$ so that G' should contain a complete graph $K(r)$ of size r? It is not impossible that this conjecture is false but our original conjecture is true.

Horak and Tuza have a paper which will soon appear in the Journal of Combinatorial Theory Ser. B where they prove among others that if G_i, $1 \leq i \leq n$, are n complete graphs of size n, then $\bigcup_{i=1}^{n} G_i$ has chromatic number at most $(1+o(1))n^{3/2}$.

W.I. Chang, E. Lowler, *Edge coloring of hypergraphs and a conjecture of Erdös, Faber, Lovász,* Combinatorica 8 (1988), 293-295.

N. Hindman, *On a conjecture of Erdös, Faber and Lovász about n-colorings,* Canadian J. of Math. 33 (1981), 563-570.

P.D. Seymour, *Packing nearly disjoint sets,* Combinatorica 2 (1982), 91-97.

2. Two or three years ago Frank Hsu told me a very simple problem due to a colleague from Mainland China and himself. Consider n vertex disjoint triangles $T_1, \ldots, T_n$ and a Hamiltonian cycle passing through these $3n$ vertices but not using any of the edges of T_i. The triangles and the Hamiltonian cycle defines a graph G of $3n$ vertices and $6n$ edges. Is it true that G has an independent set of n vertices? It is very surprising that this problem seems difficult. One would think that it is either trivial or trivially false, but this impression seems wrong. My only contribution was to ask: Is it true that perhaps G is three chromatic and perhaps this remains true if the Hamiltonian cycle is replaced by a graph of degree 2 not containing a C_4?

3. The n-dimensional cube has 2^n vertices and $n2^{n-1}$ edges. Is it true that every subgraph of $\left(\frac{1}{2}+\varepsilon\right)n2^{n-1}$ edges contains a C_4? Fan Chung proved this with $\frac{3}{5}n2^{n-1}$ (unpublished). Perhaps every subgraph of $n2^{n-2}+c2^n$ edges contains for sufficiently large c a C_4 and perhaps one can determine the smallest integer $r(n)$ so that every subgraph of the n-dimensional cube of $r(n)$ edges contains a C_4. ($r(n) = n2^{n-2}+2^{n-1}+1$?) I also conjectured that for every $\varepsilon > 0$ and $n > n_0(\varepsilon)$ every subgraph of $\varepsilon n2^{n-1}$ edges contains a C_6. Here I have no hope of guessing the best possible value. It is easy to see that the n-dimensional cube is the union of two subgraphs not containing a C_4. Denote by $f(n)$ the smallest integer for which the n-dimensional cube is the union of $f(n)$ subgraphs not containing a C_6. Estimate $f(n)$ as well as possible, the exact determination of $f(n)$ may be hopeless. Is there an Erdös-Stone-Simonovits type theorem for the subgraphs of the n-dimensional cube?

Let H be a graph. The Turán number $T(n;H)$ is the smallest integer for which every $G(n;T(n;H))$ contains H as a subgraph. ($G(n;e)$ is a graph of n vertices and e edges). The Erdös-Stone-Simonovits theorem states that if H is r-chromatic then

(1) $$T(n;H) = \frac{1}{2}\left(1-\frac{1}{r}\right)n^2 + o(n^2).$$

Tuza and I discussed this problem and we thought that the chromatic number can perhaps be replaced for the subgraphs of the cube by the smallest integer r for which our graph can be embedded in the r-dimensional cube or perhaps by the largest r for which our graph contains an r-dimensional cube. The first alternative seems more likely. The first problem would be to characterize those graphs which can be embedded into every subgraph of the n-dimensional cube having $\varepsilon n 2^n$ edges. This will probably be difficult since we do not know it for C_6 or C_{2k}, $k > 2$.

P. Erdös and A.H. Stone, *On the structure of linear graphs,* Bull. Amer. Math. Soc. 52 (1946), 1087-1091.

P. Erdös and M. Simonovits, *A limit theorem in graph theory,* Studia Sci. Math. Hungar. 1 (1966), 51-57.

4. Denote by $G(n; e)$ a graph of n vertices and e edges. Dirac called an r-chromatic graph critical if the omission of any of its edges decreases its chromatic number. Dirac told me this definition in 1949 and I immediately asked him: Denote by $f_r(n)$ the largest integer for which there is a $G(n; f_r(n))$ which has n vertices and is r-chromatic and critical. I was sure that for every r $f_r(n) = o(n^2)$. But Dirac showed

$$f_6(4n+2) \geq (2n+1)^2 + 4n + 2. \tag{2}$$

Let C_1 and C_2 be two vertex disjoint cycles of size $2n+1$ and join C_1 completely to C_2. It is easy to see that this graph is 6-chromatic and critical, perhaps (2) is in fact best possible, but this problem is still open and is perhaps difficult. Toft proved

$$f_4(n) > \frac{n^2}{16}.$$

Toft's graph has many vertices of bounded degree. This led me to ask: Is there an edge critical four chromatic graph of n vertices each vertex of which has degree $> cn$? I believe that such graphs do not exist. Simonovits and Toft showed that every vertex can be have degree $> cn^{1/3}$. The 3-chromatic critical graphs are the odd cycles but for $r \geq 4$ the r-critical graphs are very complicated. Dirac, Gallai, Toft and others have many nice results about them but

$$\lim_{n\to\infty} f_r(n)/n^2 = c_r$$

is not yet known for any $r \geq 4$. I offer (the miserly) prize of 100 dollars for the first value of $c_r (r \geq 4)$.

B. Toft, *On the maximal number of edges of critical k-chromatic graphs,* Studia Sci. Math. Hungar. 5 (1970), 461-470, *Two theorems on critical four chromatic graphs,* ibid. 7 (1972), 83-89.

M. Simonovits, *On color critical graphs,* ibid. 7 (1972), 67-81.

P. Erdös, *On some aspects of my work with Gabriel Dirac,* Annals of Discrete Math. 41 (1989), 111-116.

5. Pach and I have the following problem. Is it true that there is an absolute constant c so that for every n there is a graph $G(n)$ of diameter two the maximum degree of which is less than $n^{1/2} + c$? We thought that the answer is negative. If our $G(n)$ is triangle free and has diameter two then we thought that the maximum degree is $> cn^{1/2}$ for every c if $n > n_0(c)$.

6. Is it true that every triangle free graph of $5n$ vertices can be made bipartite by omitting at most n^2 edges? Many related problems can be posed e.g. is it true that every graph of $(2k+1)n$ vertices the smallest odd cycle of which has size $\geq 2k+1$ can be made bipartite by omitting at most n^2 edges? Is it true that such a graph can have at most n^{2k+1} cycles of size $2k+1$? E. Györi proved that for $k = 2$ such a graph can have at most $1.03n^5$ pentagons.

P. Erdös, R. Faudree, J. Pach and J. Spencer, *How to make a graph bipartite,* J. C. T. Ser. B 45 (1988), 86-98.

E. Györi, *On the number of C_5's in a triangle free graph,* Combinatorica 9 (1989), 101-102.

7. Bollobás, Simonovits and I and also many others published many papers on extremal graph and hypergraph problems. Here I just discuss a few some-what unconventional problems in this subject. Let H be a graph, $T(n; H)$ is its Turán number, i.e. the smallest integer for which every $G(n; T(n; H))$ contains H as a subgraph. Simonovits and I now asked: for which graphs is it true that every $G(n; T(n; H))$ contains more than one copies of H. This is well known if H is a complete graph. It is very annoying that we could not settle this question for $H = C_4$, we suspect that every $G(n; T(n; C_4))$ contains more than $cn^{1/2}$ copies of C_4.

Renu Laskar and I proved that every $G\left(n; \left[\frac{n^2}{4}\right] + 1\right)$ contains a triangle (x_1, x_2, x_3) whose triangle degree is $> n(1 + \varepsilon)$. The triangle degree is defined as the sum of the degrees of the vertices of the triangle. Our ε was a small positive constant independent of n. We also showed that the triangle degree does not have to be more than $2(\sqrt{3} - 1)n$. Fan in a recent paper proved that every $G\left(n; \left[\frac{n^2}{4}\right] + 1\right)$ contains a triangle whose degree is $> \frac{21n}{16}$. Perhaps $2(\sqrt{3} - 1)n$ is the correct bound.

Caccetta, Vijayan and I proved that every $G\left(n; \left[\frac{n^2}{4}\right] + 1\right)$ contains a triangle each vertex of which has degree $> \frac{n}{3}$ and $\frac{n}{3}$ is best possible. We conjectured and Faudree and I proved that every $G\left(n; \left[\frac{n^2}{4}\right] + 1\right)$ contains a triangle x_1, x_2, x_3 for which $v(x_i) > \frac{n}{3}$ and

$$\sum_{i=1}^{n} v(x_i) > n(1 + c)$$

for some absolute constant $c > 0$. Perhaps in fact $c = 2\sqrt{3} - 3$ which in view of our result with Renu Laskar is best possible if true. This conjecture may be too optimistic.

B. Bollobás, *Extremal Graph Theory,* London Math. Soc. Monographs VII, Acadmeic Press, 1978.

M. Simonovits, *Extremal Graph Theory, Selected Topics in Graph Theory II.* Edited by L. W. Beineke and R. J. Wilson, Academic Press, London, New York, San Francisco.

M. Simonovits, *Extremal Graph Problems, Degenerate Extremal Problems, and Supersaturated Graphs,* Progress in Graph Theory, Academic Press, Canada, 1989, 419-437.

P. Erdös and M. Simonovits, *Cube supersaturated graphs and related problems,* ibid. 203-218.

P. Erdös and R. Laskar, *A note on the size of a chordal subgraph,* Proc. Southeastern Conference on Combinatorics... Boca Raton Utilitas Math. Winnipeg (1985), 81-86.

G. Fan, *Degree sum for a triangle in a graph,* J. Graph Theory 12 (1988), 249-264.

L. Caccetta, P. Erdös and K. Vijayan, *Graphs with unavoidable subgraphs with large degrees,* ibid. 12 (1988), 17-29.

8. Hajanl and I have the following problem: Let H be any fixed graph and G a graph of n vertices ($n \to \infty$). Assume that G does not contain H as an induced subgraph. Is it then true that there is an ε_H so that either G contains a clique or an independnet set of size $\geq n^{\varepsilon_H}$? We proved this with exp $c(\log n)^{\frac{1}{2}}$ instead of n^{ε_H} and proved it with n^{ε_H} for many special cases, e.g. we showed that if H is C_4 then $\varepsilon_{C_4} \geq \frac{1}{3}$. We could not prove it if H is C_5 or P_5 (a path of four edges).

Our paper with Hajnal will soon appear.

9. I now state a problem of Nesetril and myself. Let G be a graph the maximum degree of which is n and the number of edges of G is $> \frac{5n^2}{4}$. Then we conjectured that G contains two edges which are strongly independent (e_1 and e_2 are strongly independent if they are vertex disjoint and there is no edge e which intersects both e_1 and e_2). It is easy to see that the conjecture if true is best possible. Fan Chung and Trotter and independnetly and simultaneously Gyárfás and Tuza proved this conjecture, their quadruple paper will soon appear in Discrete Mathematics. Nesetril and I then posed with some trepidation the following Vizing type conjecture: The Strong edge chromatic number of our graph is at most $\frac{5n^2}{4}$. In other words the edges of G can be coloured by at most $\frac{5n^2}{4}$ colors so that two edges which get the same color are strongly independent. This conjecture is still open and even if it turns out to be false it would be interesting to find the maximum possible value of the strong edge chromatic number. Faudree, Gyárfás, Schelp and Tuza have two more papers on this subject which will soon appear. Their paper contains many nice theorems and many nice problems. Here I just state one of them: Let G be a bipartite graph of maximum degree n. Can its edges be strongly coloured by n^2 colors?

10. Let $h(n)$ tend to infinity arbitrary slowly. Is there a graph G of infinite chromatic number every induced subgraph of n vertices of which can be made bipartite by the omission of $\leq h(n)$ edges? This nice problem is due to Hajnal Szemerédi and myself and I offer 100 dollars for an answer. An example of Galali-Lovász shows that for every r there is a graph of chromatic number $r+2$ with $h(n) < cn^{1-/r}$ and perhaps this is best possible. Rödl showed that for triple systems $h(n)$ can increase *arbitrary* slowly.

P. Erdös, A. Hajnal and E. Szemerédi, *On almost bipartite large chromatic graphs,* Annals of Discrete Math 12 (1983), 117-123. This paper contains several other interesting problems of infinite character.

V. Rödl, *Nearly bipartite graphs with large chromatic number,* Combinatorica 2 (1982), 377-383.

11. Here is an attractive group of problems of Pyber, Tuza and myself. Can one characterize the graphs $G(n; e)$ which have the following property: Let p be large and color the edges of $K(p)$ by e colors so that in every vertex every color has degree $\geq \frac{(1-\varepsilon)p}{e}$. Is it then true that if $p > p_0(\varepsilon)$ our $K(p)$ contains a totally multicoloured subgraph G? We only know that the answer is positive if G is a triangle or C_4 and it certainly is negative if every edge of G is contained in a triangle and e is even. Brightwell and Trotter also showed that the answer is negative if G is a C_{4k+2}. We certainly would like to know the answer if G is C_5. Also we have the following question: Color the edges of a $K(12n+1)$ by 6 colors so that every vertex should have degree $2n$ in every color. Is it then true that our graph contains a totally multicoloured C_6 and a totally multicoloured $K(4)$? Clearly many related questions can be asked but we will formulate them in a forthcoming note.

12. Burr, Graham, V. T. Sós and I in several papers investigated problems of the following type (the second paper on this subject which will soon appear is joint with P. Frankl): Let H be a fixed graph and $e(n; H)$ an integer. Determine or estimate the smallest integer $f(e(n; H))$ for which there is a graph $G(n; e(n; H))$ whose edges can be colored by $f((e; (n; H))$ colors so that every copy of H which occurs in our G should be totally multicolored (i.e. every edge should get a different color). Clearly for $e(n; H) < T(n; H)$, $f(e(n; H)) = 1$. To see this take a $G(n; e(n; H))$ which does not contain H as a subgraph all its edges can get the same color. On the other hand as soon as $e(n; H) \geq T(n; H)$ interesting and difficult problems can be posed. Let $H = C_{2k+1}$ be a cycle of size $2k+1$, $k \geq 3$. We proved that

$$f\left(\left[\frac{n^2}{4}\right]+1, C_{2k+1}\right) > cn^2,\ c \leq \frac{1}{8},$$

and conjecture

$$f\left(\left[\frac{n^2}{4}\right]+1, C_{2k+1}\right) = (1+o(1))\frac{n^2}{8}.$$

The situation for C_5 is much more complicated, we only proved

$$c_1 n < f\left(\left[\frac{n^2}{4}\right]+1, C_5\right) \leq \left[\frac{n}{2}\right]+3$$

and very recently Simonovits and I proved that the upper bound gives the correct value.

We proved that for $c_1 < \dfrac{1}{2}$

$$f(c_1 n^2; C_5) < c_2 n^2/\log n$$

but have no good lower bound for $f(c_1, n^2, C_5)$. Also we could not decide whether

$$f(cn^2; C_4)/n \to \infty$$

for every $c > 0$. Many more problems are formulated in our papers.

S. A. Burr, P. Erdös, R. L. Graham and V. T. Sós, *Maximal antiramsey graphs and the strong chromatic number,* J. of Graph Theory, 13, (1989), 263-279.

13. The following question was formulated by Colbourn and myself during a discussion at our meeting. Let G be an r-uniform hypergraph on n vertices and cn^2 edges which forms a clique i.e. every two hyperedges have exactly one vertex in common. Is it then true that our G has r edges no vertex of which is contained in 3 edges but every two edges have a vertex in common? For $r = 3$ this is a theorem of Ruzsa and Szemerédi. As far as we know the question is open for $r > 3$.

I. Ruzsa and E. Szemerédi, *Triple systems with no six points carrying three triangles,* Combinatorics, Coll. Math. Soc. Bolyai 18 (1978), 939-945. (Ed. A. Hajnal, V. T. Sós).

Now I state two very old problems which belong to extremal set theory.

14. This problem is due to Rado and myself and goes back more than 30 years. A family of sets $B_1, B_2, \ldots$ is called a Δ-system if the intersection of any two of them is the same. Peter Frankl calls such a system by the picturesque name «Sunflower». Let $f_k(n)$ be the smallest integer for which every family of sets A_i, $|A_i| = n$, $1 \leq i \leq f_k(n)$ contains a Δ system of K elements. We proved $f_k(n) < (k-1)^n n!$ and conjectured

$$f_k(n) < c_k^n. \tag{3}$$

I offer 1000 dollars for a proof or disproof of this conjecture. (3) seems simple but unless many of us missed a simple idea, it is probably very difficult.

P. Erdös and R. Rado, *Intersection theorems for systems of set I and II.* J. London Math. Soc. 35 (1960), 85-90 and 44 (1969), 467-479.

P. Erdös, E. C. Milner and R. Rado, *Intersection theorems for systems of sets III.* J. Austral. math. Soc. 18 (1974), 22-40.

15. Let $f(n)$ be the largest integer for which there is a family of sets $A_i \subset S$, $|S| = 4n$, $|A_i| = 2n$, $|A_i \cap A_j| \geq 2$, $1 \leq i < j \leq f(n)$. In our paper with Ko and Rado we conjectured

$$(4) \qquad (n) = \left(\binom{4n}{2n} - \binom{2n}{n}\right)/2.$$

It is easy to see that (4) if true is best possible. To see this let $B \cup C = S$, $B \cap C = \emptyset$, $|B| = |C| = 2n$. $|A_i \cap B| \geq n+1$, $1 \leq i \leq f(n)$. This construction gives $f(n)$ sets A_i, the intersection of any two of which has size ≥ 2. Our paper with Ko and Rado contained many unsolved problems, all but this one have been settled. I offered 250 pounds for a proof or disproof od (4).

P. Erdös, Chao Ko and R. Rado, *Intersection theorems for systems of finite sets,* Quarterly J. of Math., 12 (1961), 313-320.

Now I discuss some problems on finite geometries and block designs.

16. Is it true that there is an absolute constant c so that every finite geometry has a blocking set which meets every line in fewer than c points? It would be of some interest to show that c can not be too small. The following more general question is surely interesting. Is it true that to every c_1 there is a c_2 for which if $L(n)$ is a linar space on n vertices each line of which has length $\geq c_1\sqrt{n}$ then there is a set S which intersects every line and each of them in fewer that c_2 points? Surely c_2 tends to infinity if c_1 tends to 0. This probably will be asy to prove. Perhaps the following question is also of interest. Determine or estimate the smallest integer k for which every linear space $L(n)$ every line of which has size $\geq k$ has property B or in other words is two chromatic – i.e. has a blocking set which contains none of the lines. $k < c_1 \log n$ and probably $\log n$ is the correct order of magnitude for k.

17. Jean larson and I have the following problem. Is it true that for every n there is a linear space each line of which has length greater than $n^{1/2} - c$? (c is an absolute constant independent of n, the length (or size) of a line is the number of points on it). We proved the existence of such a linear space each line of which is greater than $n^{1/2} - n^{3/5}$, this could be replaced by $n^{1/2} - (\log n)^c$ if we would know that $p_{n+1} - p_n < (\log n)^{c_1}$ (p_n is the n-th prime). A recent result of Shrikande and Singhi makes it very likely that the answer to our question is negative.

P. Erdös and J. Larson, *On pairwise balanced block designs with the sizes of blocks as uniform as possible,* Ann. Discrete Math. 15 (1983), 129-134.

S. S. Shrikhande and N. M. Singhi, *On a problem of Erdös and Larson,* Combinatorica 5 (1985), no. 4, 351-358.

18. Is it true that for every $\varepsilon > 0$ there is an $n_0(\varepsilon)$ so that for every $n > n_0(\varepsilon)$, $|S| = n$, there is a family of sets $A_i \subset S$, $1 \leq i \leq t_n$, $t_n > n(1-\varepsilon)$, $|A_i \cap A_j| = 1$, $|A_i| = [n^{1/2}] + 1$.

In particular if $|S| = n^2 + n + 1$ are there more than $(1-\varepsilon)n^2$ sets $|A_i| = n + 1$, $|A_1 \cap A_j| = 1$?

19. Let $a_1 < a_2 < \ldots < a_t$ be a Sidon sequence of integers, in other words, the sums $a_i + a_j$ are all distinct. Can one then find a sequence $a_1 < a_2 < \ldots < a_t < a_{t+1} < \ldots < a_{p+1}$ which is a perfect difference set mod(p^2+p+1) i.e. every non-zero residue mod(p^2+p+1) can be uniquely written in the form $a_i - a_j$? I offer 250 dollars for a positive answer and 100 dollars for a counterexample. I would give 100 dollars for the construction of a sequence $a_1 < \ldots < a_t < a_{t+1} \ldots < a_n$, $a_n = (1 + o(1))n^2$ for which the sums $a_i + a_j$, $1 \leq i < j \leq n$ are all distinct.

The following question is perhaps old. Let $B_1, B_2, \ldots, B_n$ be sets, $|B_i \cap B_j| \leq 1$ for every $1 \leq i < j \leq n$. Is it true that there is a p and sets $|A_i| = p+1$, $1 \leq i \leq p^2+p+1$, $B_i \subset A_i$, $1 \leq i \leq n$, $|A_i \cap A_j| = 1$, $1 \leq i < j \leq p^2+p+1$? In other words the A's are the lines of a finite geometry.

20. The following problem is due to Lovász and myself. Let $m(n)$ be the smallest integer for which there is a family of sets A_i, $1 \leq i \leq m(n)$; $|A_i| = n$, $|A_i \cap A_j| \geq 1$, $1 \leq i \leq j \leq m(n)$ and if S is a set, $|S \cap A_i| \geq 1$, $1 \leq i \leq m(n)$ then $|S| \geq n$. We conjectured $\frac{m(n)}{n} \to \infty$ but only could prove $m(n) < n^{\frac{3}{2}+\varepsilon}$. We believe that there are $cn \log n$ lines in a finite geometry so that every set representing these lines has size $\geq n$ (i.e. every set meeting all these lines has size $\geq n$). Perhaps $cn \log n$ can be replaced by cn, but we do not believe this.

The following problem seems interesting: Is it true that for every c there is an ε for which if $|A_k| = n$, $1 \leq k \leq cn$ is a family of sets satisfying $|A_{k_1} \cap A_{k_2}| \geq 1$ for every $1 \leq k_1 < k_2 \leq n$ there always is a set $S, |S| < (1-\varepsilon)n$, $|A_k \cap S| \geq 1$? This conjecture if true would of course imply our conjecture with Lovász, $\frac{m(n)}{n} \to \infty$.

P. Erdös and L. Lovász, *Problems and results on three-chromatic hypergraphs and some related questions,* Finite and infinite sets, Coll. Math. Soc.

J. Bolyai 10 (1975), 609-627 (Editors A. Hajnal, R. Rado, V. T. Sós).

S. J. Dow, D. A Drake, Z. Füredi and J. A. Larson, *A lowar bound for the cardinality of a maximal family of mutually intersecting sets of equal size,* Congressus Numerantium 48 (1985), 47-48.

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