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ON THE SYMMETRIC BLOCK DESIGN WITH PARAMETERS $(430,78,14)$

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In this paper we have proved that a Frobenius group of order 301 cannot operate on a symmetric block design with parameters (430,78,14).

1. Introduction

A $2-(v,k,\lambda)$ design $(\mathcal{P},\mathcal{B},I)$ is said to be *symmetric* if the relation $|\mathcal{P}| = |\mathcal{B}| = v$ holds and in that case we often speak of a symmetric design with parameters (v,k,λ). The collection of the parameter sets (v,k,λ) for which a symmetric $2-(v,k,\lambda)$ design exists is often called the "spectrum". The determination of the spectrum for symmetric designs is a widely open problem. For example, a finite projective plane of order n is a symmetric design with parameters $(n^2+n+1, n+1, 1)$ and it is still unknown whether finite projective planes of non–prime–power order may exist at all.

The existence/non-existence of a symmetric design has often required "ad hoc" treatments even for a single parameter set (v,k,λ). The most famous instance of this circumstance is perhaps the non-existence of the projective plane of order 10, see [9].

It is of interest to study symmetric designs with additional properties, which often involve the assumption that a non–trivial automorphism group acts on

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the design under consideration, see for instance [4]. The present paper is concerned with a symmetric design $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ with parameters $(430, 78, 14)$: the existence/non–existence of such a design is still in doubt as far as we know. We shall further assume that the given design admits a certain automorphism group of order 301. This choice is motivated by the work of Z. Janko, see [7]. It is our purpose to prove that a symmetric design with these properties cannot exist. We shall use the method of tactical decompositions, as developed in [7]; see also [4]. We assume the reader is familiar with the basic facts of design theory, see for instance [2], [3] and [10]. If g is an automorphism of a symmetric design $\mathcal{D}$ with parameters (v, k, λ), then g fixes an equal number of points and blocks, see [10, Theorem 3.1, p.78]. We denote the sets of these fixed elements by $F_{\mathcal{P}}(g)$ and $F_{\mathcal{B}}(g)$ respectively, and their cardinality simply by $|F(g)|$. We shall make use of the following upper bound for the number of fixed points, see [10, Corollary 3.7, p. 82]:

$$|F(g)| \leq k + \sqrt{k - \lambda} \tag{1}$$

It is also known that an automorphism group G of a symmetric design has the same number of orbits on the set of points $\mathcal{P}$ as on the set of lines $\mathcal{B}$: [10, Theorem 3.3, p.79]. Denote that number by t.

2. Point- and block-orbits

We adopt the notation and terminology of Section 1 in [4]: we repeat some fundamental relations here for the reader's sake. Let $\mathcal{D}$ be a symmetric design with parameters (v, k, λ) and let G be a subgroup of the automorphism group $Aut\mathcal{D}$ of $\mathcal{D}$. Denote the point orbits of G on $\mathcal{P}$ by $\mathcal{P}_1, \mathcal{P}_2, \dots \mathcal{P}_t$ and the line orbits of G on $\mathcal{B}$ by $\mathcal{B}_1, \mathcal{B}_2, \dots \mathcal{B}_t$. Put $|\mathcal{P}_r| = \omega_r$ and $|\mathcal{B}_i| = \Omega_i$. Obviously,

$$\sum_{r=1}^{t} \omega_r = \sum_{i=1}^{t} \Omega_i = v. \tag{2}$$

Let γ_{ir} be the number of points from $\mathcal{P}_r$, which lie on a line from $\mathcal{B}_i$; clearly this number does not depend on the chosen line. Similarly, let Γ_{js} be the number of lines from $\mathcal{B}_j$ which pass through a point from $\mathcal{P}_s$. Then, obviously,

$$\sum_{r=1}^{t} \gamma_{ir} = k \text{ and } \sum_{j=1}^{t} \Gamma_{js} = k. \tag{3}$$

By [3, Lemma 5.3.1. p.221], the partition of the point set $\mathcal{P}$ and of the block set $\mathcal{B}$ forms a tactical decomposition of the design $\mathcal{D}$ in the sense of [3, p.210]. Thus, the following equations hold:

$$\Omega_i \cdot \gamma_{ir} = \omega_r \cdot \Gamma_{ir} \tag{4}$$

$$\sum_{r=1}^{t} \gamma_{ir}\Gamma_{jr} = \lambda\Omega_j + \delta_{ij}(k-\lambda) \tag{5}$$

$$\sum_{i=1}^{t} \Gamma_{ir}\gamma_{is} = \lambda\omega_s + \delta_{rs}(k-\lambda) \tag{6}$$

where δ_{ij}, δ_{rs} are the Kronecker symbols.

For a proof of these equations, the reader is referred to [3] and [4]. Equation (5), together with (4) yields

$$\sum_{r=1}^{t} \frac{\Omega_j}{\omega_r}\gamma_{ir}\gamma_{jr} = \lambda\Omega_j + \delta_{ij}(k-\lambda); \tag{7}$$

Definition 1. *The $(t\times t)$-matrix (γ_{ir}) is called the orbit structure of the design $\mathcal{D}$.*

The first step in the construction of a design is to find all possible orbit structures. The second step of the construction is usually called indexing. In fact for each coefficient γ_{ir} of the orbit matrix one has to specify which γ_{ir} points of the point orbit $\mathcal{P}_r$ lie on the lines of the block orbit $\mathcal{B}_i$. Of course, it is enough to do this for a representative of each block orbit, as the other lines of that orbit can be obtained by producing all G-images of the given representative.

3. Action of the Frobenius group of order 301

In our construction of symmetric $2-(430,78,14)$ designs we assume the existence of an automorphism group $G = \langle \rho,\sigma | \rho^{43} = \sigma^7 = 1, \rho^\sigma = \rho^4\rangle$, which is a so called Frobenius group of order 301 with Frobenius kernel of order 43 (see [6]).

Lemma 3.1. *Let ρ be an element of G with $o(\rho) = 43$. Then $\langle\rho\rangle$ acts fixed–point–free.*

Proof. We know that the number of fixed points is the same as the number of fixed blocks for the action of $\langle\rho\rangle$ on $\mathcal{D}$. Denote this number by f. Obviously $f \equiv 430 (\mathrm{mod}\ 43)$, i.e. $f \equiv 0 (\mathrm{mod}\ 43)$. The upper bound (1) for the number of fixed points yields $f \in \{0,43,86\}$. As $o(\rho) > \lambda$, an application of a result of M. Aschbacher [1, Lemma 2.6, p.274] forces the fixed structure to be a subdesign of $\mathcal{D}$. But there is no symmetric design with $v = 43$ or $v = 86$ and $\lambda = 14$. Hence, f is equal to 0. □

Our next task is to determine the lengths of the orbits of G on the sets of points and blocks of the symmetric block design $\mathcal{D}$. The possible orbit lengths are $7, 43, 301$.

Lemma 3.2. *There is no orbit of length* 7.

Proof. If false, then ρ would have at least seven fixed points or seven fixed lines, which is not possible. $\square$

Lemma 3.3. *There is no orbit of length* 301.

Proof. Up to reordering, there are precisely two possibilities for the arrays expressing the lengths of the G–orbits on points and blocks, namely:

$$\mathcal{O}_1 = [43,43,43,301], \mathcal{O}_2 = [43,43,43,43,43,43,43,43,43,43].$$

The case $\mathcal{O}_1$ does not occur, as then there is no orbit structure (there is no solutions for equations (7) and (3)). Thus, we are in case $\mathcal{O}_2$. $\square$

Thus we have

Theorem 3.4. *The action of the group G yields* 10 *orbits on points and* 10 *orbits on blocks, each of length* 43.

In what follows we assume $|\mathcal{P}_i| = |\mathcal{B}_i| = 43$, $i = 1,2,\ldots,10$. From the structure of G it follows that G acts faithfully on each line and point orbit. For $i = 1,2,\ldots,10$, we put $\mathcal{P}_i = \{p^i_0, p^i_1, \ldots, p^i_{42}\}$. Thus, G acts on each such point orbit as a permutation group in a unique way, up to point–labelling. Hence, for the two generators of G we may put:

$\rho = (0,1,2,3,4,5,6,7,8,9,0,11,12,13,14,15,16,17,18,19,20,21,22,23,$
$24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42)$

and

$\sigma = (0)(1,4,16,21,41,35,11)(2,8,32,42,39,27,22)(5,20,37,19,33,3,12)$
$(6,24,10,40,31,38,23)(7,28,26,18,29,30,34)(9,36,15,17,25,14,13).$

where, it is clear that, the above indicated numbers refer to point–subscripts.

We immediately obtain the following

Corollary 3.5. *The element σ of G of order* 7 *fixes precisely* 10 *points and* 10 *blocks of $\mathcal{D}$. Each block orbit contains a unique line stabilized by σ.*

The following definition is basic for our construction of designs.

Definition 2. *The set of indices of the points of $\mathcal{P}_r$, which lie on a fixed representative of block orbit $\mathcal{B}_i$, is called the index set for the position (i,r) of the orbit structure.*

In what follows, we are going to construct a representative for each block orbit: namely, the line fixed by σ. Clearly σ acts on the intersection of $\mathcal{P}_r$ with the representative of the block orbit $\mathcal{B}_i$. Therefore, the numbers γ_{ir} are all congruent to 0 or 1 modulo 7. This assumption together with the system (7)-(3) gives a unique solution for the orbit structure of the design $\mathcal{D}$:

OS	43	43	43	43	43	43	43	43	43	43
43	15	7	7	7	7	7	7	7	7	7
43	7	15	7	7	7	7	7	7	7	7
43	7	7	15	7	7	7	7	7	7	7
43	7	7	7	15	7	7	7	7	7	7
43	7	7	7	7	15	7	7	7	7	7
43	7	7	7	7	7	15	7	7	7	7
43	7	7	7	7	7	7	15	7	7	7
43	7	7	7	7	7	7	7	15	7	7
43	7	7	7	7	7	7	7	7	15	7
43	7	7	7	7	7	7	7	7	7	15

An automorphism of a matrix is a permutation of rows followed by a permutation of columns leaving the matrix unchanged. It is clear that the set of all such automorphisms is a group, which we call the automorphism group of that matrix. The automorphism group of the above matrix is isomorphic to the full symmetric group Σ_{10} of degree ten. We shall use this fact to eliminate isomorphic designs during the indexing process.

4. Indexing of the Representatives for Each Block Orbit

Denote the matrix of the orbit structure by A and its coefficients by $a_{ij}, 1 \leq i, j \leq 10$. Obviously, $a_{ij} \in \{7, 15\}$. We want to find all possibilities for the index sets for all positions of the matrix A. We obtain these possibilities from the cycles of the permutation representation of σ. In case $a_{ij} = 7$ the index set is obtained from a single cycle of σ, yielding precisely six possibilities; in case $a_{ij} = 15$ the index set is obtained from a pair of cycles of σ together with its fixed element 0, yielding precisely $\binom{6}{2} = 15$ possibilities. Altogether, we obtain precisely 21 index sets. We write them down and denote them by the non-negative integers from $0'$ to $20'$:

$0' = \{1,4,16,21,41,35,11\}$,
$1' = \{2,8,32,42,39,27,22\}$,
$2' = \{5,20,37,19,33,3,12\}$,
$3' = \{6,24,10,40,31,38,23\}$,
$4' = \{7,28,26,18,29,30,34\}$,
$5' = \{9,36,15,17,25,14,13\}$,
$6' = \{0,1,4,16,21,41,35,11,2,8,32,42,39,27,22\}$,
$7' = \{0,1,4,16,21,41,35,11,5,20,37,19,33,3,12\}$,
$8' = \{0,1,4,16,21,41,35,11,6,24,10,40,31,38,23\}$,

$$
\begin{aligned}
9' &= \{0,1,4,16,21,41,35,11,7,28,26,18,29,30,34\},\\
10' &= \{0,1,4,16,21,41,35,11,9,36,15,17,25,14,13\},\\
11' &= \{0,2,8,32,42,39,27,22,5,20,37,19,33,3,12\},\\
12' &= \{0,2,8,32,42,39,27,22,6,24,10,40,31,38,23\},\\
13' &= \{0,2,8,32,42,39,27,22,7,28,26,18,29,30,34\},\\
14' &= \{0,2,8,32,42,39,27,22,9,36,15,17,25,14,13\},\\
15' &= \{0,5,20,37,19,33,3,12,6,24,10,40,31,38,23\},\\
16' &= \{0,5,20,37,19,33,3,12,7,28,26,18,29,30,34\},\\
17' &= \{0,5,20,37,19,33,3,12,9,36,15,17,25,14,13\},\\
18' &= \{0,6,24,10,40,31,38,23,7,28,26,18,29,30,34\},\\
19' &= \{0,6,24,10,40,31,38,23,9,36,15,17,25,14,13\},\\
20' &= \{0,7,28,26,18,29,30,34,9,36,15,17,25,14,13\}.
\end{aligned}
$$

For each of the block representatives corresponding to the rows of A we obtain $15 \cdot 6^9 = 151165440$ possibilities for the index sets arising from the various point–orbits.

Now, one constructs the possible orbits of length 43 one by one. To do this, one considers the rows of A and replaces the numbers a_{ij} by index sets of appropriate size, using the integer name for these index sets. For example, let us take the first row of A. By making use of the ordering of the index sets, the first possibility for an orbit to check would be $L_1:\ 6'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'$.

One applies the group $\langle\rho\rangle$ of order 43 to the index sets occurring in L_1 and checks whether two dierent ρ–images have the right intersection number, namely 14. If the intersection condition is satisfied, then we retain L_1; otherwise, we discard it. The next possibility to check would be $L_1:\ 6'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 0'\ 1'$, and the last one for the first row of A would be $L_1:\ 20'\ 5'\ 5'\ 5'\ 5'\ 5'\ 5'\ 5'\ 5'\ 5'$.

In this way, one obtains ten sets which exhaust all the possibilities for the ten block orbits, respectively, and which then have to be checked against each other for the intersection property.

To reduce the number of possibilities and to eliminate isomorphic designs as soon as possible, we make use of the group generated by the mapping

$$\alpha : x \mapsto 3x \pmod{43}.$$

Clearly, α induces an automorphism of order 42 of $\langle\rho\rangle$ which commutes with σ. It is well known that such a group produces isomorphic designs [4, Lemma 1.8 p.54]. The cycle decomposition of α on the 21 index sets is

$$(0',2',5',1',3',4')(6',15',20')(7',17',14',12',18',9')(8',16',10',11',19',13').$$

Hence, α acts as an element of order 6 on the set of index sets.

We have thus used three means for reducing the output of isomorphic symmetric designs, namely: the automorphism group of the orbit structure; the lexicographical ordering of the index sets to get an ordering of the orbit or block types and an ordering of designs (for a precise explanation of how one introduces such an ordering the reader is referred to [4]); the group generated by α (one only needs to consider one index set for each cycle of α).

5. Result

The computations outlined here have been carried out by a computer. Our main result is contained in the following.

Theorem 5.1. *There is no symmetric designs with parameters* $(430, 78, 14)$ *which is faithfully acted upon by G.*

Proof. An exhaustive investigation carried out by computer shows that there is no possibility for indexing the first representative block. We have implemented the search by a C++ program, available from the authors, which ran 7 minutes on a Personal Computer of the Department of Mathematics of the University of Prishtina. The computer was equipped with a 32 bit PENTIUM IV processor and a 1GB RAM under a Windows operating system. See the C++ program given in the Appendix. □

6. Appendix: the computation

```
/* indexing the first representative block L1 (430,78,14) with Frob_43x7 */
#include <stdio.h>
#define MOD 43
#define LAMBDA 14
int h[MOD]; int a[78]; int i,j,m; int a1,a2,a3,a4,a5,a6,a7,a8,a9,a10; double wz,
w2;
    /* Index sets */
int i1[1]={{0}};
int i7[6][7]={{1,4,16,21,41,35,11},{2,8,32,42,39,27,22},{5,20,37,19,33,3,12},
              {6,24,10,40,31,38,23},{7,28,26,18,29,30,34},{9,36,15,17,25,14,13}};
int i15[15][15]={{0,1,4,16,21,41,35,11,2,8,32,42,39,27,22},
                 {0,1,4,16,21,41,35,11,5,20,37,19,33,3,12},
                 {0,1,4,16,21,41,35,11,6,24,10,40,31,38,23},
                 {0,1,4,16,21,41,35,11,7,28,26,18,29,30,34},
                 {0,1,4,16,21,41,35,11,9,36,15,17,25,14,13},
                 {0,2,8,32,42,39,27,22,5,20,37,19,33,3,12},
                 {0,2,8,32,42,39,27,22,6,24,10,40,31,38,23},
                 {0,2,8,32,42,39,27,22,7,28,26,18,29,30,34},
                 {0,2,8,32,42,39,27,22,9,36,15,17,25,14,13},
```

```
                    {0,5,20,37,19,33,3,12,6,24,10,40,31,38,23},
                    {0,5,20,37,19,33,3,12,7,28,26,18,29,30,34},
                    {0,5,20,37,19,33,3,12,9,36,15,17,25,14,13},
                    {0,6,24,10,40,31,38,23,7,28,26,18,29,30,34},
                    {0,6,24,10,40,31,38,23,9,36,15,17,25,14,13},
                    {0,7,28,26,18,29,30,34,9,36,15,17,25,14,13}};
FILE *fo;
int mod();
/* ―――――――――――――――――――― */
void main() {
    if((fo=fopen("I_L1.TXT","w"))==NULL) { printf("\a\a Can not open the
file \n ","I_L1.TXT"); exit(0); }
/* 1 */
for (a1=0;a1<3 ;a1++) { for (i=0; i<15;i++) a[i]=i15[a1][i];
 /* 2 */
 for (a2=0;a2<6;a2++) { for (i=0; i<7;i++) a[15+i]=i7[a2][i];
  /* 3 */
  for (a3=0;a3<6;a3++) { for (i=0; i<7;i++) a[22+i]=i7[a3][i];
   /* 4 */
   for (a4=0;a4<6 ;a4++) { for (i=0; i<7;i++) a[29+i]=i7[a4][i];
    /* 5 */
    for (a5=0;a5<6;a5++) { for (i=0; i<7;i++) a[36+i]=i7[a5][i];
     /* 6 */
     for (a6=0;a6<6;a6++) { for (i=0; i<7;i++) a[43+i]=i7[a6][i];
      /* 7 */
      for (a7=0;a7<6;a7++) { for (i=0; i<7;i++) a[50+i]=i7[a7][i];
       /* 8 */
       for (a8=0;a8<6;a8++) { for (i=0; i<7;i++) a[57+i]=i7[a8][i];
        /* 9 */
        for (a9=0;a9<6;a9++) { for (i=0; i<7;i++) a[64+i]=i7[a9][i];
         /* 10 */
         for (a10=0;a10<6;a10++) { for (i=0; i<7;i++) a[71+i]=i7[a10][i];
                w2++;
                /* check for compatibility */
                for (m=1;m<MOD;m++) h[m]=0;
                for (i=0; i<15;i++) { for (j=0; j<15;j++)
                        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
                for (i=15; i<22;i++) { for (j=15; j<22;j++)
                        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
                for (i=22; i<29;i++) { for (j=22; j<29;j++)
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        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=29; i<36;i++) { for (j=29; j<36;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=36; i<43;i++) { for (j=36; j<43;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=43; i<50;i++) { for (j=43; j<50;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=50; i<57;i++) { for (j=50; j<57;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=57; i<64;i++) { for (j=57; j<64;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=64; i<71;i++) { for (j=64; j<71;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (i=71; i<78;i++) { for (j=71; j<78;j++)
        { if (i==j) continue; h[mod(a[i]-a[j],MOD)]++; } }
    for (m=1;m < MOD;m++){ if (h[m]>LAMBDA) goto sta10 ; }
    wz++;
    /* Solution */
    fprintf(fo,"%8.0f,%8.0f \n",wz,w2);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[0], a[1], a[2], a[3], a[4], a[5], a[6],a[7], a[8], a[9]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[10], a[11], a[12], a[13], a[14], a[15], a[16],a[17], a[18], a[19]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[20], a[21], a[22], a[23], a[24], a[25], a[26],a[27], a[28], a[29]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[30], a[31], a[32], a[33], a[34], a[35], a[36],a[37], a[38], a[39]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[40], a[41], a[42], a[43], a[44], a[45], a[46],a[47], a[48], a[49]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[50], a[51], a[52], a[53], a[54], a[55], a[56],a[57], a[58], a[59]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,",
    a[60], a[61], a[62], a[63], a[64], a[65], a[66],a[67], a[68], a[69]);
    fprintf(fo,"%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d,%2d\n",
    a[70], a[71], a[72], a[73], a[74], a[75], a[76],a[77]);
sta10:;
      } /* for a10 */
     } /* for a9 */
    } /* for a8 */
   } /* for a7 */
```

```
          } /* for a6 */
         } /* for a5 */
        } /* for a4 */
       } /* for a3 */
      } /* for a2 */
     } /* for a1 */
     fclose(fo);
     } /* end */
/* ———————————————— */
/* modulo function */
mod(a,b)
int a,b;
{ int i,j; i=a/b; j=a - b*i; if(j<0) j=j+b; return(j); }
```

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