«LE MATEMATICHE»
Vol. XLIV (1989) - Fasc. II, pp. 333-344

CERTAIN TRANSFORMATIONS OF BASIC HYPERGEOMETRIC FUNCTIONS OF TWO VARIABLES

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In this note the authors derive three transformations for the basic hypergeometric functions of two variables by making use of certain known summation formula. As $q \to 1$, known transformations for ordinary hypergeometric functions given earlier by Carlitz [4], Jackson [7] and Slater [11] are obtained.

1. Introduction and Preliminaries.

Let $|q| < 1$, and

$$(a;q)_n = \prod_{j=0}^{\infty} \frac{(1-aq^j)}{(1-aq^{n+j})}; \; n > 0 \tag{1}$$

for arbitrary a (real or complex).

Then the basic hypergeometric series is defined by Gasper and

(*) Entrato in Redazione il 2 ottobre 1990

Rahman [6] in the form

$$(2)\quad {}_r\emptyset_s\left[\begin{matrix} a_1,\ldots,a_r; \\ b_1,\ldots,b_s; \end{matrix} q;x\right] = \sum_{n\geq 0}\frac{(a_1;q)_n\ldots(a_r;q)_n x^n\{(-1)^n q^{\frac{n}{2}(n-1)}\}^{1+s-r}}{(q;q)_n(b_1;q)_n\ldots(b_s;q)_n}$$

where, for convergence, $|x| < 1$.

Also, the generalized basic hypergeometric series of two variables defined by Jain [9] (see also Srivastava and Karlsson [13, p. 347, eqn. 272]) is given by

$$(3)\quad \emptyset^{A:B';B''}_{C:D';D''}\left(\begin{matrix}(a):(b');(b'');q;x,y\\(c):(d');(d'');i,j,k\end{matrix}\right)$$

$$= \sum_{m,n\geq 0}\frac{\prod_{t=1}^{A}(a_t;q)_{m+n}\prod_{t=1}^{B'}(b'_t;q)_m\prod_{t=1}^{B''}(b''_t;q)_n x^m y^n}{\prod_{t=1}^{C}(c_t;q)_{m+n}\prod_{t=1}^{D'}(d'_t;q)_m\prod_{t=1}^{D''}(d''_t;q)_n} \times\frac{q^{im(m-1)/2+jn(n-1)/2+kmn}}{(q;q)_m(q;q)_n}$$

where $|x| < 1$, $|y| < 1$, and i, j, k are arbitrary integers.

The symbol (a) will stand for the sequence of $'A'$ parameters

$$a_1,\ldots,a_A.$$

Further, if we employ the notation

$$(4)\quad \prod\left[\begin{matrix}(a);\\(b);\end{matrix} q\right] = \prod_{t=0}^{\infty}\frac{(1-(a)q^t)}{(1-(b)q^t)}$$

then

$$(5)\quad \lim_{q\to 1}\left\{\frac{(q^\alpha;q)_n}{(q^\beta;q)_n}\right\} = \frac{(\alpha)_n}{(\beta)_n}$$

where

$$(6)\quad (\alpha)_n = \alpha(\alpha+1)\ldots(\alpha+n-1);$$

for arbitrary α and β; $\beta\neq 0, -1, -2,\ldots$

2. The following three summations for double series are to be proved in this section. These summations will be used to derive certain new transformations for basic hypergeometric functions of two variables.

First summation:

$$(7)\qquad \sum_{r,s\geq 0}\frac{(q^{-m};q)_r(q^{-n};q)_s(\beta;q)_r(\gamma;q)_s(\alpha;q)_{r+s}\gamma^{-r}\beta^{-s}q^{r+s-rs}}{(q;q)_r(q;q)_s(\alpha;q)_r(\alpha;q)_s(\beta q^{1-m}/\gamma;q)_r(\gamma q^{1-n}/\beta;q)_s}=$$

$$=\frac{(\alpha;q)_{m+n}(\gamma;q)_m(\beta;q)_n q^{-mn}q^{-mn}\beta^{-m}\gamma^{-n}}{(\alpha;q)_m(\alpha;q)_n(\gamma/\beta;q)_m(\beta/\gamma;q)_n}$$

Second summation:

$$(8)\qquad \sum_{r,s\geq 0}\frac{(q^{-m};q)_r(q^{-n};q)_s(\gamma/\alpha;q)_{r+s}(\alpha)^{r+s}q^{(m+n)r+ns}}{(q;q)_r(q;q)_s(\gamma;q)_{r+s}}=\frac{(\alpha;q)_{m+n}}{(\gamma;q)_{m+n}}$$

Third sumamtion:

$$(9)\qquad \sum_{r+s\leq n}\frac{(q^{-n};q)_{r+s}(\alpha;q)_r(\alpha';q)_r(\alpha';q)_s(\beta;q)_r(\beta';q)_s q^{r+s}}{(q;q)_r(q;q)_s(\beta\beta';q)_{r+s}(\alpha q^{1-n}/\beta';q)_r(\alpha' q^{1-n}/\beta;q)_s}=$$

$$=\frac{(\beta,q)_n(\beta';q)_n(\beta\beta'/\alpha\alpha';q)_n}{(\beta\beta';q)_n(\beta/\alpha';q)_n(\beta'/\alpha;q)_n}$$

By an appeal to the q-analogue of Vandermonde's theorem, namely,

$$\frac{(\alpha;q)_{r+s}}{(\alpha;q)_r(\alpha;q)_s}=\sum_{j=0}^{\min(r,s)}\frac{(q^{-r};q)_j(q^{-s};q)_j q^{rs+j}}{(q;q)_j(\alpha;q)_j}$$

and then replacing r by $(r+j)$ and s by $(s+j)$, the L.H.S. of (7) becomes

$$\sum_{j=0}^{\min(m,n)} \frac{(q^{-m};q)_j(q^{-n};q)_j(\beta;q)_j(\gamma;q)_j(\gamma\beta)^{-j}q^{2j-j^2}}{(q;q)_j(\beta q^{1-m}/\gamma;q)_j(\gamma q^{1-n}/\beta;q)_j(\alpha;q)_j}$$

$$\times \sum_{r=0}^{m-j} \frac{(q^{-m+j};q)_r(\beta q^j;q)_r(q^{1-j}/\gamma)^r}{(q;q)_r(\beta q^{1-m+j}/\gamma;q)_r}$$

$$\times \sum_{s=0}^{n-j} \frac{(q^{-n+j};q)_s(\gamma q^j;q)_s(q^{1-j}/\beta)^s}{(q;q)_s(\gamma q^{1-n+j}/\beta;q)_s}$$

Now, summing the inner ${}_2\emptyset_1(\cdot)$ series, with the help of q-analogue of Gauss's theorem, Slater [11, p. 24, IV-3], namely,

$$(10) \qquad {}_2\emptyset_1\left[\begin{matrix} a, q^{-n}; \\ c; \end{matrix} \; q; \frac{cq^n}{a}\right] = \frac{(c/a;q)_n}{(c;q)_n}$$

We arrive at the desired result (7).

The remaining results (8) and (9) can be established by making use of the result (10) and Saalschützian theorem, Slater [11, p. 247, IV-4]

$$(11) \qquad {}_3\emptyset_2\left[\begin{matrix} a, b, q^{-n}; \\ c, d; \end{matrix} \; q; q\right] = \frac{(c/a;q)_n(c/b;q)_n}{(c;q)_n(c/ab;q)_n}$$

where $cd = abq^{1-n}$.

It is interesting to observe that as $q \to 1$ in (7) and (8), we obtain respectively the following known summation formulas for the ordinary hypergeometric function due to Carlitz [4, p. 140, eqn. 9] and Sharma [10, p. 256].

$$(12) \qquad \sum_{r,s\geq 0} \frac{(-m)_r(-n)_s(\beta)_r(\gamma)_s(\alpha)_{r+s}}{r!s!(\alpha)_r(\alpha)_s(1+\beta-m-\gamma)_r(1+\gamma-n-\beta)_s}$$

$$= \frac{(\alpha)_{m+n}(\gamma)_m(\beta)_n}{(\alpha)_m(\alpha)_n(\gamma-\beta)_m(\beta-\gamma)_n}$$

and

$$\sum_{r,s\geq 0} \frac{(-m)_r(-n)_s(\gamma-\alpha)_{r+s}}{(\gamma)_{r+s}r!s!} = \frac{(\alpha)_{m+n}}{(\gamma)_{m+n}} \tag{13}$$

The result (9), was given by Al-Salam [1, p. 458, eqn. 3.1]. It is the q-analogue of a result due to Carlitz [3, p. 202, eq. 7].

3. In this section the following transformations are to be established

$$\phi^{A+1:B'+1;B''+1}_{C:D'+2;D''+2}\begin{pmatrix} (a), \alpha : (b'), \gamma; (b''), \beta; q; \dfrac{x}{\beta}, \dfrac{y}{\gamma} \\ (c) : (d'), \alpha, \dfrac{\gamma}{\beta}; (d''), \alpha, \dfrac{\beta}{\gamma}; -, -, -1 \end{pmatrix} = \tag{14}$$

$$= \sum_{r,s\geq 0} \frac{\prod_{t=1}^{A}(a_t;q)_{r+s}(\alpha;q)_{r+s}\prod_{t=1}^{B'}(b'_t;q)_r(\beta;q)_r\prod_{t=1}^{B''}(b''_t;q)_s}{\prod_{t=1}^{C}(c_t;q)_{r+s}\prod_{t=1}^{D'}(d'_t;q)_r(\alpha;q)_r(\gamma/\beta;q)_r\prod_{t=1}^{D''}(d''_t;q)_s}$$

$$\times \frac{(\gamma;q)_s(x/\beta)^r(y/\gamma)^s q^{-rs}}{(\alpha;q)_s(\beta/\gamma;q)_s(q;q)_r(q;q)_s}$$

$$\times \phi^{A:B'+1;B''+1}_{C:D'+1;D''+1}\begin{pmatrix} (a)q^{r+s} : (b')q^r, \dfrac{\gamma}{\beta}; (b'')q^s, \dfrac{\beta}{\gamma}; q; x, y \\ (c)q^{r+s} : (d')q^r, \dfrac{\gamma q^r}{\beta}; (d'')q^s, \dfrac{\beta q^s}{\gamma}; -, -, - \end{pmatrix}$$

provided that $|x| < 1$, $|y| < 1$.

$$\phi^{A+1:B';B''}_{C+1:D';D''}\begin{pmatrix} (a), \alpha : (b'); (b''); q; x, y \\ (c); \gamma : (d'); (d''); -, -, - \end{pmatrix} = \tag{15}$$

$$= \sum_{r,s\geq 0} \frac{\prod_{t=1}^{A}(a_t;q)_{r+s}\left(\dfrac{\gamma}{\alpha};q\right)_{r+s}\prod_{t=1}^{B'}(b'_t;q)_r\prod_{t=1}^{B''}(b''_t;q)_s}{\prod_{t=1}^{C}(c_t;q)_{r+s}(\gamma;q)_{r+s}\prod_{t=1}^{D'}(d'_t;q)_r\prod_{t=1}^{D''}(d''_t;q)_s}$$

$$\times \frac{(-\alpha x)^r(-\alpha y)^s q^{\frac{r}{2}(r-1)+\frac{s}{2}(s-1)+rs}}{(q;q)_r(q;q)_s}$$

$$\times \emptyset^{A:B';B''}_{C:D';D''}\left(\begin{matrix}(a)q^{r+s}:(b')q^r;(b'')q^s;q;x,yq^r\\(c)q^{r+s}:(d')q^r;(d'')q^s;-,-,-\end{matrix}\right)$$

provided that $|x|<1$, $|y|<1$.

(16)
$$\emptyset^{A:B';B''+3}_{C:D';D''+3}\left(\begin{matrix}(a):(b');(b''),\beta,\beta',\beta\beta'/\alpha\alpha';q;x,y\\(c):(d');(d''),\beta\beta',\beta/\alpha',\beta'/\alpha;-,-,-\end{matrix}\right)=$$

$$=\sum_{r,s\geq 0}\frac{\prod_{t=1}^{A}(a_t;q)_{r+s}\prod_{t=1}^{B''}(b''_t;q)_{r+s}(\alpha;q)_r(\beta;q)_r(\alpha';q)_s}{\prod_{t=1}^{C}(c_t;q)_{r+s}\prod_{t=1}^{D''}(d''_t;q)_{r+s}(\beta\beta';q)_{r+s}\left(\frac{\beta' q^s}{\alpha};q\right)_r}$$

$$\times\frac{(\beta';q)_s(y\beta'/\alpha)^r(y\beta/\alpha')^s q^{rs}}{(\beta q^r/\alpha';q)_s(q;q)_r(q;q)_s}$$

$$\times\emptyset^{A:B';B''+2}_{C:D';D''+2}\left(\begin{matrix}(a)q^{r+s}:(b');(b'')q^{r+s},\frac{\beta q^r}{\alpha'},\frac{\beta' q^s}{\alpha};q;x,y\\(c)q^{r+s}:(d');(d'')q^{r+s},\frac{\beta q^{r+s}}{\alpha'},\frac{\beta q^{r+s}}{\alpha};-,-,-\end{matrix}\right)$$

provided that $|x|<1$, $|y|<1$.

To prove (14), we consider the L.H.S., and replace it by its equivalent series (3), then on using (7), it becomes

$$\sum_{m,n\geq 0}\frac{\prod_{t=1}^{A}(a_t;q)_{m+n}\prod_{t=1}^{B'}(b'_t;q)_m\prod_{t=1}^{B''}(b''_t;q)_n x^m y^n}{\prod_{t=1}^{C}(c_t;q)_{m+n}\prod_{t=1}^{D'}(d'_t;q)_m\prod_{t=1}^{D''}(d''_t;q)_n(q,q)_m(q;q)_n}$$

$$\times\sum_{r,s\geq 0}\frac{(q^{-m};q)_r(q^{-n};q)_s(\beta;q)_r(\gamma;q)_s(\alpha;q)_{r+s}\gamma^{-r}\beta^{-s}q^{r+s-rs}}{(q;q)_r(q;q)_s(\alpha;q)_r(\beta q^{1-m}/\gamma;q)_r(\alpha;q)_s(\gamma q^{1-n}/\beta;q)_s}$$

Now replacing m by $(m+r)$ and n by $(n+s)$, and interchanging the order of double summations, which is justified under the given conditions. The result (14) now readily follows.

The proof of (15) can be developed by making use of (8).

The remaining result (16) can be proved expanding its L.H.S., using (9) and the making use of then following series rearrangement techniques, Scrivastava and Manocha [12]

$$\sum_{r=0}^{n}\sum_{s=0}^{n-r} B(s, r+s) = \sum_{r=0}^{n}\sum_{s=0}^{r} B(s, r) \tag{17}$$

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n} B(r, n) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} B(r, n+r) \tag{18}$$

$$\sum_{r=0}^{\infty}\sum_{s=0}^{r} B(s, r) = \sum_{r=0}^{\infty}\sum_{s=0}^{\infty} B(r, r+s) \tag{19}$$

We obtain that

$$\sum_{r,s\geq 0} \frac{\prod_{t=1}^{A}(a_t;q)_{r+s}\prod_{t=1}^{B''}(b_t'';q)_{r+s}(\alpha;q)_r(\beta;q)_r(\alpha';q)_s(\beta';q)_s(yq)^{r+s}}{\prod_{t=1}^{C}(c_t;q)_{r+s}\prod_{t=1}^{D''}(d_t'';q)_{r+s}(q;q)_{r+s}(q;q)_r(q;q)_s(\beta\beta';q)_{r+s}}$$

$$\sum_{m,n\geq 0} \frac{\prod_{t=1}^{A}(a_tq^{r+s};q)_{m+n}\prod_{t=1}^{B'}(b_t';q)_m\prod_{t=1}^{B''}(b_t''q^{r+s};q)_n}{\prod_{t=1}^{C}(c_tq^{r+s};q)_{m+n}\prod_{t=1}^{B'}(d_t';q)_m\prod_{t=1}^{D''}(d_t''q^{r+s};q)_n}$$

$$\times \frac{(q^{n-r-s};q)_r(q^{-n-s};q)_s x^m y^n}{\left(\frac{\alpha q^{1-n-r-s}}{\beta'};q\right)_r\left(\frac{\alpha' q^{1-n-r-s}}{\beta};q\right)_s(q;q)_m(q^{1+r+s};q)_n}$$

which simplifies to R.H.S. of (16).

Special Cases:

If we set $A = C = D' = D'' = 0$; $B' = B'' = 1$, $b'_1 = \gamma/\beta$, $b''_1 = \beta/\gamma$ in (14) and sum the inner ${}_1\emptyset_0(\cdot)$ series with the help of Heine's theorem, Slater [11, p. 248, IV. 11], namely,

$$(20) \qquad {}_1\emptyset_0[a; -; q; z] = \prod \left[\begin{matrix} az; \\ z; \end{matrix} \; q \right]$$

and make use of the identity

$$(21) \qquad \emptyset^{(2)}[\alpha; \beta, \gamma; \alpha, \alpha; q; x, y] = \emptyset^{(2)}[\alpha; \gamma; \beta; \alpha, \alpha; q; y, x]$$

We obtain the following interesting transformation

$$(22) \qquad \emptyset^{(2)}[\alpha; \gamma, \beta; \alpha, \alpha', q; x, y] = \prod \left[\begin{matrix} \frac{\gamma x}{\beta}, & \frac{\beta y}{\gamma}; \\ x, & y; \end{matrix} \; q \right] \cdot$$

$$\cdot \emptyset^{(2)}[\alpha; \gamma, \beta; \alpha, \alpha; q; y, x]$$

where $\emptyset^{(2)}(\cdot)$ is defiend as in Jackson [8], namely

$$(23) \qquad \emptyset^{(2)}[\alpha; \beta, \beta'; \gamma, \gamma'; q; x, y] =$$

$$= \sum_{m,n \geq 0} \frac{(\alpha; q)_{m+n} (\beta; q)_m (\beta'; q)_n x^m y^n}{(q; q)_m (q; q)_n (\gamma; q)_m (\gamma'; q)_n}$$

As $q \to 1$ in (22) we get known transformation for F_2 function given by Carlitz [4, p. 140, eq. 10], namely,

$$(24) \qquad F_2[\alpha; \gamma, \beta; \alpha, \alpha; x, y] =$$

$$= \left(\frac{1-x}{1-y} \right)^{\beta - \gamma} F_2[\alpha; \gamma, \beta; \alpha, \alpha; x, y]$$

Next, if we take $A = C = D' = D'' = 0$; $B' = B'' = 1$, $b_1' = \beta$, $b_1'' = \mu$ in (15) and sum the inner ${}_1\emptyset_0(\cdot)$ series by using (20), we obtain the following intersting transformation

(25)
$$\emptyset^{(1)}[\alpha;\beta,\mu;\gamma;q;x,y] =$$
$$= \prod\left[\begin{matrix}\beta x, \mu y; \\ x, \ y;\end{matrix}\ q\right] \emptyset^{1:2;1}_{2:1;-}\left(\begin{matrix}\dfrac{\gamma}{\alpha} : \beta, y; \mu; q; x, y \\ \gamma, \mu y : \beta x; -; 1, 1, 1\end{matrix}\right)$$

where $\emptyset^{(1)}$ is defined by Jackson [8] as

(26)
$$\emptyset^{(1)}[\alpha;\beta,\beta';\gamma;q;x,y] = \sum_{m,n\geq 0} \frac{(\alpha;q)_{m+n}(\beta;q)_m(\beta';q)_n x^m y^n}{(q;q)_m(q;q)_n(\gamma;q)_{m+n}}$$

If we let $y \to 0$ in (25), we obtain the known transformation due to Jackson [7, p. 145, eq. 4], namely

(27)
$${}_2\emptyset_1\left[\begin{matrix}\alpha, \beta; \\ \gamma;\end{matrix}\ q; x\right] = \prod\left[\begin{matrix}\beta x; \\ x;\end{matrix}\ q\right]\ {}_2\emptyset_2\left[\begin{matrix}\dfrac{\gamma}{\alpha}, \beta; \\ \gamma, \beta x;\end{matrix}\ q; -\alpha x\right]$$

If we replace α by γ/α, then as $q \to 1$, we get known transformation due to Slater [11, p. 31, eq. 1.7.1.3], namely.

(28)
$${}_2F_1\left[\begin{matrix}\alpha, \beta; \\ \gamma;\end{matrix}\ x\right] = (1-x)^{-\beta}\ {}_2F_1\left[\begin{matrix}\gamma-\alpha, \beta; \\ \gamma;\end{matrix}\ \frac{x}{1-x}\right]$$

For $\beta \to 0$, (27) reduces to yet another transformation

(29)
$${}_1\emptyset_1\left[\begin{matrix}\alpha; \\ \gamma;\end{matrix}\ q; x\right] = \prod\left[\begin{matrix}-; \\ x;\end{matrix}\ q\right]\ {}_1\emptyset_1\left[\begin{matrix}\gamma/\alpha; \\ \gamma;\end{matrix}\ q; -\alpha x\right]$$

which is q-analogue of Euler's transformation

(30)
$${}_1F_1\left[\begin{matrix}\alpha; \\ \gamma;\end{matrix}\ x\right] = e^x\ {}_1F_1\left[\begin{matrix}\gamma-\alpha; \\ \gamma;\end{matrix}\ -x\right]$$

Finally, we set $A = C = B' = D' = 0$; $B'' = 2$, $b_1'' = \beta/\alpha'$, $b_2'' = \beta'/\alpha$; $D'' = 1$, $d_1'' = \mu$, and let $x \to 0$ in (16), then we obtain an interesting transformation

$$(31)\qquad {}_3\phi_2\left[\begin{matrix}\beta,\beta',\beta\beta'/\alpha\alpha';\\ \mu,\beta\beta';\end{matrix}\ q;y\right] = \sum_{r,s\geq 0}\frac{(\beta/\alpha;q)_r(\beta'/\alpha;q)_s}{(\mu;q)_{r+s}}$$
$$\times\frac{(\alpha;q)_r(\alpha';q)_s(\beta;q)_r(\beta';q)_s\left(\frac{\beta' qy}{\alpha}\right)^r\left(\frac{\beta qy}{\alpha'}\right)^s}{(\beta\beta';q)_{r+s}(q;q)_r(q;q)_r(q;q)_s}$$
$$\times{}_2\phi_1\left[\begin{matrix}\frac{\beta q^r}{\alpha'},\ \frac{\beta' q^s}{\alpha};\\ \mu q^{r+s};\end{matrix}\ q;y\right]$$

which is q-analogue of a result given by Carlitz [3, p. 202, eq. 9], namely

$$(32)\qquad {}_3F_2\left[\begin{matrix}\beta,\beta',\beta+\beta'-\alpha-\alpha';\\ \mu,\beta+\beta';\end{matrix}\ y\right] = \sum_{r,s\geq 0}\frac{(\beta-\alpha')_r(\beta'-\alpha)_s(\alpha)_r}{(\mu)_{r+s}(\beta+\beta')_{r+s}}$$
$$\times\frac{(\alpha')_s(\beta)_r(\beta')_s y^{r+s}}{r!s!}\ {}_2F_1\left[\begin{matrix}\beta-\alpha'+r,\beta'-\alpha+s;\\ \mu+r+s;\end{matrix}\ y\right]$$

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