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LANGUAGE THEORY AND EXPERT SYSTEMS

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Some remarks on the problems of knowledge representation and processing, as recognized in connection with the use of computers in the scientific research work, emphasizes the relevance of these problems for the studies on both the theory of languages and the expert system. A consideration of the common traits in the recent history of these studies, with reference to the use of computers on texts in natural language, motivates the introduction of set theoretic and algebraic methods, suitable for applications in the analysis and in the automatic treatment of languages, based on the concept of model sets and on relational structures suggested from the connections between syntax and semantics evidenced in some examples of sublanguages corresponding to theories of different classes of physical phenomena. Some details of these methods are evidenced, wich have been already succesfully used or whose application appears suggestive of interesting development.

1. Introduction.

In the early sixties Ljapunow [1] suggested that our understanding of any phenomenon or process, be it natural or artificial, might be controlled by the adequacy of some computer program, suitably conceived to give a model representation of that process or phenomenon.

Though one might argue about the non uniqueness and the intrinsic limitations of this kind of control, it turns out that some of the current trends of the research work in physics and in biology, in mathematics and in the social sciences, in chemistry and in the sciences concerning both the design and/or the management of complex systems, and the information processing required in several kinds of decision problems, all seem to conspire in giving new relevance, possibly with new meaning, to that suggestion.

Let me quote, just to recall some piece of evidence, the studies on the behaviour of non-hamiltonian mechanical systems, those on the classical and quantal chaotic processes, mainly carried out by computer simulation or emulation, and the studies in the lattice approximations for solving problems of quantum chromodynamics and of statistical physics, which led to the design of *ad hoc* computer systems [2].

As a matter of fact, when the need arises of using the full computer power available, one is led to try out some sort of mutual adjustment between the coding of the basic informations and that of the instructions for their processing: a task requiring to put at work any available knowledge about programming techniques as well as about the effective functional capabilities of the hardware employed to get the results of a suitably structured set of operations on the basic informations. It happens in this way that the scientific user gets involved in some typical issues of computer science and technology.

Having mentioned that physical problems have motivated the design of *ad hoc* computers, let me remember also that these problems, while calling the attention of physicists on matters like numerical analysis and computability theory [3], also have suggested new ways of thinking about such fundamental subjects as the connection of quantum theory with the observational conditions defining its domain, or the selection of basic elements apt to qualify a physical theory [4].

Remark that in any scientific area the theory can be thought of as the discourse expressing or showing our understanding of some suitably specified domain of facts, and of the elements of reality evidenced by them. The syntactic and semantic features af any such discourse give rise to a specialized use of the language, as it has been emphasized by I.D.J. Bross et al. and by myself [5], with reference to the concept of a scientific sublanguage, related to that of sublanguage grammars originally introduced by Z.S. Harris [6]. The evolution of scientific knowledge is in a sense inescapably reflected in the evolution of some scientific sublanguage supporting the representation and the processing of that knowledge, so as to allow a critical control of both, by using methods partly specific of a scientific area and partly more generally effective. The computer modelling problems suggested by Ljapunow can be directly connected with those of the computer processing of a scientific sublanguage, at least in so far these concern the implementation of a structured set of transformations on a basic set of data to faithfully represent a phenomenon as we claim to know it, say by recognizing the elements of evidence determining which of its features can be derived from general laws or rules and which are only conditionally allowed in a particular realization.

As the problems of the representation and processing of knowledge are faced in the development of scientific resarch, in the automatic treatment of texts in natural language, in the design of data base management systems and of expert systems, any experience gained in these areas can be a valuable tool to get a new insight into what might be called their common ground.

To give some sort of historical qualification to my view, let me remember that one of the first expert systems was presented at a meeting in 1956 at Dartmouth College: it was named Logic Theorist and proved theorems in logic [7]. With the General Problem Solver [8] and other early work on automatic theorem proving [9], the Logic Theorist derives from the basic idea that general methods of problem solving can be found, and can be made computational in a way essentially independent of the task at hand. This resembles rather well the initial trends in the automatic treatment of natural languages and of the related automatic translation problems [10]. And in both cases it is well known that the early implementation efforts oriented towards general purpose systems were definitely inefficient in practice.

One might argue that, according to Ljapunow, this was an indication of insufficient understanding. As a matter of fact the difficulties led in both areas to try a critical revision of the research projects, mainly aimed at solving problems of more limited scope, possibly exploiting the peculiar features of their relevance in the applications giving them a practical value. I am referring here as well to studies on the automatic treatment of texts in natural language using a reduced dictionary or involving a limited semantic domain, as to studies on expert systems set out to gain insight in the declarative and procedural aspects of human knowledge by working on specific tasks [11].

These studies, while achieving some practical results, induced as a by-product the growth of some king of empirical and theoretical know-how about concepts and methods suitable for clarifying the problems initially encountered and hinting at their possible solutions. Let me recall, e.g., some natural language question-answering systems (about flight schedules, about an ATN grammar, about the Apollo 11 moon rocks [12]), where several special purpose techniques were put at work, which later were used in many different systems and applications. Also let me quote, for having similar qualifications, the DENDRAL program [13], being a *smart assistant* for a chemist concerned with structure determination in organic chemistry.

The growth of experience and know-how seems to be quite effective in stimulating efforts to uncover *ordering principles* paving the way to transform a catalogue of data records, or parts of it, in a family of basic elements wherefrom the others can be generated according to well defined rules: in every area of scientific enterprise there is plently of historical evidence supporting this view, and the fields of automatic natural language processing and of knowledge engineering are both no exception.

So the idea that many tasks happen to have requirements in common, and that they can be met by an expert system *shell*, to which knowledge can be *added*, so to speak, about a selection of particular tasks, led to the realization of such typical shells as Emycin [14] and OPS5 [15], each of them covering a range of tasks (though no one covering them all). There are variations on this line, one of which is to provide a *toolkit* containing many of the methods used in the various expert system shells. The Expert System Environment/VM [16] KEE [17] and LOOPS [18] can be regarded as examples of such toolkits.

An expert system is designed to act as an intelligent assistant in some task or to solve a certain problem as efficiently as a human expert: so its performance reflects how well its built-in capabilities of recognizing and using knowledge effectively reproduce an expert behaviour. In turn, this raises the questions of representation and processing of knowledge in a way rather directly accessible to the control suggested by Ljapunow. But, of course, since the language is a basic tool in representing knowledge both for reasoning about its implications and for communication purposes, the research work on knowledge representation languages for expert systems and on the general theory of languages are clearly related to each other. Moreover, as new language uses have the effect of transforming a language, that relationship can be expected to suggest new ways of thinking about knowledge and language.

I'll try in the following to clarify the intended meaning of this statement by outlining some ideas and results in language theory that may be useful in the design of expert systems.

2. Model sets in language theory.

A theory can be studied by exploring its consequences or its models: if its status as a deductive theory is not well established, then the second approach is to be preferred. This is the case, I think, of language theory, where one must be prepared to employ several partial or approximate models somehow as in the empirical sciences instruments are employed to get informations on new or poorly known phenomena. The model sets, as I understand them, are a sort of basic tools for such investigations. They have been introduced in the theory of language by J. Hintikka [19], who clarified and developed ideas previously advanced by R. Carnap and L. Wittgenstein.

Let us consider the set M of all senteces being true in a possible world, say describing elements of evidence gained in some definite domain of experience. This set can be characterized by a number of conditions, reformulating the truth conditions of propositional connectives and of quantifiers. They are stated as a sort of downwards closure conditions, requiring that if some complex sentence belongs to M, then other simpler sentences also must belong to M. These conditions, to be considered as a definition of a model set, can be specified as follows:

$C\neg$: if F is an atomic formula and belongs to M, then its negation $\neg F$ does not belong to M

$C\wedge$: if $F \wedge G$ belongs to M, then both F and G belong to M

$C\vee$: if $F \vee G$ belongs to M, then either F or G or both belong to M

$C\exists$: if $(\exists x)F$ belongs to M, then the sentence $F(a/x)$, obtained by substituition of x with a in F, belongs to M for at least one free singular term a

$C\forall$: if $(\forall x)F$ belongs to M, then $F(b/x)$ belongs to M for each free singular term b occurring in the sentences of M.

It has been assumed that the negation sign «¬» only occurs immediately to the left of atomic formulae: but this does not involve essential limitations in applying the concepts introduced above.

Remark that a sentence is true iff it belongs to a given model set [19]. In other words it is satisfiable iff it can be immersed in a model set. In terms of satisfiability one can define then, according to well known patterns, all the most important metalogical concepts: a sentence is inconsistent iff it is not satisfiable, it is logically true iff its negation is not satisfiable, F follows logically from G iff $F \wedge \neg G$

is not satisfiable, and so on.

I'll not argue here about the advantages of the model set approach with respect to, say, Carnap's state descriptions, or others. More on this can be found in the work of Hintikka [19].

But I want to emphasize that model sets can be exploited in several ways. It is well known that a proof of F can be given as a refutation of its negation $\neg F$, so we can limit our attention to refutation procedures. A refutation of G amounts to showing that it cannot be immersed in a model set. In order to do this we can start with the set $\{G\}$ and expand it by «addition», one by one, of the missing elements, as required to make it a model set.

To outline the method for carrying out this task, let M be some approximation to a model set; an improved approximation can be obtained by applying one of the following rules:

$A\wedge$: if $F_1 \wedge F_2 \in M$ then F_1 and F_2 are added to M

$A\vee$: if $F_1 \vee F_2 \in M$ and if neither F_1 nor F_2 are in M, then either F_1 or F_2 is added to M

$A\exists$: if $(\exists x)F \in M$ and if M does not contain sentences in the form $F(b/x)$, then $F(a/x)$ is added to M, with a being an arbitrary new singular term

$A\forall$: if $(\forall x)F \in M$ and if b occurs in the elements of M, then $F(b/x)$ is added to M.

When $A\vee$ is applied the construction tree has a two-ways branching: if both lead to a violation of the $C\neg$ rule, then G is not satisfiable.

The refutation procedure can be transformed into its dual demonstration procedure, constituting a systematic search for a counterexample of G, that is for a model set including $\neg G$. It turns out that the rules involved in this search are essentially a version of the Herbrand's type rules in quantification theory. Moreover, the central part of the metatheory of first order logic can be recovered, quite simply, by exploiting the model set approach to language theory.

More advanced results can be obtained as well: e.g. one can show that if $\{M_1 + M_2\}$ is refutable, while M_1 and M_2 are not, then a sentence F exists satisfying (i) F contains only predicates and individual constants occurring in the elements of both M_1 and M_2, (ii) $\{M_1 + \{F\}\}$ and $\{M_2 + \{\neg F\}\}$ are both refutable. This is a sort of *separation* lemma corresponding to the well known *interpolation* lemma of W. Craig [20]. The information thus obtained on F can be further qualified by suitably strengthening Craig's lemma, as discussed by L. Henkin [21].

The model set approach to the theory of languages has several features which might be exploited to improve the automatic treatment of languages in knowledge based systems. Let me recall the flexible modularity introduced with the method of describing *possible worlds* by limited expressive means, e.g. by considering «several individuals in their mutual simultaneous relations» within a given domain of experience. This originates the concept of a *constituent*, roughly a description of some possible world in terms of a finite number of individuals and of a relational structure comprising them, without ever mentioning a particular individual (that is, only *generic* individuals appear in the relational structure) [19]. In a sense the constituents may be thought of as a tool to construct a *finitary* semantic theory, allowing successive approximations, of a sort perhaps alluded to in J. Herbrand [22], whose relevance for the problems of knowledge acquisition and processing in expert systems and in data base management and query-answering systems seems quite clear.

It is worth mentioning that such a theory gives a basis for effectively exploring new possibilities in the treatment of sentences with quantifiers established by induction, hinting at alternative approaches to the inductive methods in the search for general laws in large sets of data and in the definition of the information encoded in the semantic and syntactic elements of their descriptions.

The distributive normal forms, being disjunctions of constituents, appear in this theory as a direct generalization of the complete disjunctive normal forms of sentential logic, but, by contrast to the latter, they do not admit a decision method. This is due to the fact that some constituents (of degree larger than one) may be inconsistent, and there is no recursive general method to uncover which of the constituents are inconsistent, in spite of the fact that a simple automatic procedure can be given for excluding those whose inconsistency is *trivial* [19].

This difficulty can be considered as arising from the very nature of the language, reflecting somehow its incompleteness and indeterminacy features, seemingly related to its remarkable flexibility to quite a variety of user views and intents. It led Hintikka to introduce the concepts of *surface information* and of *depth information* and to show that the source of the above mentioned undecidability lies in the fact that one does not know how deep the analysis must go on to uncover possible inconsistencies hidden in a given constituent [19].

It can be shown that several logical procedures, say the demonstrations of logical truth, those from premises and those of equivalence, amongst others, are based on the elimination of inconsistent constituents, and therefore, in all non trivial cases, involve a growth of Hintikka's surface information.

Remark that the model set approach to the treatment of language admits as well an algebraic as a set theoretic formalization, both suitable for computer representation and processing.

As a matter of fact, the *C*-rules, corresponding to Carnap's [23] *truth conditions* are directly related to the method developed by W.A. Woods for defining and processing semantic information in Query-Answering systems [12]. He designed indeed a computer excutable procedure which can be regarded as a notational version of the standard predicate calculus. More recently, the *Discourse Representation Theory* introduced by F. Guenthner et al. [24] with the aim of improving the treatment of natural langauge involved in knowledge based systems, includes *meaning rules* and deduction procedures, (the latter based on the construction of counter-examples by means of a *tableau calculus*), appearing as clear images of elements peculiar to the model set theory of languages.

Hintikka's distributive normal forms and their *constituents* have been shown by D. Scott [25] to admit a purely set theoretical definition, wherein the constituents correspond to certain sets of finite rank, which could be considered quite apart from any chosen formal language. Though the translation back to their original, quite syntactical description, say in the framework of first-order logic, is very quick, Scott's formulation contributes to better understanding what exactly is expressed in those normal forms. I think that the set theoretic definition given by D. Scott might be quite useful in a new approach to the automatic treatment of languages by exploiting the existing computer systems designed for set processing.

To outline Scott's formulation, let $R \subseteq A \times A$ be a relation and $a \in A^m$, a being an $m - terms$ (finite) sequence of elements of A. There is then an obviously *induced relation* on the indices of the terms of a:

$$R[a] = \{(i,j) \in m \times m | a_i R a_j\}$$

with $m = |a|$. Remark, as the notation above suggests, that the integer m is interpreted as an *ordinal*, and that the set $R[a]$ is of finite rank no matter what the set A is. For a relational system $\mathbf{A} = \langle A, R \rangle$ let us define

$$\mathbf{A}[a] = \langle m, R[a] \rangle$$

In the more general case, when $\mathbf{A} = \langle A, R_0, R_1, \ldots \rangle$, with each $R_i \subseteq A^{k_i}$, an analogous definition can be given, with $R_i[a]$ so redefined that $R_i[a] \subseteq |a|^{k_i}$ and $\mathbf{A}[a] = \langle |a|, R_0[a], R_1[a], \ldots \rangle$. Remark that as long as a finite number of finitary relations is involved in the above definition the induced structure $\mathbf{A}[a]$ is an object of finite rank in V_ω. If $\mathbf{A} = \langle A, R \rangle$ and $\mathbf{B} = \langle B, S \rangle$ are two relational systems, with $a \in A^m$ and $b \in B^k$, then the equation

$$\mathbf{A}[a] = \mathbf{B}[b]$$

holds true iff $m = k$ and $a_i R a_j \Leftrightarrow b_i S b_j$. The corresponding statement in the formal language could be phrased by saying that a and b

satisfy the same atomic formulae (and hence the same quantifier free formulae).

Given a finite sequence $a \in A^m$ in a relational system $\mathbf{A} = \langle A, R \rangle$ how can it be extended to a longer one, say $a \frown a'$, being the concatenation of a and another finite sequence a'? The totality of possible *types* of extensions determines how a *sits* in A, while $\mathbf{A}[a]$ describes how the a_i's relate among themselves. The answer to the stated question is built up stepwise, with *types* defined inductively:

$$\tau_0^{\mathbf{A}}[a] = \mathbf{A}[a]$$

$$\tau_{n+1}^{\mathbf{A}}[a] = \{\tau_n^{\mathbf{A}}[a \frown x] | x \in A\}$$

Let us call $\tau_n^{\mathbf{A}}[a]$ the *n-type* of the sequence a within the structure $\mathbf{A}$. It is useful to classify the various types in certain *levels* by defining:

$$C_0^m = \{\langle m, r \rangle | r \subseteq m \times m\}$$

$$C_{n+1}^m = PC_n^{m+1} = P^{n+1} C_0^{m+n+1}$$

where P is the operation of forming the family of all non empty subsets, and P^{n+1} its $(n+1)$-fold iterate. It can be verified that

$$\tau_n^{\mathbf{A}}[a] \in C_n^m$$

for all n, m, where $a \in A^m$ and $\mathbf{A} = \langle A, R \rangle$ any relational system. This shows that, given n and m, there are only finitely many possible values of $\tau_n^{\mathbf{A}}[a]$, even if the set A is infinite. Following Hintikka, the elements of C_n^m are called *constituents of depth n*.

The main results of D. Scott can be reported as follows:

1) the classes C_n^m are pairwise disjoint;
2) if $\mathbf{A}$ and $\mathbf{B}$ are relational structures and a, b finite sequences, then the equation $\tau_n^{\mathbf{A}}[a] = \tau_q^{\mathbf{B}}[b]$ always implies $|a| = |b|$ and $n = q$;
3) If $\mathbf{A} = \langle A, R \rangle$ and $\mathbf{B} = \langle B, S \rangle$ are relational systems, with $a \in A^k$ and $b \in B^k$, and if $\pi : m \to k$ is any mapping on integers, then the equation

$$\tau_n^{\mathbf{A}}[a] = \tau_n^{\mathbf{B}}[b]$$

always implies

$$\tau_n^{\mathbf{A}}[a \circ \pi] = \tau_n^{\mathbf{B}}[b \circ \pi]$$

4) the equation $\tau_q^{\mathbf{A}}[a] = \tau_q^{\mathbf{B}}[b]$ always implies

$$\tau_n^{\mathbf{A}}[a] = \tau_n^{\mathbf{B}}[b]$$

provided $n \leq q$.

It is well known that the subsets $\Phi \subseteq C_n^m$ form a *Boolean algebra*. Hence the predicates (of m-term sequences) defined by $\tau_n^{\mathbf{A}}[a] \in \Phi$ are closed under the Boolean operations. For *quantification*, however, one has to increase the depth of the types. D. Scott shows that

5) if $\Phi \subseteq C_n^{m+1}$, then $P\Phi \subseteq C_{n+1}^m$ and $\tau_{n+1}^{\mathbf{A}}[a] \in P\Phi$ iff $\forall x \in A \cdot \tau_n^{\mathbf{A}}[a \frown x] \in \Phi$
6) $\tau_{n+1}^{\mathbf{A}}[a] \cap \Phi \neq \emptyset$ iff $\exists x \in A \cdot \tau_n^{\mathbf{A}}[a \frown x] \in \Phi$
7) a sequence $a \in A^m$ satisfies in $\mathbf{A}$ a formula ψ in prenex form with n quantifiers and $m + n$ variables altogether iff $\tau_n^{\mathbf{A}}[a] \in \Psi$, with Ψ the subset of C_n^m corresponding to ψ.

The significance of the equation $\tau_n^{\mathbf{A}}[a] = \tau_n^{\mathbf{B}}[b]$, where $\mathbf{A}$ and $\mathbf{B}$ are as specified under 3), and $a \in A^m$ and $b \in B^m$, can be phrased by saying that the two sequences satisfy the same formulae of quantifier depth $\leq n$, corresponding to the fact that the types belong to the same subsets of C_n^m. This is the answer to the question on the comparison of types, and Scott [25] shows how directly his set-theoretic analysis can be related to that with Hintikka's distributive normal forms.

A remarkable feature of the set theoretic formulation can be recognized in the fact that its coding up of the syntax in quite a few definitions introduces a noticeable simplification in the analysis of the satisfaction of formulae, as compared with its usual presentations.

3. Relational structures in language theory.

In the formal, or «mathematical», approach to the theory of languages [26], a language is defined as a subset L of the set Σ^* of all finite sequences on some finite alphabet Σ, together with some *method of description*. Three such methods are mainly used:

(1) *Generation*: when a set of rules, a *grammar*, is given for generating just the elements of Σ^* belonging to L.

(2) *Acceptance*: when there is a device (e.g. a finite state machine with auxiliary storage) such that if $s \in \Sigma^*$ is given as input, then the corresponding output belongs to some designated set iff $s \in L$.

(3) *Algebraic*: when a basic family of sets and a list of algebraic operations on them are given, and L is the subset of Σ^* built up from the basis by means of any finite number of those operations.

To think about the connection between language and knowledge one is confronted with the question of assigning a *semantic valence* to the elements of any given language L, as defined above. In the formal theory this assignment may be described in terms of some *morphisms* relating elements of L and elements of the domain one is speaking about in the language L.

Let me remark that no general agreement exists among language theorists and philosophers, on how well such a description captures the concept of *meaning*, though, of course, from a logical point of view, its basic structure has been thoroughly investigated in model theory [27].

The problems discussed in this connection, in addition to some sort of subtle complexity, perhaps reflecting the well known limitations of a formal semantics, also have an intriguing variety of aspects, appearing as well when working on the computer processing of natural languages as when studying the relationship of a discourse about elements of reality with one about our knowledge of them. In particular they seem to involve the methods of definition and inference in a context where the equivalence of different representations of the same piece of evidence, in the same language or in different languages, apparently determines a complex relationship among the meaning assignment procedures and the language structures. It turns out that this relationship has to do with the constraints imposed by any chosen coding of data or informations on the possibilities open to their processing, which seems to me of fundamental relevance both in the sciences and in the design of expert systems.

I'll try in the following to illustrate this point by means of an example [28].

Let us consider a set F whose elements are *facts*, a set LD whose elements are *descriptions* (in the language L) and a set LR whose elements are *reference rules* (of the language L). The reference rules are introduced to abstractly represent the meaning assignment procedures in the sense that given a fact $f \in F$, there is a rule $Lr \in LR$ generating a description Ld of the fact in L, $Ld \in LD$. Also, given a description $Ld \in LD$, there is a rule $Lr \in LR$ to recognize the fact $f \in F$ described by Ld.

By assumption, different descriptions of the same fact are allowed, generated by different reference rules, reflecting the possibility of alternative *views*, as e.g., in describing the motion of a mass-point, they correspond to the choice of a specific reference frame. Conversely, a given description may refer to several facts, iff they are determined by different reference rules, again reflecting different possible *views* or reference frames.

Remark that a reference rule Lr is a complex object built up of factual and intentional elements, as, in general, the meaning assignment procedures cannot be reduced to formal or *syntactic* operations. E.g. the position of a mass point at a given time is determined by coordinate values only when both their measurement procedures and a reference frame have been effectively specified.

The above is formally synthesized in a basic relation B, being a subset of $LD \times F \times LR$, such that $(Ld, f, Lr) \in B$ iff Ld is the description of f generated by Lr and, conversely, f is the fact described by Ld according to Lr. Remark that Lr stands for both the interpretation rule giving f from Ld and the description rule giving Ld from f, so that if $(Ld, f, Lr) \in B$ any two elements of the triple (Ld, f, Lr) uniquely determine the third.

Let us define now an equivalence relation on the set LR of reference rules in L: two such rules Lr and Lr' are said to be *coherent*, in symbols $(Lr, Lr') \in C_{LR}$, iff

$$\forall f \exists Ld, Ld' \cdot (Ld, f, Lr) \in B \wedge (Ld', f, Lr') \in B$$

and

$$\forall Ld \exists f, f' \cdot (Ld, f, Lr) \in B \wedge (Ld, f', Lr') \in B$$

The equivalence class of C_{LR} containing Lr will be indicated as $[r]C_{LR}$.

Remark that in any language L one can describe the facts of several *possible worlds*: «two electric charges of the same sign attract each other» is the (shortened) description of a fact which is not realized in our physical world. So let us introduce the selection of a possible world by considering in F the subset TF of *true facts*, say of those effectively belonging to the world of our observational experience, or, if you like, to a suitably chosen *possible world*.

Let us define then the restriction B_T of the basic relation B to the subset of true facts

$$(Ld, f, Lr) \in B_T \Leftrightarrow (Ld, f, Lr) \in B \wedge f \in TF$$

and the set $\Delta(Lr)$ of descriptions being *reliable* in Lr

$$\Delta(Lr) \equiv \{Ld | \exists f \cdot (Ld, f, Lr) \in B_T\}$$

With the given definitions it turns out that any mapping

$$\rho_r : [r]C_{LR} \rightarrow [r]C_{LR}$$

uniquely defines a mapping

$$\delta_r : \Delta(Lr) \rightarrow \Delta(Lr')$$

such that if $\rho_r Lr \rightarrow Lr'$ then $\delta_r Ld \rightarrow Ld'$ connects descriptions of the same (true) fact, for all $Lr' \in [r]C_{LR}$. Also ρ_r uniquely determines a mapping $\phi_r : F \to F$ such that $\phi_r f \to f'$ iff f has the same description by Lr as f' by Lr'. This reflects an important property of the connection between the *stability* of descriptions and a *relativity* requirement on the meaning assignment procedures.

If $\phi_r : TF \to TF$, then, for all $Lr' \in [r]C_{LR}$, one has

$$\forall Ld \cdot Ld \in \Delta(Lr) \leftrightarrow Ld \in \Delta(Lr')$$

which amounts to say that $\Delta(Lr) = \Delta(Lr')$ whenever Lr and Lr' belong to the same equivalence class of C_{LR}, or, otherwise stated, that a reliability condition in B_T is invariant against the ρ_r transformations and their δ_r counterparts.

This clearly shows a basic feature of the relationship between the descriptions and the meaning assignment procedures of L.

As a practical example one might recall that physical laws can be regarded as reliability conditions in the language of physics [28], and that, e.g. the relativistic laws can be encoded in a *format* invariant against transformations from a reference frame to another, in the same equivalence class, say, of *inertial* reference frames.

Let us introduce the notation $\rho_{r',r}$ for an operation defined on some equivalence class of C_{LR} and such that $\rho_{r',r} Lr = Lr'$. The corresponding operation induced on LD, in the way stated above, will be denoted $\delta_{r',r}$, so that if Ld describes some true fact by Lr, then $Ld' = \delta_{r',r} Ld$ describes the same fact by Lr'.

Let me emphasize that the pair $(\rho_{r',r}, \delta_{r',r})$ can be considered as a sort of *translation rule* from the Lr- to the Lr'-descriptions within $[r]C_{LR}$.

Now an interesting question arises from the physical theory of relativity: do the $\rho_{r',r}$ operations form a group?

The question is interesting because the answer sheds some light on the relationship among the meaning assignment procedures and the language structures.

To show how this happens, we want to consider, in the following, only Lr's belonging to a fixed $[r]C_{LR}$. Let us point out that

$$\rho_{r',r} = I \Leftrightarrow r' = r$$

having indicated with I the identity. Moreover

$$(\rho_{r',r})^{-1} = \rho_{r,r'}$$

$$\rho_{r'',r'}\rho_{r',r} = \rho_{r'',r}$$

but $\rho_{r''',r''}\rho_{r',r}$ is not yet defined, in general. To make it definite, so as to enforce a group structure on the ρ's, it is required that, for any choice of r, r', r'',

$$\exists r''' \cdot \rho_{r''',r''} = \rho_{r',r}.$$

Now let us interpret the last statement as a description by Lr'' of a fact: it is a true fact when $r'' = r$, because then $r''' = r'$. For any other choice of r'' it must be true as well, because a description reliable in r belongs to $\Delta(Lr)$ and $\Delta(Lr) = \Delta(Lr'')$ for all r'' in $[r]C_{LR}$.

I want to stress the point that, while in the case of physical relativity the interpretation given above appears as a natural recognition of the fact that the reference frames involved in the meaning assignment procedures are also physical objects, in general it seems quite possible to distinguish, in the so called reference rules, components of factual and of intentional content, so that by analogy, new instances of that interpretation might be found.

Remark that the set LR of reference rules, determining the interpretation of a description, enters in the formal analysis partitioned in equivalence classes, which may be thought of as corresponding to sets of equivalent *language users*. Within each class the LR rules are controlled by the same reliability conditions, and the latter are invariant against transformations corresponding to any *translation* of an Lr-description in an Lr'-description of the same fact.

The semantic implications of such transformations depend, so to speak, on *how much* of meaning is encoded in declarative and in procedural form. I recall that Prolog, the well known *logic programmming* language first implemented to support natural language processing [29], and then chosen as a basis for the Japanese Fifth Generation Project [30], contains both some declarative features from computational mathematical logic and some procedural aspects from conventional programming. This is important, I think, to understand its power and flexibility in applications.

The relationship among syntactic and semantic structures, in the example we are considering, can be analyzed in terms of the ρ-and of the δ-transformations. Clearly they have a special rôle in the definition of an algebra on $LD \times TF \times LR$, reflecting the compositional features of its linguistic and factual elements. This looks like a particular instance of the many-sorted algebras [31] employed in the studies on programming languages, and, therefore, its computer implementation is quite feasible.

To show the power of the chosen method of analysis let us consider two languages L_1 and L_2, and, correspondingly, the relations $B_{iT} \subseteq B_i$, with $i = 1,2$, defined in the same way as $B_T \subset B$, so that, e.g.,

$$(L_i d_i, f_i, L_i r_i) \in B_{iT} \subset L_i D_i \times TF_i \times L_i R_i$$

with $L_i d_i \in L_i D_i$, $f_i \in TF_i$ and $L_i r_i \in L_i R_i$, for $i = 1,2$. With the definition

$$f_i L_i r_i = L_i d_i \Leftrightarrow (L_i d_i, f_i L_i r_i) \in B_{iT}$$

for $i = 1,2$, let us assume that a mapping θ from TF_1 to TF_2 can be given so that $\theta TF_1 \subseteq TF_2$ and, consequently, $(\theta f) L_2 r_2 = L_2 d_2$. One can get now, for any θ defined as above and any $f \in TF_1$, a *generalized* translation rule

$$\delta^{(1,2)}_{r_2;r_1} f L_1 r_1 = (\theta f) L_2 r_2$$

from a description reliable in L_1 to a description reliable in L_2. This translation rule has a remarkable flexibility, due to its θ-dependence, and, of course, to show that it can be used, I could exhibit again several examples from my own experience in physics. Let me quote, for one, the analysis of energy conversion phenomena involving electromagnetic fields and electric charges in the language of electrodynamics and in the language of thermodynamics, the two languages involving different selections of observational elements ($TF_1 \neq TF_2$) and different theoretical processing methods. The generalized translation rule, by construction, embodies the consistency conditions separately imposed by the laws of thermodynamics and of electrodynamics, and, possibly, the thermodynamic equivalence of different electrodynamic conditions.

Note that the translation rules defined within a given language L may be considered as a special case of the generalized translation rules from L_1 to L_2, connecting different languages. In the second case, of course, the transformations corresponding to the ρ's, say

$$\rho^{(1,2)}_{r_2,r_1} L_1 r_1 = L_2 r_2$$

can acquire, in a natural way, only a semigroup structure, which entails an analogous limitation on the induced structure of the $\delta^{(1,2)}$'s. I'll just add here that the generalized translation rules are relevant not only for studying the connections of scientific sublanguages with the natural language, but also for clarifying some patterns of language evolution whose computer implementation might improve the *learning capabilities* of expert systems.

The θ-mapping introduced above is not arbitrary: it must be chosen so that some consistency conditions are satisfied, which have been already investigated in the study of relational structures. In particular let me quote here the contribution of G. Dantoni [32], whose relevance is, in my opinion, of very high value.

The suggestion of a more general setting for the algebraic treatment of some problems in the theory of languages, outlined above, came to me from another piece of Dantoni's work [33].

I'll try to describe its basic ideas as a conclusion.

Let M be a non empty set and R an $n-ary$ relation defined on M. The *modulus* of R, say $|R|$, is defined as

$$|R| = M_1 \times M_2 \times \ldots \times M_n$$

with $M_s (s = 1, 2, \ldots, n)$ the set of the $s-th$ place terms in the n-terms sequences being the elements of R.

Given any such R, an equivalence relation E_r can be defined on it: any two elements of R, say $r \equiv (m_1, m_2, \ldots, m_n)$ and $r' \equiv (m'_1, m'_2, \ldots, m'_n)$, are said to be *intersecting* when, for at least one

$s(s = 1, 2, \ldots, n)$ it is $m_s = m'_s$; $(r, r') \in E_R$ holds true when either r and r' are intersecting or there is a finite sequence of elements in R, say $r_1, r_2, \ldots, r_h$ such that r_i and r_{i+1} are intersecting $(i = 1, 2, \ldots, h - 1)$ while r and r_1 as well as r' and r_h are intersecting.

The equivalence classes of E_R are called the *connected parts* of R. If $R_j, (j \in J)$, is an element in the family of the connected parts of R, then the relation $\bar{R} = \bigcup_{j \in J} |R_j|$ will be referred to as the *associated relation* to R. So, in general, given R, also the following items are given

a) the equivalence relation E_R; b) the sets $M_s(s = 1, 2, \ldots, n)$; c) on any M_s an equivalence relation $E_R^{(s)}$, whose classes comprise elements belonging to the same connected part of R; the sets $M_s/E_R^{(s)}$ $(s = 1, 2, \ldots, n)$ all have the same cardinality; d) a family of one-to-one mappings $\varphi_2, \varphi_3, \ldots, \varphi_n$ of $M_1/E_R^{(1)}$ on $M_2/E_R^{(2)}$, $M_3/E_R^{(3)}, \ldots, M_n/E_R^{(n)}$, respectively; e) the associated relation $\bar{R}$.

If we consider now the relation $B \subset LD \times F \times LR$, it turns out that its restriction to $[r]C_{LR}$ obviously belongs to one of its connected parts. In the previous analysis we have implicitely assumed that all of $TF \subset F$ can be described by any $Lr' \in LR$. This assumption might reflect the thesis that all physical phenomena can be described in terms of spacetime-ordered sets of point-like events. The assumption simplifies the presentation of the chosen example, and it is useful in the comparison of reference rules, within the same language, belonging to different equivalence classes of C_{LR}, e.g. corresponding to inertial and rotating reference frames. But it is not necessary.

In the domain of physics there are fields and particles, patterns of events distinguishable for different kinds of observable features, irreducible to a space-time description, and whose understanding is expressed in different (sub-) languages. Also it is well known that the knowledge about biological or chemical phenomena cannot be reduced to that of their physical aspects. Thus there are conditions suggesting the introduction of reference rules whose operation is defined on a limited domain of facts, and allowing different languages for describing different aspects of the same domain of facts. This can be done by assuming that the basic relation B_T in L has several connected parts. Let us denote with B_j, for any $j \in J$, a connected part of the relation $B \subset LD \times F \times LR$. Let b_j denote any triple (Ld, f, Lr) in B_j. With $|B_j|$ being the *associated relation* to B_j defined above, let us consider the family of relational structures $\mathbf{B}_j = \langle |B_j|, B_j \rangle$, $(j \in J)$. For any mapping θ from all of $|B_j|$ to (all or part of) $|B_{j'}|$, satisfying $\theta b_j \in B_{j'} \cap (\theta |B_j|)$ for all $b_j \in B_j$, one gets a class of translation rules from the sublanguage formalized in B_j to that formalized in $B_{j'}$. This is just one of the five types of homomorphic correspondences analyzed by Dantoni [32]. Remark that, in general, a connected part of B may comprise several classes of *coherent* reference rules, inducing new details of structure on the meaning assignment procedures within L.

Of course, on the strength of a previous remark, the generalization outlined in the case of a single language L is an instance of that involving two different languages. While in the first case one is considering only the transformations within the same set $L \subset \Sigma^*$ of *coding elements* for the relevant information, in the second case also transformations can be analyzed involving a change of these *coding elements*, with the additional advantage deriving from the several possibilites open to the choice of the θ-morphism. If this morphism is chosen, with reference to two languages L_1 and L_2, so that for any triple $(L_1 d_1, f_1, L_1 r_1) \equiv b_1 \in B_1$, it is $\theta b_1 \in B_2$, then a correspondence is obtained relating, in general, any connected part of B_1 to several connected parts of B_2. Examples of applications can be indicated in the studies on congruence-classes geometries [34] and on the local and global symmetries characterizing the physics of gauge fields, in general relativity [28] and in the theory of fundamental interactions.

In connection with the problems of knowledge representation, let me quote also another sort of interesting examples, with reference to the possibility of interpreting some geometric patterns in an Euclidean manifold as models of a non-Euclidean geometry, or of translating, so to speak, results from potential theory to the theory of Brownian motions and of other random processes (and conversely), or else of clarifying which conditions determine a complementarity of languages analogous to that well known in the description of quantum phenomena.

I think that the algebraic analysis of relational structures is a powerful tool yet to be exploited in the development of knowledge representation languages: while giving some critical insight in what a language is or may be, the research in this filed should be interesting for the science scholars and for the designers of expert systems, and might be a good opportunity of interdisciplinary collaboration.

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