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THE INTERSECTION OF THREE DISTINCT LATIN SQUARES

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In this paper, we mainly solve the intersection problem of three distinct latin squares of two different types: LS and ILS, and the intersections of ICLS and CLSH can be obtained by a similar way.

1. Introduction.

A latin square of order n is an $n \times n$ array such that each of the integers $1, 2, \ldots, n$ (or any set of n distinct symbols) occurs exactly once in each row and each column. A latin square $L = [l_{i,j}]$ is called *idempotent* if $l_{i,i} = i$ for each i, and is said to be *commutative* provided that $l_{i,j} = l_{j,i}$ for each pair $\{i, j\}$. It is well known that an idempotent commutative latin square of order n exists if and only if n is odd.

Let $S = \{1, 2, \ldots, 2m\}$ and $H = \{\{1,2\}, \{3,4\}, \ldots, \{2m-1, 2m\}\}$. The 2-element subsets in H are called holes. Let $(S, *)$ be a quasigroup with the property that, for each hole $h \in H, (h, *)$ is a subquasigroup. Such a quasigroup $(S, *)$ is called a quasigroup with 2×2 holes, and

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is also referred as a latin square with 2×2 holes. Moreover, if the latin square is commutative, we have a commutative latin square with 2×2 holes (CLSH). It was shown in [1] that a CLSH exists for each even order except 4.

Two latin squares of order $n, L = [l_{i,j}$ and $M = [m_{i,j}$, are said to have k entries in common if there exist exactly k cells (i,j) such that $l_{i,j} = m_{i,j}$. When we consider the intersection of idempotent commutative latin squares (ICLS) or CLSH, we only count the common entries in the upper triangular part of the latin squares (not including the diagonal or the holes).

Let $J_1[v] = \{k|$ there exist two latin squares (LS) which have k entries in common.$\}$, and $J_2[v], J_3[v], J_4[v]$ be the corresponding definition of ILS, ICLS, and CLSH respectively. The following results are known:

(1) $J_1[v] = I_1[v] = \{0, 1, 2, \ldots, v^2 - 6, v^2 - 4, v^2\}$ for each $v \leq 5$. [3]

(2) $J_2[v] = I_2[v] = \{v, v+1, v+2, \ldots, v^2 - 6, v^2 - 4, v^2\}$ for each $v \geq 6$. [3]

(3) $$J_3[v] = \left\{0, 1, 2, \ldots, \frac{v(v-1)}{2} - 6, \frac{v(v-1)}{2} - 4, \frac{v(v-1)}{2}\right\}$$ for each old $v \geq 7$. [11]

(4) $$J_4[v] = I_4[v] = \left\{0, 1, 2, \ldots, \frac{v(v-2)}{2} - 6, \frac{v(v-2)}{2} - 4, \frac{v(v-2)}{2}\right\}$$ for each even $v \geq 10$. [2]

Since we can construct a lot of different designs by using different type of latin suqares. Hence, the intersection problems of certain designs can be tackled by counting the intersections of latin squares. [1, 4, 5] Recently, the problem of λ-fold designs with prescribed λ repeated blocks has been studied in [6, 9]. Due to the reason that the mutual intersections of three distinct latin squares can be utilized to study the 3-fold designs with prescribed 3 times repeated blocks, we are intrested in this problem.

In this paper, we mainly solve the intersection problem of three distinct latin squares of two different types: LS and ILS, and the intersection of ICLS and CLSH can be obtained by a similar way, we will not go through them.

2. The Intersections of Latin Squares.

It was shown in [3] that there exist two latin suqares of order v ($LS(v)$) which have k entries in common for each $k \in \{0, 1, 2, \ldots, v^2-6, v^2-4, v^2\}$ and $v \geq 5$. It is interesting to know the mutual intersections of three distinct latin squares.

In what follows we will denote the set of possible intersections of i distinct latin squares of jth type by $J_j^i[v]$. For example, we are dealing with $i=3$ and $j=1$ now. More clearly, let $J_1^3[v] = \{k|$ there exist three distinct latin squares of order v which have mutually k entries in common$\}$. Figure 2.1 is an example of $5 \in J_1^3$ [6].

6	1	4	3	5	2		1	6	2	5	3	4		1	6	2	5	3	4
2	6	3	4	1	5		4	2	5	1	6	3		4	2	5	1	6	3
4	2	5	1	6	3		2	4	3	6	1	5		2	4	3	6	1	5
5	4	2	6	3	1		5	3	6	4	2	1		3	5	6	4	2	1
1	3	6	5	2	4		6	1	4	3	5	2		6	1	4	3	5	2
3	5	1	2	4	6		3	5	1	2	4	6		5	3	1	2	4	6

Figure 2.1.

LEMMA 2.1. *For each* $v \geq 3$, $J_1^3[v] \subseteq I_1^3[v] = I_1[v] \setminus \{v^2-6, v^2-4, v^2\}$.

Proof. It is not difficult to see that if there exist t entries in a row (or column) which are the common entries of three distinct latin squares, then $t \neq v-1$. This implies that v^2-1, v^2-2, v^2-5 are not in $J_1^3[v]$. If v^2-4 is in $J_1^3[v]$, because v^2-3, v^2-2 and v^2-1 are not in $J_1^2[v]$, then three $LS(v)$ must be of the form as in Figure 2.2, but it is not possible. Thus $v^2-4 \notin J_1^3[v]$. Finally, assume that $v^2-6 \in J_1^3[v]$. It is a direct result that the 6 entries can be arrange in one of the three shapes in Figure 2.3. By trying all the possible cases, we conclude that $v^2-6 \in J_1^3[v]$ is not possible either. Hence we have the proof.

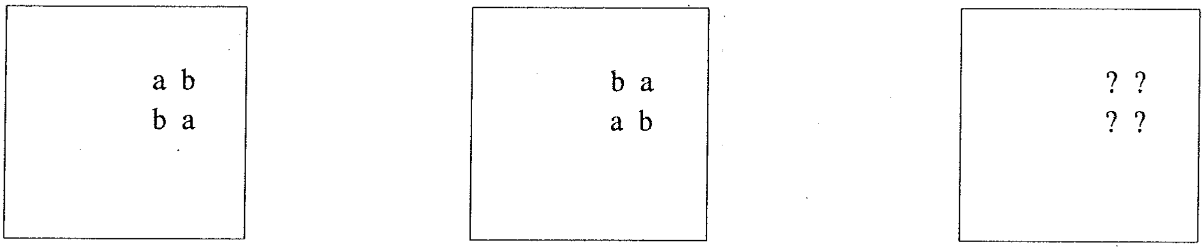

Figure 2.2.

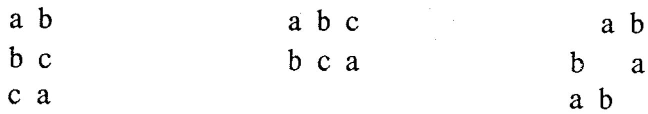

Figure 2.3.

Since $J_1^3[v] \subseteq I_1^3[v]$, in order to prove that $J_1^3[v] = I_1^3[v]$, we need to show that $I_1^3[v] \subseteq J_1^3[v]$ only.

Because that there are exactly two distinct latin squares of order 2, we will start our arguments with LS (3).

LEMMA 2.2. *$J_1^3[3] = \{0, 1\}$, $J_1^3[4] = I_1^3[4]$, and $J_1^3[5] = I_1^3[5] \backslash \{17\}$.*

Proof. By direct computing and constructions. (Appendix A).

LEMMA 2.3. $J_1^3[6] = I_1^3[6]$.

Proof. From Appendix B, we obtain $\{6, 7, 8, \ldots, 29\} \subseteq J_1^3$ [6] by using idempotent latin squares of order 6. Since a latin square of order 6 can be in the form as in Figure 2.4, and $J_1^3[3] = \{0, 1\}$, hence we have $\{0, 1, 2, 3, 4\} \subseteq J_1^3$ [6]. Furthermore $5 \in_1^3$ [6] (Figure 2.1), we conclude the proof;

LS (3) A	LS (3) B
LS (3) C	LS (3) D

A, D: Based on {1,2,3}.

B, C: Based on {4,5,6}.

Figure 2.4.

LEMMA 2.4. $J_1^3[7] = I_1^3[7]$.

Proof. From Appendix B, we obtain $\{7, 8, 9, \ldots, 42\} \subseteq J_1^3[7]$ by using ILS(7). Since a latin square of order 3 can be embedded in a latin square of order 7, (Figure 2.5) and we can permute the entries 1, 2, 3; 4, 5, 6, 7 independently, hence we have $\{0, 1, 3, 4, 5\} \subseteq J_1^3[7]$ by using Lemma 2.2 and suitable permutations. With the ararys in Figure 2.6 for B, we are able to construct three distinct latin squares of order 7 which have either 2 or 6 entries in common. Thus we are done.

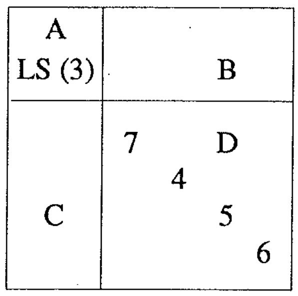

Figure 2.5.

	6	5	4	7		4	5	6	7		4	6	7	5
B_1:	4	6	7	5	B_2:	5	6	7	4	B_3:	6	5	4	7
	5	7	6	4		6	7	4	5		5	7	6	4

Figure 2.6.

It is well-known that a latin square of order v can be embedded in latin square of order u provided that $u \geq 2v$. Hence we obtain the following recursive constructions.

LEMMA 2.5. *For every $v \geq 4$, if $J_1^3[v] = I_1^3[v]$ then $J_1^3[2v] = I_1^3[2v]$ and $J_1^3[2v+1] = I_1^3[2v+1]$.*

Proof. First, we consider the latin squares of order $2v$. Since a latin square of order v can be embedded in a latin square of order $2v$, and this latin square of order $2v$ must be of the form as in Figure 2.7, hence we have $J_1^3[v] \cup \{v^2\} + J_1^3[v] \cup \{v^2\} + J_1^3[v] \cup \{v^2\} + J_1^3[v] \subseteq J_1^3[2v]$. This implies that $I_1^3[2v] \subseteq J_1^3[2v]$ whenever $J_1^3[v] = I_1^3[v]$. Thus $J_1^3[2v] = I_1^3[2v]$. For the case $2v+1$, since we can embed a latin square of order v in a latin square of order $2v+1$ and the entries $1, 2, \ldots, v; v+1, v+2, \ldots, 2v+1$ can be permuted independnetly, therefore $J_1^3[v] + \{0, v+1, 2(v+1), \ldots, (v-2)(v+1), v(v+1)\} + \{0, 2v+1, \ldots, (v-1)(2v+1), (v+1)(2v+1)\} \subseteq J_1^3[2v+1]$. This implies that $I_1^3[2v+1] \subseteq J_1^3[2v+1]$ and we have the proof.

LS (v) A	LS (v) B
LS (v) C	LS (v) D

A, D: Based on {1,2,...,v}.

B, C: Based on {v+1v+2...2v}.

Figure 2.7.

THEOREM 2.6. *$J_1^3[v] = I_1^3[v]$ for every $v \geq 6$.*

Proof. By Lemma 2.2, Lemma 2.3, Lemma 2.4, and Lemma 2.5, we have $J_1^3[v] = I_1^3[v]$ for every $v \geq 6$ provided that we can prove that $J_1^3[10] = I_1^3[10]$ and $J_1^3[11] = I_1^3[11]$. Because of the fact that we can embed a latin square of order 4 in a latin square of order 10 (or 11), we have $92 \in J_1^3[10]$ (or $113 \in J_1^3[11]$). Moreover, $I_1^3[10]\backslash\{92\} \subseteq J_1^3[10]$ can be obtained by embedding a latin square of order 5 in a latin square of order 10 and $I_1^3[11]\backslash\{113\} \subseteq J_1^3[11]$ can be obtained by embedding a latin square of order 5 in a latin square of oreder 11. Thus we have the proof.

3. The Intersections of Idempotent Latin Squares.

The intersection problem of two idempotent latin squares has been solved by Fu in [3]. In this section we will study the mutual intersections of three distinct idempotent latin squares. By the same reason as we mentioned in Lemma 2.1, we have the following lemma.

LEMMA 3.1. $J_2^3[v] \subseteq I_2^3[v] = I_2[v]\backslash\{v^2-6, v^2-4, v^2\}$.

Since there are exactly one idempotent latin square of order 3 and two idempotent latin squares of order 4, we start with ILS(5). We note here that in order to show $J_2^3[v] = I_2^3[v]$, it suffices to prove that $I_2^3[v] \subseteq J_2^3[v]$.

LEMMA 3.2. $J_2^3[5] = \{5, 6, 7, \ldots, 13\}$.

Proof. See Appendix B.

LEMMA 3.3. *$J_2^3[6] = I_2^3[6], J_2^3[7]$, and $J_2^3[8] = I_2^3[8]$.*

Proof. See Appendix B.

LEMMA 3.4. *$J_2^3[9] = I_2^3[9], J_2^3[10] = I_2^3[10]$, and $J_2^3[11] = I_2^3[11]$.*

Proof. It is well known that an ILS(v) can be embedded in an ILS(U) for each $u \geq 2v+1$. Thus, we can obtain the intersections of three ILS(u) by way of the intersections in their corresponding parts: A, B, C, and D (Figure 2.8). We note here that we may use the same A's (or B's or C's or D's) provided the other parts are distinct. First, let $u=9$ and $v=4$. The intersections on B (similarly C) are actually the intersections of three 4×5 latin rectangles where i is missing in the ith column, $i = 1, 2, \ldots, 5$. In Appendix C, we have the intersections of three distinct such 4×5 latin rectangles, which are $0, 1, 2, \ldots, 11, 13$. Then we have $(\{4,16\} + \{0,1,2,\ldots,11,13,20\} + \{0,1,2,\ldots,11,13,20\} + \{5,10,15,25\}) \backslash \{81\} \subseteq J_2^3[9]$, i.e;, $\{9,10,\ldots,72,74\} \subseteq J_2^3[9]$. Since $73 \in J_2^3[9]$ can be obtained by letting L_1 and L_2 have exactly four different entries in B, L_2 and L_3 have exactly four different entries in part C, hence we have $J_2^3[9] = I_2^3[9]$. For the case $u=11$ and $v=5$, the proof of $J_2^3[11] = I_2^3[11]$ is similar to the above except we will use 5×6 rectangles in B and the intersections can be found in Appendix C. Finally, we consider $u=10$ and $v=4$. It is not difficult to see that the intersections of the cells in B and D which are filled with $5,6,7,8,9,10$ are equal to the intersections of a latin square with its first row fixed as $(5,6,7,8,9,10)$;, Thus we can use the intersections of three 5×6 rectangles as in Appendix C. In the part C, we will consider the intersections of three 6×4 rectangles with two fixed columns and this is the same as the intersections of three 4×6 rectangles. Hence we conclude that $(\{4,16\}+\{0,1,2,\ldots,23,30\}+\{0,1,2,\ldots,17,24\}+\{3,12,18,30\}) \backslash \{100\} \subseteq J_2^3[10]$, i.e., $I_2^3[10] \subseteq J_2^3[10]$.

Q.E.D.

LEMMA 3.5. $J_2^3[12] = I_2^3[12]$.

ILS (v) A	B
C	D

Figure 2.8.

Proof. We first consider a special type of idempotent latin square of order 12. (Figure 2.9) It is not difficult to see that by Lemma 2.2 and the two ILS(4) have exactly 4 entries (diagonal) in common we have $\{4,16\}+\{4,16\}+\{4,16\}+\{0,1,2,\ldots,9\}+\{0,1,2,\ldots,9\}+\{0,1,2,\ldots,9\}+\{0,1,2,\ldots,9\}+\{0,1,2,\ldots,9\}+\{0,1,2,\ldots,9\}\subseteq J_2^3[12]$, i.e., $\{12,13,14,\ldots,102\}\subseteq J_2^3[12]$. By the fact that we can select the same LS(4) in A_2, A_3, B_2, B_3 and C_2, C_3 (not all of them), we have $\{103,104,\ldots,137\}\subseteq J_2^3[12]$. Thus we have the proof.

ILS (4) A_1	LS (4) C_2	LS (4) B_2
LS (4) C_3	ILS (4) B_1	LS (4) A_2
LS (4) B_3	LS (4) A_3	ILS (4) C_1

A_1,A_2,A_3: Based on $\{1,2,3,4\}$.

B_1,B_2,B_3: Based on $\{5,6,7,8\}$.

C_1,C_2,C_3: Based on $\{9,10,11,12\}$.

Figure 2.9.

Before we prove that $J_2^3[v]=I_2^3[v]$ for each $v\geq 6$, we need the following lemmas.

LEMMA 3.6. *If* $J_2^3[v] = I_2^3[v]$ *and* $v \geq 6$, *then* $J_2^3[2v+1] = I_2^3[2v+1]$.

Proof. Since an idempotent latin square of order v can be embedded in an idempotent latin square of order $2v+1$, we have $J_2^3[v]+\{0, v+1, 2(v+1), \ldots, (v-2)(v+1), v(v+1)\}+\{0, v+1, 2(v+1), \ldots, (v-2)(v+1), v(v+1)\}+\{v+1, 2(v+1), \ldots, (v-1)(v+1), (v+1)^2\} \subseteq J_2^3[2v+1]$ by suitably selecting distinct ILS(v), permuting the rows of B, permuting the columns of C, and permuting the entries $1, 2, \ldots, v$ in D. (Figure 2.10) Thus we conclude the proof.

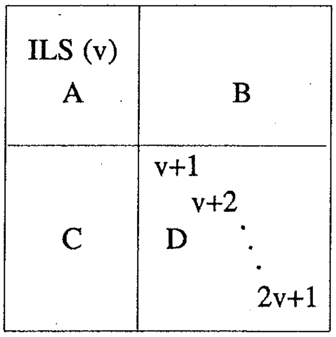

Figure 2.10.

LEMMA 3.7. *If* $J_2^3[v] = I_2^3[v]$ *and* $v \geq 6$, *then* $J_2^3[2v+2] = I_2^3[2v+2]$.

Proof. It is well-known that an ILS(v) can be embedded in an ILS($2v+2$). (Figure 2.11.) By a similar argument as in Lemma 2.5, we have $J_2^3[v]+\{0, v+2, 2(v+2), \ldots, (v-2)(v+2), v(v+2)\}+\{0, v+2, 2(v+2), \ldots, (v-2)(v+2), v(v+2)\}+\{2(v+2)\}+\{0, v+2, \ldots, (v-2)(v+2), v(v+2)\} \subseteq J_2^3[2v+2]$, i.e., $\{3v+4, 3v+5, \ldots, (2v+2)^2-7\} \subseteq J_2^3[2v+2]$. In [3], it was shown that there exists a pair of ILS($2v+2$) which have an idempotent latin subsquare of order v and they have exactly $v+2$ entries in common outside of the subsquare. By letting one of the two ILS($2v+2$) constructed as above be the third ILS($2v+2$) and suitably replace the subsquare with distinct ILS(v), then we have $\{v+2\}+J_2^3[v] \subseteq J_2^3[2v+2]$. Thus $I_2^3[2v+2] \subseteq J_2^3[2v+2]$ and this concludes the proof.

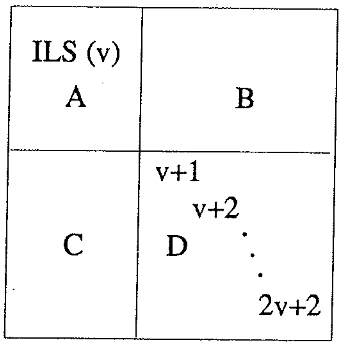

Figure 2.11.

THEOREM 3.8. *$J_2^3[v] = I_2^3[v]$ for each $v \geq 6$.*

Proof. By Lemma 3.2, Lemma 3.4, Lemma 3.5, Lemma 3.6, and Lemma 3.7.

4. Remarks.

With the results in section 1 and section 2, we can easily obtain the mutual intersections of three distinct half-idempotent latin squares which can be referred to [2]. Also, by a similar technique we should be able to obtain the intersections of three ICLS and CLSH. As to the applications of λ-fold designs with λ prescribed repeated blocks, we simply collect λ designs which have k blocks in common mutually, then these k blocks are repeated λ times. Particularly, from the results of this paper, we are able to construct certain 3-fold designs with a prescribed number of three times repeated blocks.

Appendix A

No.	of latin	sq. (3)	No. of entries in common
3	2	1	1
5	4	3	0

No.	entries of latin square (3)								
1	1	2	3	2	3	1	3	1	2
2	1	2	3	3	1	2	2	3	1
3	1	3	2	2	1	3	3	2	1
4	1	3	2	3	2	1	2	1	3
5	2	1	3	1	3	2	3	2	1
6	2	1	3	3	2	1	1	3	2
7	2	3	1	1	2	3	3	1	2
8	2	3	1	3	1	2	1	2	3
9	3	1	2	1	2	3	2	3	1
10	3	1	2	2	3	1	1	2	3
11	3	2	1	1	3	2	2	1	3
12	3	2	1	2	1	3	1	3	2

No.	of latin	sq. (4)	No. of entries in common
3	2	13	10
6	5	13	1
10	9	13	2
12	8	13	3
12	11	13	4
7	6	5	5
5	4	3	6
7	5	3	7
3	2	1	8
5	3	1	9

Appendix A (*Continued*)

No.	entries of latin square (3)															
1	1	2	3	4	2	1	4	3	3	4	1	2	4	3	2	1
2	1	2	3	4	2	1	4	3	3	4	2	1	4	3	1	2
3	1	2	3	4	2	1	4	3	4	3	1	2	3	4	2	1
4	1	2	3	4	2	1	4	3	4	3	2	1	3	4	1	2
5	1	2	3	4	2	3	4	1	3	4	1	2	4	1	2	3
6	1	2	3	4	2	3	4	1	4	1	2	3	3	4	1	2
7	1	2	3	4	2	4	1	3	3	1	4	2	4	3	2	1
8	1	2	3	4	3	1	4	2	4	3	2	1	2	4	1	3
9	1	2	3	4	3	4	1	2	2	1	4	3	4	3	2	1
10	1	2	3	4	3	4	1	2	2	3	4	1	4	1	2	3
11	1	2	3	4	3	4	1	2	4	1	2	3	2	3	4	1
12	1	2	3	4	3	4	1	2	4	3	2	1	2	1	4	3
13	4	3	1	2	3	4	2	1	1	2	3	4	2	1	4	3

No.	of latin	sq. (4)	No. of entries in common
3	2	21	0
5	4	21	1
3	2	20	2
5	4	20	3
4	2	20	4
7	5	20	5
16	15	10	6
13	12	11	7
6	2	19	8
3	2	19	9
5	2	19	10
5	4	1	11
5	4	3	12
3	2	1	13
16	14	1	14
9	8	19	15
17	2	22	16
18	17	22	18

Appendix A (*Continued*)

No.	entries of latin square (3)
1	1 2 3 4 5 2 1 4 5 3 3 4 5 1 2 4 5 2 3 1 5 3 1 2 4
2	1 2 3 4 5 2 1 4 5 3 3 4 5 1 2 5 3 1 2 4 4 5 2 3 1
3	1 2 3 4 5 2 1 4 5 3 3 4 5 2 1 4 5 1 3 2 5 3 2 1 4
4	1 2 3 4 5 2 1 4 5 3 3 4 5 2 1 5 3 2 1 4 4 5 1 3 2
5	1 2 3 4 5 2 1 4 5 3 3 5 1 2 4 4 3 5 1 2 5 4 2 3 1
6	1 2 3 4 5 2 1 4 5 3 3 5 1 2 4 5 4 2 3 1 4 3 5 1 2
7	1 2 3 4 5 2 1 4 5 3 3 5 2 1 4 4 3 5 2 1 5 4 1 3 2
8	1 2 3 4 5 2 1 4 5 3 4 5 1 3 2 3 4 5 2 1 5 3 2 1 4
9	1 2 3 4 5 2 1 4 5 3 4 5 2 3 1 3 4 5 1 2 5 3 1 2 4
10	1 2 3 4 5 2 1 4 5 3 5 3 1 2 4 3 4 5 1 2 4 5 2 3 1
11	1 2 3 4 5 2 1 4 5 3 5 3 2 1 4 3 4 5 2 1 4 5 1 3 2
12	1 2 3 4 5 2 1 4 5 3 5 4 2 3 1 4 3 5 1 2 3 4 1 2 4
13	1 2 3 4 5 2 1 5 5 4 3 4 1 5 2 4 5 2 1 3 5 3 4 2 1
14	1 2 3 4 5 2 1 5 5 4 3 5 4 1 2 4 3 2 5 1 5 4 1 2 3
15	1 2 3 4 5 2 1 5 3 4 5 4 2 1 3 4 2 1 5 2 3 5 4 2 1
16	1 2 3 4 5 2 3 1 5 4 3 4 5 1 2 4 5 2 3 1 5 1 4 2 3
17	1 2 3 4 5 2 3 1 5 4 3 4 5 1 2 5 1 4 2 3 4 5 2 3 1
18	1 2 3 4 5 2 3 5 1 4 3 4 1 5 2 5 1 4 2 3 4 5 2 3 1
19	1 2 3 4 5 2 3 4 5 1 4 5 1 2 3 3 4 5 1 2 5 1 2 3 4
20	5 4 3 2 1 4 3 2 1 5 3 2 1 5 4 2 1 5 4 3 1 5 4 3 2
21	5 4 2 3 1 4 5 3 1 2 2 1 4 5 3 1 3 5 2 4 3 2 1 4 5
22	1 2 3 5 4 2 3 1 4 5 3 4 5 1 2 5 1 4 2 3 4 5 2 3 1

Appendix B

No.	entries of latin sq. (5) outside diagonal
1	3 2 5 4 4 5 1 3 5 4 2 1 3 5 1 2 2 1 4 3
2	3 2 5 4 4 5 3 1 5 4 1 2 2 5 1 3 3 1 4 2
3	3 2 5 4 5 4 1 3 4 5 2 1 3 1 5 2 2 4 1 3
4	3 2 5 4 5 4 3 1 4 5 1 2 2 1 5 3 3 4 1 2
5	3 4 5 2 3 5 1 4 4 5 2 1 5 1 2 3 2 4 1 3
6	3 4 5 2 3 5 1 4 5 4 2 1 2 5 1 3 4 1 2 3
7	3 4 5 2 4 5 1 3 5 1 2 4 3 5 2 1 2 4 1 3
8	3 4 5 2 4 5 3 1 2 5 1 4 5 1 2 3 3 4 1 2
9	3 4 5 2 4 5 3 1 5 1 2 4 2 5 1 3 3 4 2 1
10	3 4 5 2 5 1 3 4 4 5 2 1 2 1 5 3 3 4 2 1
11	3 5 2 4 3 4 5 1 4 5 1 2 5 1 2 3 2 4 1 3
12	3 5 2 4 3 4 5 1 5 4 1 2 2 5 1 3 4 1 2 3
13	3 5 2 4 4 1 5 3 5 4 1 2 3 5 2 1 2 1 4 3
14	3 5 2 4 5 4 1 3 2 4 5 1 3 5 1 2 4 1 2 3
15	3 5 2 4 5 4 1 3 4 1 5 2 3 5 2 1 2 4 1 3
16	3 5 2 4 5 4 3 1 2 1 5 2 2 5 1 3 3 4 2 1
17	4 2 5 3 3 5 1 4 4 5 2 1 5 3 1 2 2 1 4 3
18	4 2 5 3 4 5 3 1 2 5 1 4 5 3 1 2 3 1 4 2
19	4 2 5 3 4 5 3 1 5 1 2 4 3 5 1 2 2 3 4 1
20	4 2 5 3 5 1 3 4 4 5 1 2 2 3 5 1 3 1 4 2
21	4 2 5 3 5 1 3 4 4 5 2 1 3 1 5 2 2 3 4 1
22	4 2 5 3 5 4 3 1 2 5 1 4 3 1 5 2 4 3 1 2
23	4 5 2 3 3 1 5 4 4 5 1 2 5 3 2 1 2 1 4 3
24	4 5 2 3 3 4 5 1 2 5 1 4 5 3 1 2 4 1 2 3
25	4 5 2 3 5 1 3 4 4 1 5 2 3 5 2 1 2 3 4 1
26	4 5 2 3 5 4 3 1 2 1 5 4 3 5 1 2 4 3 2 1
27	4 5 3 2 3 1 5 4 4 5 2 1 5 1 2 3 2 3 4 1
28	4 5 3 2 3 4 5 1 2 5 1 4 5 1 2 3 4 3 1 2
29	4 5 3 2 3 4 5 1 5 1 2 4 2 5 1 3 4 3 2 1
30	4 5 3 2 4 1 5 3 2 5 1 4 5 3 2 1 3 1 4 2
31	4 5 3 2 4 1 5 3 5 1 2 4 3 5 2 1 2 3 4 1
32	4 5 3 2 5 4 1 3 2 1 5 4 3 5 2 1 4 3 1 2
33	5 2 3 4 3 4 5 1 5 4 1 2 2 1 5 3 4 3 1 2
34	5 2 3 4 4 1 5 3 5 4 1 2 2 3 5 1 3 1 4 2
35	5 2 3 4 4 1 5 3 5 4 2 1 3 1 5 2 2 3 4 1
36	5 2 3 4 4 5 1 3 2 4 5 1 5 3 1 2 3 1 4 2
37	5 2 3 4 5 4 1 3 2 4 5 1 3 1 5 2 4 3 1 2
38	5 2 3 4 5 4 1 3 4 1 5 2 2 3 5 1 3 4 1 2
39	5 4 2 3 3 1 5 4 5 4 1 2 2 3 5 1 4 1 2 3
40	5 4 2 3 3 5 1 4 2 4 5 1 5 3 1 2 4 1 2 3
41	5 4 2 3 3 5 1 4 4 1 5 2 5 3 2 1 2 4 1 3
42	5 4 2 3 4 5 3 1 2 1 5 4 5 3 1 2 3 4 2 1
43	5 4 2 3 5 1 3 4 2 4 5 1 3 1 5 2 4 3 2 1
44	5 4 2 3 5 1 3 4 4 1 5 2 2 3 5 1 3 4 2 1
45	5 4 3 2 3 1 5 4 5 4 2 1 2 1 5 3 4 3 2 1
46	5 4 3 2 3 5 1 4 2 4 5 1 5 1 2 3 4 3 1 2
47	5 4 3 2 4 1 5 3 5 1 2 4 2 3 5 1 3 4 2 1
48	5 4 3 2 4 5 1 3 2 1 5 4 5 3 2 1 3 4 1 2

Appendix B (*Continued*)

No.	of Id. latin	sq. (5)	No. of entries in common outside diagonal
17	16	13	0
11	10	1	1
5	4	1	2
5	4	2	3
3	2	1	4
7	6	3	5
6	5	3	6
5	4	3	7
6	2	1	8

No.	of Id. latin	sq. (6)	No. of entries in common outside diagonal
21	13	18	0
9	8	18	1
7	3	18	2
3	2	18	3
5	3	18	4
7	6	18	5
6	4	18	6
12	11	10	7
12	9	7	8
12	10	7	9
12	10	8	10
7	5	1	11
7	6	5	12
5	4	1	13
7	4	2	14
4	3	1	15
7	6	4	16
3	2	1	17
4	3	2	18
5	3	2	19
19	17	16	20
15	3	2	21
20	14	13	22
8	3	2	23

Appendix B (*Continued*)

No.	entries of latin sq. (5) outside diagonal
1	3 2 5 6 4 3 4 6 1 5 5 6 1 4 2 2 5 6 3 1 6 4 1 2 3 4 1 5 3 2
2	3 2 5 6 4 3 4 6 1 5 5 6 1 4 2 6 1 5 2 3 2 4 6 3 1 4 5 1 2 3
3	3 2 5 6 4 3 4 6 1 5 5 6 1 4 2 6 5 1 2 3 2 4 6 3 1 4 1 5 2 3
4	3 2 5 6 4 3 4 6 1 5 5 6 2 4 1 3 1 5 3 2 2 4 3 1 3 4 5 1 3 2
5	3 2 5 6 4 3 4 6 1 5 5 6 2 4 1 6 5 1 2 3 4 1 6 3 2 2 4 5 1 3
6	3 2 5 6 4 3 4 6 1 5 5 6 2 4 1 6 5 1 3 2 2 4 6 1 3 4 1 5 3 2
7	3 2 5 6 4 3 4 6 1 5 6 5 1 4 2 2 6 5 3 1 4 1 6 2 3 5 4 1 3 2
8	3 2 5 6 4 3 4 6 1 5 6 5 1 4 2 5 6 1 2 3 2 4 6 3 1 4 1 5 2 3
9	3 2 5 6 4 3 4 6 1 5 6 5 2 4 1 5 1 6 2 3 4 6 1 3 2 2 4 5 1 3
10	3 2 5 6 4 3 4 6 1 5 6 5 2 4 1 5 6 1 2 3 4 1 6 3 2 2 4 5 1 3
11	3 2 5 6 4 3 4 6 1 5 6 5 2 4 1 5 6 1 3 2 2 4 6 1 3 4 1 5 3 2
12	3 2 5 6 4 3 5 6 4 1 4 6 1 2 5 2 5 6 1 3 6 1 4 3 2 5 4 1 2 3
13	3 2 5 6 4 3 6 1 4 5 5 4 6 1 2 6 1 5 2 3 2 6 4 3 1 4 5 1 2 3
14	3 2 5 6 4 3 6 1 4 5 5 4 6 1 2 6 5 1 2 3 2 6 4 3 1 4 1 5 2 3
15	3 2 5 6 4 4 1 6 3 5 5 6 1 4 2 6 1 5 2 3 2 4 6 3 1 3 5 4 2 1
16	3 2 5 6 4 4 1 6 3 5 6 5 1 4 2 2 6 5 1 3 3 4 6 2 1 5 1 4 3 2
17	3 2 5 6 4 4 1 6 3 5 6 5 2 4 1 2 6 5 1 3 3 4 6 1 2 5 1 4 3 2
18	6 5 3 4 2 3 1 5 6 4 2 4 6 1 5 6 5 2 3 1 4 1 6 2 3 5 3 4 1 2
19	3 2 5 6 4 4 5 6 1 3 6 5 1 4 2 2 6 1 3 5 3 4 6 2 1 5 1 4 3 2
20	3 2 5 6 4 4 6 1 3 5 5 4 6 1 2 4 1 5 2 3 2 6 4 3 1 3 5 1 2 4
21	3 2 5 6 4 4 1 6 3 5 5 6 1 4 2 3 5 6 2 1 6 1 4 2 3 2 4 5 3 1

Appendix B(*Continued*)

No.	of Id. latin	sq. (7)	No. of entries in common outside diagonal
13	8	25	0
3	2	25	1
4	2	25	2
6	5	25	3
5	4	25	4
6	4	24	5
3	2	24	6
4	3	24	7
9	3	24	8
21	21	15	9
20	11	15	10
18	12	15	11
17	10	15	13
10	8	15	14
5	4	15	15
3	2	23	16
11	10	23	17
4	2	23	18
4	3	23	19
14	13	15	20
4	3	15	21
13	3	15	22
19	16	10	23
14	12	6	24
17	16	10	25
10	8	6	26
17	14	10	27
9	8	6	28
7	6	4	29
17	14	13	30
5	4	,	31
17	7	2	32
7	5	2	33
3	2	1	34
5	2	1	35

Appendix B (*Continued*)

No.	entries of latin sq. (7) outside diagonal
1	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 1 3 2 7 4 5 2 3 1 6 5 4 3 2 1
2	3 2 5 4 4 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 1 3 2 7 4 5 3 2 1 6 5 4 2 3 1
3	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 1 3 2 7 5 4 2 3 1 6 4 5 3 2 1
4	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 1 3 2 7 5 4 3 2 1 6 4 5 2 3 1
5	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 2 3 1 7 4 5 1 3 2 6 5 4 3 2 1
6	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 2 3 1 7 5 4 1 3 2 6 4 5 3 2 1
7	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 3 1 2 7 4 5 2 3 1 6 5 4 1 2 3
8	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 4 7 6 3 1 2 7 5 4 2 3 1 6 4 5 1 2 3
9	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 6 7 4 1 3 2 7 4 5 2 3 1 4 5 6 3 2 1
10	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 6 7 4 2 3 2 7 4 5 3 2 1 4 5 6 2 3 1
11	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 6 7 4 2 3 1 7 4 5 1 3 2 4 5 6 3 2 1
12	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 6 7 4 3 1 2 7 4 5 2 3 1 4 5 6 1 2 3
13	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 7 4 6 1 3 2 4 7 5 2 3 1 6 5 4 3 2 1
14	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 7 4 6 1 3 2 4 7 5 3 2 1 6 5 4 2 3 1
15	3 2 5 7 4 6 3 1 7 6 5 4 2 1 6 4 7 5 5 7 6 1 2 3 7 6 4 1 3 2 4 5 7 2 3 1 6 4 5 3 2 1
16	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 2 3 7 4 6 3 1 2 4 7 5 2 3 1 6 5 4 1 2 3
17	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 3 2 4 7 6 1 2 3 7 4 5 2 3 1 6 5 4 3 2 1
18	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 3 2 4 7 6 1 2 3 7 4 5 3 2 1 6 5 4 2 3 1
19	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 3 2 4 7 6 1 2 3 7 5 4 2 3 1 6 4 5 3 2 1
20	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 3 2 4 7 6 2 1 3 7 4 5 3 2 1 6 5 4 1 3 2
21	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 1 3 2 6 7 4 3 2 1 7 4 5 1 2 3 4 5 6 2 3 1
22	3 2 5 4 7 6 3 1 6 7 4 5 2 1 7 6 5 4 5 6 7 2 1 3 4 7 6 1 3 2 7 4 5 2 3 1 6 5 4 3 1 2
23	1 2 5 4 7 6 3 1 6 7 4 5 5 6 7 1 2 4 7 1 6 3 5 2 6 7 4 2 1 3 4 5 7 3 2 1 2 4 5 1 6 3
24	5 6 7 3 2 4 5 4 1 7 3 6 6 4 2 1 7 5 7 1 2 6 5 3 3 7 1 6 4 2 2 3 7 5 4 1 4 6 5 3 2 1
25	5 6 7 3 2 4 5 2 1 6 7 3 6 4 2 7 1 5 3 7 1 2 5 6 2 3 7 6 4 1 7 1 5 3 4 2 4 6 2 5 1 3

Appendix B (*Continued*)

No.	of Id. latin	sq. (8)	No. of entries in common outside diagonal
11	8	27	0
6	4	27	1
11	10	23	2
11	8	23	3
11	3	23	4
6	5	23	5
6	3	23	6
4	3	23	7
3	2	23	8
5	4	23	9
3	2	24	10
14	12	24	11
12	11	24	12
17	14	24	13
20	19	24	14
22	21	24	15
21	20	24	16
15	12	25	17
4	3	25	18
4	2	25	19
5	4	25	20
8	7	25	21
3	2	25	22
9	6	25	22
10	9	25	24
8	6	26	25
3	2	25	22
9	6	25	23
10	9	25	24
8	6	26	25
6	5	26	26
6	2	26	27
4	2	26	28
4	3	26	29
11	9	8	30
11	10	8	31
11	10	5	32
11	10	6	34
6	5	4	35
6	5	2	36
7	4	1	37
6	5	3	38
4	3	2	39
4	2	1	40
12	3	2	41
5	3	2	42
13	10	1	43
6	3	1	44
6	3	2	45
9	6	3	46
9	3	2	47
18	16	15	48
3	2	1	49

Appendix B (*Continued*)

No.	entries of Id. latin sq. (8) outside diagonal
1	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 1 2 3 7 8 4 1 3 5 2 8 4 5 2 6 3 1 6 5 7 3 2 4 1
2	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 1 2 3 7 8 4 2 3 5 1 8 4 5 1 6 3 2 6 5 7 3 2 4 1
3	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 1 2 3 7 8 4 3 2 5 1 8 4 5 1 6 3 2 6 5 7 2 3 4 1
4	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 1 2 3 8 4 7 1 3 5 2 6 8 5 3 2 4 1 7 5 4 2 6 3 1
5	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 1 2 3 8 4 7 3 2 5 1 6 8 5 1 3 4 2 7 5 4 2 6 3 1
6	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 3 1 2 7 8 4 3 2 5 1 8 4 5 2 6 1 3 6 5 7 1 3 4 2
7	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 3 1 2 8 4 7 1 2 5 3 6 8 5 2 3 4 1 7 5 4 3 6 1 2
8	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 3 1 2 8 4 7 2 3 5 1 6 8 5 1 2 4 3 7 5 4 3 6 1 2
9	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 3 2 1 7 8 4 1 2 5 3 8 4 5 3 6 1 2 6 5 7 2 3 4 1
10	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 4 7 6 8 3 2 1 7 8 4 1 3 5 2 8 4 5 2 6 1 3 6 5 7 3 2 4 1
11	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 4 7 8 1 2 3 7 8 4 1 3 5 2 8 5 6 3 2 4 1 4 7 5 2 6 3 1
12	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 4 7 8 1 2 3 7 8 4 3 2 5 1 8 5 6 1 3 4 2 4 7 5 2 6 3 1
13	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 4 7 8 1 2 3 8 7 4 1 3 5 2 4 8 5 2 6 3 1 7 5 6 3 2 4 1
14	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 4 7 8 1 2 3 8 7 4 3 2 5 1 4 8 5 1 6 3 2 7 5 6 2 3 4 1
15	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 7 4 8 1 2 3 4 8 7 1 3 5 2 8 4 5 2 6 3 1 7 5 6 3 2 4 1
16	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 7 4 8 1 2 3 4 8 7 2 3 5 1 8 4 5 1 6 3 2 7 5 6 3 2 4 1
17	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 7 4 8 1 2 3 4 8 7 3 2 5 1 8 4 5 1 6 3 2 7 5 6 2 3 4 1
18	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 7 4 8 1 2 3 8 4 7 1 3 5 2 4 8 5 2 6 3 1 7 5 6 3 2 4 1
19	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 8 7 1 4 2 3 4 7 5 8 3 1 2 8 5 4 2 6 3 1 7 4 6 3 2 1 5
20	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 8 7 1 4 2 3 7 4 5 8 3 1 2 8 5 4 2 6 3 1 4 7 6 3 2 1 5
21	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 8 7 2 4 1 3 7 4 5 8 3 2 1 8 5 4 1 6 3 2 4 7 6 3 2 1 5
22	3 2 5 4 7 8 6 3 1 6 7 8 4 5 2 1 7 8 5 6 4 5 6 8 1 2 3 7 6 8 7 2 4 1 3 7 4 5 8 3 2 1 8 5 4 3 6 1 2 4 7 6 1 2 3 5
23	8 7 6 4 5 3 2 8 6 5 3 4 1 7 7 6 1 2 8 5 4 6 5 2 7 3 8 1 4 7 8 2 1 6 3 2 3 1 7 8 4 5 3 4 5 8 1 2 6 5 1 4 3 6 7 2
24	3 4 5 6 8 2 7 4 8 7 3 5 1 6 5 7 8 4 2 6 1 8 6 2 1 7 3 5 6 8 7 3 1 4 2 7 4 5 1 2 8 3 2 5 1 6 8 3 4 3 1 6 2 7 4 5
25	3 2 5 4 7 8 6 4 1 6 7 8 3 5 7 8 1 2 5 6 4 8 5 6 1 3 2 7 3 6 8 7 2 4 1 5 7 4 8 3 1 2 6 4 5 2 8 1 3 2 1 7 3 6 4 5
26	3 2 5 4 7 8 6 3 1 6 7 8 4 5 7 8 1 2 5 6 4 8 5 6 1 2 3 7 4 6 8 7 1 2 3 2 7 4 3 8 5 1 6 1 5 8 3 4 2 5 4 7 2 6 3 1
27	4 5 7 8 3 6 2 4 7 8 6 1 5 3 8 7 6 4 2 1 5 6 3 2 7 5 8 1 3 6 8 1 7 2 4 5 8 1 3 2 4 7 2 1 4 5 3 8 6 7 5 6 2 1 4 3

Appendix C

No. of latin sq. (5) (with the first row fixed)			No. of entries in common
1	2	7	5
18	17	10	6
15	14	11	7
15	14	13	8
15	14	12	9
9	8	6	10
5	4	1	11
5	4	3	12
3	2	1	13
18	16	1	14
21	20	19	15
22	21	19	16
23	22	21	18

No.	entries of latin sq. (5) outside diagonal																								
1	1	2	3	4	5	2	1	4	5	3	3	4	5	1	2	4	5	2	3	1	5	3	1	2	4
2	1	2	3	4	5	2	1	4	5	3	3	4	5	1	2	5	3	1	2	4	4	5	2	3	1
3	1	2	3	4	5	2	1	4	5	3	3	4	5	2	1	4	5	1	3	2	5	3	2	1	4
4	1	2	3	4	5	2	1	4	5	3	3	4	5	2	1	5	3	2	1	4	4	5	1	3	2
5	1	2	3	4	5	2	1	4	5	3	3	5	1	2	4	4	3	5	1	2	5	4	2	3	1
6	1	2	3	4	5	2	1	4	5	3	3	5	1	2	4	5	4	2	3	1	4	3	5	1	2
7	1	2	3	4	5	2	4	5	1	2	2	1	4	5	3	4	5	2	3	1	5	3	1	2	4
8	1	2	3	4	5	2	1	4	5	3	3	5	2	1	4	5	4	1	3	2	4	3	5	2	1
9	1	2	3	4	5	2	1	4	5	3	4	3	5	1	2	3	5	1	2	4	5	4	2	3	1
10	1	2	3	4	5	2	1	4	5	3	5	3	1	2	4	3	4	5	1	2	4	5	2	3	1
11	1	2	3	4	5	2	1	4	5	3	5	3	2	1	4	3	4	5	2	1	4	5	1	3	2
12	1	2	3	4	5	2	1	4	5	3	5	4	1	3	2	4	3	5	2	1	3	5	2	1	4
13	1	2	3	4	5	2	1	4	5	3	5	4	2	3	1	3	5	1	2	4	4	3	5	1	2
14	1	2	3	4	5	2	1	4	5	3	5	4	2	3	1	4	3	5	1	2	3	5	1	2	4
15	1	2	3	4	5	2	1	5	3	4	3	4	1	5	2	4	5	2	1	3	5	3	4	2	1
16	1	2	3	4	5	2	1	5	3	4	3	5	4	1	2	4	3	2	5	1	5	4	1	2	3
17	1	2	3	4	5	2	1	5	3	4	5	4	2	1	3	4	3	1	5	2	3	5	4	2	1
18	1	2	3	4	5	2	3	1	5	4	3	4	5	1	2	4	5	2	3	1	5	1	4	2	3
19	1	2	3	4	5	2	3	1	5	4	3	4	5	2	1	5	1	4	3	2	4	5	2	1	3
20	1	2	3	4	5	2	3	1	5	4	3	5	4	2	1	4	1	5	3	2	5	4	2	1	3
21	1	2	3	4	5	2	3	4	5	1	3	1	5	2	4	4	5	1	3	2	5	4	2	1	3
22	1	2	3	4	5	2	3	4	5	1	3	1	5	2	4	5	4	1	3	2	4	5	2	1	3
23	1	2	3	4	5	2	3	4	5	1	3	5	1	2	4	4	1	5	3	2	5	4	2	1	3

Appendix C (*Continued*)

No. of latin sq. (6) (with the first and second row fixed except #24)			No. of entries in common
24	23	22	6
24	12	10	7
24	13	10	8
24	14	3	9
24	20	3	10
24	20	19	11
21	20	1	12
20	21	23	13
20	21	22	14
16	15	12	15
16	15	13	16
16	15	14	17
11	10	9	18
8	7	6	19
8	7	5	20
5	4	3	21
3	2	1	22
16	6	5	23
8	7	2	24
16	3	1	25
19	18	5	26
20	18	16	27
5	3	1	28
19	18	16	29

Appendix C (*Continued*)

No.	entries of latin sq. (5) outside diagonal
1	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 1 2 3 5 6 1 2 3 4 6 4 5 3 1 2
2	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 1 2 3 6 4 5 3 1 2 5 6 1 2 3 4
3	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 1 3 2 5 6 1 3 2 4 6 4 5 2 1 3
4	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 1 3 2 6 4 5 2 1 3 5 6 1 3 2 4
5	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 2 1 3 5 6 1 3 2 4 6 4 5 1 3 2
6	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 3 1 2 5 6 1 2 3 4 6 4 5 1 2 3
7	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 5 6 3 1 2 6 4 5 1 2 3 5 6 1 2 3 4
8	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 6 5 1 2 3 5 4 6 3 1 2 6 5 1 2 3 4
9	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 6 5 3 1 2 5 4 6 1 2 3 6 5 1 2 3 4
10	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 4 6 5 3 1 2 6 5 1 2 3 4 5 4 6 1 2 3
11	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 5 4 6 1 2 3 4 6 5 3 1 2 6 5 1 2 3 4
12	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 6 4 5 2 1 3 4 5 6 1 3 2 5 6 1 3 2 4
13	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 6 5 1 3 2 4 4 6 5 1 3 2 5 4 6 2 1 3
14	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 6 5 1 3 2 4 5 4 6 1 3 2 4 6 5 2 1 3
15	1 2 3 4 5 6 2 3 4 5 6 1 3 1 2 6 4 5 6 5 1 3 2 4 5 4 6 2 1 3 4 6 5 1 3 2
16	1 2 3 4 5 6 2 3 4 5 6 1 3 1 5 6 2 4 4 5 6 1 3 2 5 6 1 2 4 3 6 4 2 3 1 5
17	1 2 3 4 5 6 2 3 4 5 6 1 3 1 5 6 2 4 4 5 6 1 3 2 6 4 2 3 1 5 5 6 1 2 4 3
18	1 2 3 4 5 6 2 3 4 5 6 1 3 1 5 6 2 4 4 5 6 2 1 3 5 6 1 3 4 2 6 4 2 1 3 5
19	1 2 3 4 5 6 2 3 4 5 6 1 3 1 5 6 2 4 4 5 6 3 1 2 5 6 1 2 4 3 6 4 2 1 3 5
20	1 2 3 4 5 6 2 3 4 5 6 1 3 1 5 6 2 4 4 5 6 3 1 2 5 6 2 1 4 3 6 4 1 2 3 5
21	1 2 3 4 5 6 2 3 4 5 6 1 6 4 5 3 1 2 5 6 1 2 3 4 4 5 6 1 2 3 3 1 2 6 4 5
22	1 2 3 4 5 6 2 3 4 5 6 1 5 6 1 2 3 4 6 4 5 3 1 2 4 5 6 1 2 3 3 1 2 6 4 5
23	1 2 3 4 5 6 2 3 4 5 6 1 6 4 5 3 1 2 5 6 1 2 3 4 3 1 2 6 4 5 4 5 6 1 2 3
24	1 2 3 4 5 6 3 1 2 6 4 5 4 5 3 1 2 3 2 3 4 5 6 1 5 6 1 2 3 4 6 4 5 3 1 2

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