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LOCAL DIFFERENTIABILITY FOR THE SOLUTIONS TO BASIC SYSTEMS OF HIGHER ORDER

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In this paper we show a local differenziability result and a fundamental estimate for the solutions to non linear basic systems of higher order.

These results are preliminary to the study of the quasi-holder continuity for non linear parabolic systems of higher order.

1. Introduction.

Let Ω be a bounded open subset of $\mathbb{R}^n$, with $n \geq 1$ whose boundary is sufficiently smooth; let x be a point of $\mathbb{R}^n$, $t \in \mathbb{R}$ and $X = (x, t)$ a point of $\mathbb{R}^n \times \mathbb{R}$. Let N be a integer > 1, $(|)_k$ and $\|\cdot\|_k$ the scalar product and the norm in $\mathbb{R}^k$ respectively.

If T is a positive number we set $Q = \Omega \times (-T, 0)$ and we define

$$B(x^0, \sigma) = \{x \in \mathbb{R}^n : \|x - x^0\| < \sigma\}$$

$$Q(X_0, \sigma) = B(x^0, \sigma) \times (t_0 - \sigma^{2m}, t_0)$$

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where $X_0 = (x^0, t_0)$.

Moreover we say that $Q(X_0, \sigma) \subset\subset Q$ if

$$B(x^0, \sigma) \subset\subset \Omega \quad \text{and} \quad \sigma^{2m} < t_0 + T \leq T.$$

Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be a multindex and $|\alpha| = \alpha_1 + \alpha_2 + \ldots + \alpha_n$; we denote by $\mathcal{R}, \mathcal{R}^*$ and $\mathcal{R}'$ respectively the cartesian products $\prod_{|\alpha| \leq m} \mathbb{R}^N_\alpha$, $\prod_{|\alpha| \leq m-1} \mathbb{R}^N_\alpha$ and $\prod_{|\alpha| = m} \mathbb{R}^N_\alpha$, while $p = \{p^\alpha\}_{|\alpha| \leq m}$, $p^* = \{p^\alpha\}_{|\alpha| \leq m-1}$ and $p' = \{p^\alpha\}_{|\alpha| = m}$, $p^\alpha \in \mathbb{R}^N$, are respectively points of $\mathcal{R}$, $\mathcal{R}^*$ and $\mathcal{R}'$.

If $u : Q \to \mathbb{R}^n$, we set

$$Du = \{D^\alpha u\}_{|\alpha| \leq m}, \quad \delta u = \{D^\alpha u\}_{|\alpha| \leq m-1}.$$
$$D^{(k)} u = \{D^\alpha u\}_{|\alpha| = k} \quad k = 1, \ldots, m.$$

Let $A^\alpha(p')$ $|\alpha| = m$ be vectors of R^N defined in $\mathcal{R}'$, continuous in p' and such that

$$A^\alpha(0) = 0 \quad |\alpha| = m;$$

we shall call «basic system» the non linear differential system

$$E_0 u = (-1)^m \sum_{|\alpha| = m} D^\alpha A^\alpha(D^{(m)} u) + \frac{\partial u}{\partial t} = 0. \tag{1.1}$$

We suppose that the vectors $p' \to A^\alpha(p')$ are differentiable with derivatives $\dfrac{\partial A^\alpha}{\partial p_k^\beta}$, $|\alpha| = |\beta| = m$, $k = 1, 2, \ldots, N$ continuous in p' and bounded in $\mathcal{R}'$:

$$\left\{ \sum_{h,k=1}^{N} \sum_{|\alpha|=m} \sum_{|\beta|=m} \left| \frac{\partial A_h^\alpha}{\partial p_k^\beta} \right|^2 \right\}^{1/2} \leq M \quad \forall p' \in \mathcal{R}', \tag{1.2}$$

then if we denote by $A_{\alpha\beta} = \{A_{\alpha\beta}^{hk}\}$ with $A_{\alpha\beta}^{hk}(p') = \displaystyle\int_0^1 \frac{\partial A_h^\alpha(\tau p')}{\partial p_k^\beta} d\tau$, thanks to the fact that $A^\alpha(0) = 0$, we have

$$\sum_{k=1}^{N} \sum_{|\beta|=m} A_{\alpha\beta}^{hk}(p') p_k^\beta = \int_0^1 \sum_{k=1}^{N} \sum_{|\beta|=m} \frac{\partial A_h^\alpha(\tau p')}{\partial p_k^\beta} p_k^\beta d\tau = \tag{1.3}$$
$$= \int_0^1 \frac{\partial A_h^\alpha(\tau p')}{\partial \tau} d\tau = A_h^\alpha(p')$$

and therefore

$$A^{\alpha}(p') = \sum_{|\beta|=m} A_{\alpha\beta}(p')p^{\beta}$$

and

(1.4)
$$E_0 u = (-1)^m \sum_{|\alpha|=m} \sum_{|\beta|=m} D^{\alpha}[A_{\alpha\beta}(D^{(m)}u)D^{\beta}u] + \frac{\partial u}{\partial t}.$$

We also suppose that the operator E_0 is strongly parabolic in the following sense: there exists $\nu > 0$ such that

(1.5)
$$\sum_{h,k=1}^{N} \sum_{|\alpha|=m} \sum_{|\beta|=m} \frac{\partial A_h^{\alpha}}{\partial p_k^{\beta}} \xi_h^{\alpha} \xi_k^{\beta} \geq \nu \sum_{|\alpha|=m} ||\xi^{\alpha}||^2$$

for every $p' \in \mathcal{R}'$ and for any system $\{\xi^{\alpha}\}_{|\alpha|=m}$ of vectors of $\mathbb{R}^N$.

$H^{k,p}(\Omega, \mathbb{R}^N)$ and $H_0^{k,p}(\Omega, \mathbb{R}^N)$, k integer ≥ 0, and $p \geq 1$, are the usual Sobolev space of the vectors $u : \Omega \to \mathbb{R}^N$; if $p = 2$ we shall simply write H^k and H_0^k respectively.

If $u \in H^{k,p}(\Omega, \mathbb{R}^N)$, $1 \leq p < +\infty$, we define

$$|u|_{k,p,\Omega} = \left\{ \int_{\Omega} \left(\sum_{|\alpha|=k} ||D^{\alpha}u||^2 \right)^{p/2} dx \right\}^{1/p}$$

$$||u||_{k,p,\Omega} = \left\{ \int_{\Omega} \left(\sum_{|\alpha|\leq k} ||D^{\alpha}u||^2 \right)^{p/2} dx \right\}^{1/p}$$

that, if $p = 2$ we shall simply write $|\cdot|$ and $||\cdot||$ respectively.

The object of the present work is to show a result of a local differentiability for the solutions to basic systems of higher order similar to ones obtained in [2]. Such result has value in itself and together with the ones demonstrated in [1] will be able to achieve further regularity results for the solutions of nonlinear parabolic system of higher order of general type.

We also observe that our result cannot obtained as a particular case of the results shown in [4], because we do not require that the solutions to the basic systems belong to $C^{m-1,\lambda}(\bar{Q}, \mathbb{R}^N)$, $0 < \lambda < 1$. In fact the type of solution that we consider for the basic systems is the following: a vector $u \in L^2(-T, 0, H^m(\Omega, \mathbb{R}^N))$ is a solution of the system (1.1) if it results

$$\int_Q \left\{ \sum_{|\alpha|=m} (A^\alpha(D^{(m)}u)|D^\alpha\varphi) - \left(u|\frac{\partial\varphi}{\partial t}\right) \right\} dX = 0 \tag{1.6}$$

$\forall\varphi \in L^2(-T, 0, H_0^m(\Omega, \mathbb{R}^N)) \cap H^1(-T, 0, L^2(\Omega, \mathbb{R}^N))$: $\varphi(x, -T) = \varphi(x, 0) = 0$ in Ω.

Particularly by our result is useful to achieve a fundamental estimate (see sect. n. 3) which plays an important role in the regularity theory.

2. Local differentiability for the basic systems.

In this section we show a results of local differentiability for the solutions to basic systems similar to the ones obtained in [2].

The following theorem holds:

THEOREM 2.1. *Let $u \in L^2(-T, 0, H^m(\Omega, \mathbb{R}^N))$ be a solution of the system*

$$(-1)^m \sum_{|\alpha|=m} D^\alpha A^\alpha(D^{(m)}u) + \frac{\partial u}{\partial t} = 0; \tag{2.1}$$

then it exists

$$D^\alpha u \in L^2_{\text{loc}}(Q, \mathbb{R}^N), \quad \forall\alpha : |\alpha| = m + 1,$$

and $\forall B(x^0, 2\sigma) \subset\subset \Omega$, $\forall 2a \in (0, T)$ we have:

$$\int_{-a}^0 dt \int_{B(x^0,\sigma)} \sum_{|\alpha|=m+1} \|D^\alpha u\|^2 dx \leq \tag{2.2}$$

$$\leq c(\nu, M) \sum_{0<|\alpha|\leq m} \left(\frac{1}{a^{\left(1-\frac{|\alpha|-1}{m}\right)}} + \frac{1}{\sigma^{2m\left(1-\frac{|\alpha|-1}{m}\right)}} \right) \cdot$$

$$\cdot \int_{-2a}^{0} dt \int_{B(x^0,2\sigma)} ||D^\alpha u||^2 dX.$$

Proof. To achieve this result let us assume in (1.6), as test function, the function φ constructed in the following way:
let $\vartheta(x) \in C_0^\infty(\mathbb{R}^n)$ be a function with the following properties

$$0 \leq \vartheta \leq 1, \quad \vartheta = 1 \in \text{ in } B(\sigma), \quad \vartheta = 0 \text{ in } \mathbb{R}^n \backslash B\left(\frac{3}{2}\sigma\right)$$

$$|D^\gamma \vartheta| \leq c\sigma^{-|\gamma|} \quad \forall \gamma$$

where for the sake of brevity we set $B(x^0, \sigma) = B(\sigma)$

Let $\rho_p(t)$, with p integer $> 2/a$, $2a \in (0, T)$, be a function defined in $\mathbb{R}$ as follows

$$\begin{cases} \rho_p(t) = 1 & \text{if } -a \leq t \leq -2/p \\ \rho_p(t) = 0 & \text{if } t > -1/p \text{ or } t < -2a \\ \rho_p(t) = \dfrac{t}{a} + 2 & \text{if } -2a \leq t \leq -a \\ \rho_p(t) = -(pt+1) & \text{if } -2/p \leq t \leq -1/p. \end{cases}$$

Let $\{g_s(t)\}$ be a sequence of symmetric molliflying functions such that

$$\begin{cases} g_s(t) \in C_0^\infty(\mathbb{R}), \; g_s(t) \geq 0 \; g_s(t) = g_s(-t) \\ \text{supp } g_s(t) \subset [-1/s, 1/s] \\ \displaystyle\int_{\mathbb{R}} g_s(t)dt = 1 \end{cases}$$

Then for every $p > 2/a$, $\forall s > \max\left\{p, \dfrac{1}{T-2a}\right\}$ and for $|h| < \dfrac{\sigma}{2}$ we consider the function

$$\varphi = \tau_{r,-h}\{\vartheta^{2m}\rho_p[(\rho_p\tau_{r,h}u) * g_s]\} \tag{2.3}$$

where

$$\tau_{r,h}u = u(x + he^r, t) - u(x,t),\ r \text{ integer, } 1 \leq r \leq n,$$

and, setting (2.3) in (1.6), we obtain:

$$\int_Q \sum_{|\alpha|=m} (\tau_{r,h} A^\alpha(D^{(m)}u) | D^\alpha\{\vartheta^{2m}\rho_p(\rho_p\tau_{r,h}u) * g_s]\})dX = \tag{2.4}$$

$$= \int_Q (\tau_{r,h}u|\vartheta^{2m}\{\rho_p[(\rho_p\tau_{r,h}u) * g_s]\}')dX.$$

Let us set

$$A_{\alpha\beta} = \{A^{rs}_{\alpha\beta}\} \quad \text{with} \quad A^{rs}_{\alpha\beta} = \int_0^1 \frac{\partial A^\alpha_r(D^{(m)}u + \eta\tau_{r,h}D^{(m)}u)}{\partial p^\beta_s} d\eta$$

and let us observe that

$$\sum_{|\beta|=m} A_{\alpha\beta}\tau_{r,h}D^\beta u = \int_0^1 \sum_{|\beta|=m} \frac{\partial A^\alpha(D^{(m)}u + \eta\tau_{r,h}D^{(m)}u)}{\partial p^\beta} \tau_{r,h}D^\beta u d\eta =$$

$$= \int_0^1 \frac{\partial A^\alpha(D^{(m)}u + \eta\tau_{r,h}D^{(m)}u)}{d\eta} d\eta =$$

$$= A^\alpha(D^{(m)}u(x + he^r, t)) - A^\alpha(D^{(m)}u(x,t)) =$$

$$= \tau_{r,h}A^\alpha(D^{(m)}u(x,t));$$

and therefore we have

$$\tau_{r,h}A^\alpha(D^{(m)}u) = \sum_{|\beta|=m} A_{\alpha\beta}\tau_{r,h}D^\beta u. \tag{2.5}$$

Taking into account that in virtue of symmetry of $g_s(t)$ it results

$$\int_Q (\tau_{r,h}u|\vartheta^{2m}\rho_p[(\rho_p\tau_{r,h}u) * g_s]')dX = 0, \tag{2.6}$$

from (2.4), (2.5) and (2.6), we obtain

$$\text{(2.7)} \qquad \int_Q \vartheta^{2m} \sum_{|\alpha|=m} \sum_{|\beta|=m} (A_{\alpha\beta}\rho_p\tau_{r,h}D^\beta u|(\rho_p\tau_{r,h}D^\alpha u) * g_s)dX +$$

$$+\int_Q \vartheta^m \sum_{|\alpha|=m} \sum_{|\beta|=m} (A_{\alpha\beta}\rho_p\tau_{r,h}D^\beta u| \sum_{\gamma<\alpha} C_{\alpha\gamma}(\vartheta)[(\rho_p\tau_{r,h}D^\gamma u) * g_s])dX =$$

$$= \int_Q \vartheta^{2m} \rho_p'(\tau_{r,h}u|(\rho_p\tau_{r,h}u) * g_s)dX$$

where $C_{\alpha\gamma}(\vartheta)$ are suitable function such that

$$D^\alpha\{\vartheta^{2m}\rho_p[(\rho_p\tau_{r,h}u) * g_s]\} = \vartheta^{2m}\rho_p[(\rho_p\tau_{r,h}D^\alpha u) * g_s] +$$

$$+\vartheta^m\rho_p \sum_{\gamma<\alpha} C_{\alpha\gamma}(\vartheta)[(\rho_p\tau_{r,h}D^\gamma u) * g_s]$$

and

$$|C_{\alpha\gamma}(\vartheta)| \leq c\sigma^{|\gamma|-m}.$$

It is well known that if $s \to +\infty$ it results

$$(\rho_p\tau_{r,h}u) * g_s \longrightarrow \rho_p\tau_{r,h}u \quad \text{in} \quad L^2(-T, 0, H^m(\Omega, \mathbb{R}^N)),$$

therefore from (2.7), taking the limit for $s \to +\infty$, we obtain

$$\text{(2.8)} \qquad \int_Q \vartheta^{2m} \sum_{|\alpha|=m} \sum_{|\beta|=m} (A_{\alpha\beta}\rho_p\tau_{r,h}D^\beta u|\rho_p\tau_{r,h}D^\alpha u)dX +$$

$$+\int_Q \vartheta^m \sum_{|\alpha|=m} \sum_{|\beta|=m} (A_{\alpha\beta}\rho_p\tau_{r,h}D^\beta u| \sum_{\gamma<\alpha} \rho_p\tau_{r,h}C_{\alpha\gamma}(\vartheta)D^\gamma u)dX =$$

$$= \int_Q \vartheta^{2m}\rho_p'(\tau_{r,h}u|\rho_p\tau_{r,h}u)dX.$$

By the ellipticity condition and Holder inequality, for every $\varepsilon > 0$, (2.8) becomes

$$\nu \int_Q \vartheta^{2m} \rho_p^2 \sum_{|\alpha|=m} ||\tau_{r,h} D^\alpha u||^2 dX \leq \varepsilon \int_Q \rho_p^2 \vartheta^{2m} \sum_{|\alpha|=m} ||\tau_{r,h} D^\alpha u||^2 dX +$$

$$+ c(\varepsilon, M) \int_Q \rho_p^2 \sum_{|\alpha|=m} \sum_{\gamma<\alpha} |C_{\alpha\gamma}(\vartheta)|^2 ||\tau_{r,h} D^\gamma u||^2 dX +$$

$$+ \int_Q \vartheta^{2m} \rho_p \rho_p' ||\tau_{r,h} u||^2 dX.$$

At last, since it results

$$\rho_p \rho_p' \begin{cases} \leq 0 & \text{for } t \geq -2/p \\ = 0 & \text{for } t \leq -2a \text{ and for } -a \leq t \leq -2/p \\ \leq 1/a & \text{for } -2a \leq t \leq -a \end{cases}$$

for ε sufficiently small we obtain

$$\int_{-a}^{-2/p} dt \int_{B(\sigma)} \sum_{|\alpha|=m} ||\tau_{r,h} D^\alpha u||^2 dx \leq \frac{c(\nu)}{a} \int_{-2a}^{0} dt \int_{B(\frac{3}{2}\sigma)} ||\tau_{r,h} u||^2 dx +$$

$$+ \sum_{|\alpha|=m} \sum_{\gamma<\alpha} \frac{c(\nu, M)}{\sigma^{2(m-|\gamma|)}} \int_{-2a}^{0} dt \int_{B(\frac{3}{2}\sigma)} ||\tau_{r,h} D^\gamma u||^2 dx.$$

and taking the limit for $p \to \infty$ we have

$$\int_{-a}^{0} dt \int_{B(\sigma)} \sum_{|\alpha|=m} ||\tau_{r,h} D^\alpha u||^2 dx \leq \frac{c(\nu)}{a} \int_{-2a}^{0} dt \int_{B(\frac{3}{2}\sigma)} ||\tau_{r,h} u||^2 dx +$$

$$+ \sum_{|\alpha|=m} \sum_{\gamma<\alpha} \frac{c(\nu, M)}{\sigma^{2(m-|\gamma|)}} \int_{-2a}^{0} dt \int_{B(\frac{3}{2}\sigma)} ||\tau_{r,h} D^\gamma u||^2 dx.$$

Applying the lemma 3.VI of [3], Chap. I, we obtain

$$\int_{-a}^{0} dt \int_{B(\sigma)} \sum_{|\alpha|=m} ||\tau_{r,h} D^{\alpha} u||^2 dx \leq c(\nu) \frac{|h|^2}{a} \int_{-2a}^{0} dt \int_{B(2\sigma)} ||D^{(1)} u||^2 dx +$$

$$+ \sum_{0 \leq |\alpha| \leq m} \frac{|h|^2 c(\nu, M)}{\sigma^{2(m-|\alpha|+1)}} \int_{-2a}^{0} dt \int_{B(2\sigma)} ||D^{\alpha} u||^2 dx$$

and by means of well known result (see theorem 3.X of [3], Chap. I) we get that it exists $D^{\alpha}u \in L^2(B(\sigma) \times (-a, 0), \mathbb{R}^N)$, $\forall \alpha : |\alpha| = m + 1$, and that the inequality holds:

$$\int_{-a}^{0} dt \int_{B(\sigma)} \sum_{|\alpha|=m+1} ||D^{\alpha} u||^2 dx \leq \frac{c(\nu)}{a} \int_{-2a}^{0} dt \int_{B(2\sigma)} ||D^{(1)} u||^2 dx +$$

$$+ \sum_{0 \leq |\alpha| \leq m} \frac{c(\nu, M)}{\sigma^{2(m-|\alpha|+1)}} \int_{-2a}^{0} dt \int_{B(2\sigma)} ||D^{\alpha} u||^2 dx$$

from which (2.2) follows.

3. A fundamental estimate.

We may use theorem 2.1 to give a foundamental estimate. In fact the following result holh:

THEOREM 3.1. *Let $u \in L^2(-T, 0, H^m(\Omega, \mathbb{R}^N))$ be a solution of the system*

(3.1) $$(-1)^m \sum_{|\alpha|=m} D^{\alpha} A^{\alpha}(D^{(m)} u) + \frac{\partial u}{\partial t} = 0.$$

Then there exists an $\varepsilon \in (0, 1)$ such that $\forall Q(X_0, \sigma) \subset\subset Q$ and $\forall \lambda \in (0, 1)$:

(3.2) $$\int_{Q(X_0, \lambda\sigma)} \sum_{|\alpha|=m+1} ||D^{\alpha} u||^2 dX \leq c(\nu, M) \lambda^{\varepsilon(n+2m)}.$$

$$\cdot\int_{Q(X_0,\sigma)} \sum_{|\alpha|=m+1} ||D^\alpha u||^2 dX.$$

Proof. Let us choose in (1.6) $\varphi = D_s\psi$, $s = 1,\dots,n$ with $\psi \in C_0^\infty(Q(X_0,\sigma),\mathbb{R}^N)$. We obtain

$$\int_{Q(X_0,\sigma)} \sum_{|\alpha|=m} (D_s A^\alpha(D^{(m)}u)|D^\alpha\psi) - \left(D_s u|\frac{\partial\psi}{\partial t}\right) dX = 0$$

and, taking into account that it results

$$D_s A^\alpha(D^{(m)}u) = \sum_{k=1}^{m} \sum_{|\beta|=m} \frac{\partial A^\alpha(D^{(m)}u)}{\partial p_k^\beta} D_s D^\beta u_k, \tag{3.4}$$

we also have

$$\int_{Q(X_0,\sigma)} \left\{ \sum_{|\alpha|=m} \sum_{|\beta|=m} (A_{\alpha\beta}(D^{(m)}u) D^\beta D_s u|D^\alpha\psi) - \left(D_s u|\frac{\partial\psi}{\partial t}\right) \right\} dX = 0 \tag{3.5}$$

$$\forall\psi \in C_0^\infty(Q(X_0,\sigma),\mathbb{R}^N) \text{ and } s = 1,2,\dots,n,$$

with

$$A_{\alpha\beta} = (A_{\alpha\beta}^{hk}), \quad A_{\alpha\beta}^{hk}(p') = \frac{\partial A_h^\alpha(p)}{\partial p_k^\beta}.$$

Setting

$$U = D^{(1)}u$$

$$\mathcal{A}_{\alpha\beta}(D^{(m)}u) = \left\{ \begin{array}{ccccc} A_{\alpha\beta}(D^{(m)}u) & \vdots & 0 & \vdots & 0 \\ 0 & \vdots & & \vdots & 0 \\ 0 & \vdots & 0 & \vdots & A_{\alpha\beta}(D^{(m)}u) \end{array} \right\} n^2 \text{ blocks}$$

from theorem 2.1 it follows that $U \in L^2(t_0-\sigma^{2m}, t_0, H^m(B(x^0,\sigma),\mathbb{R}^{nN}))$ and in viertue of (3.5) we have:

$$\int_{Q(X_0,\sigma)} \left\{ \sum_{|\alpha|=m} \sum_{|\beta|=m} (\mathcal{A}_{\alpha\beta}(D^{(m)}u) D^\beta U|D^\alpha\psi) - \left(U|\frac{\partial\psi}{\partial t}\right) \right\} dX \tag{3.6}$$

$$\forall\psi \in C_0^\infty(Q(X_0,\sigma),\mathbb{R}^{nN}).$$

Because $\mathcal{A}_{\alpha\beta}(p')$ are $nN \times nN$ matrices bounded and elliptic we can apply the theorem (2.1) of [1] that ensures the estimate

$$\int_{Q(\lambda,\sigma)} ||D^{(m)}U||^2 dX \leq c(\nu, M)\lambda^{\varepsilon(n+2m)} \int_{Q(\sigma)} ||D^{(m)}U||^2 dX$$

and so the assert is achieved.

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