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SOME SUBORDINATION THEOREMS ASSOCIATED WITH A NEW OPERATOR

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In this paper we introduce a linear operator and obtain certain differential subordination properties associated with this linear operator. Some relevant consequences of the main results including new variations of earlier known results are also pointed out.

1. Introduction and Preliminaries

Let $\mathscr{H}(\mathbb{U})$ represent a space of analytic functions in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, then for $a \in \mathbb{C}$ and $n \in \mathbb{N}$, we let $\mathscr{A}_n$ and $\mathscr{H}[a,n]$ denote, respectively, the subclasses of the class $\mathscr{H}(\mathbb{U})$ defined by

$$\mathscr{A}_n = \left\{f \in \mathscr{H}(\mathbb{U});\ f(z) = z + a_{n+1}z^{n+1} + \dots ; z \in \mathbb{U}\right\}$$

and

$$\mathscr{H}[a,n] = \left\{f \in \mathscr{H}(\mathbb{U});\ f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \dots ;\ z \in \mathbb{U}\right\},$$

with $\mathscr{A}_1 = \mathscr{A}$. A function f analytic in $\mathbb{U}$ is said to be convex if it is univalent and $f(\mathbb{U})$ is convex. We denote by $\mathscr{K}$ the class of convex functions in $\mathbb{U}$ defined by

$$\mathscr{K} = \left\{f \in \mathscr{A}, \Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0, z \in \mathbb{U}\right\}.$$

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If $f,g \in \mathscr{H}$, then the function f is said to be subordinate to g, written as $f(z) \prec g(z)$, $z \in \mathbb{U}$, if there exists a Schwarz function $w \in \mathscr{H}$ with $w(0) = 0$ and $|w(z)| < 1$, $z \in \mathbb{U}$ such that $f(z) = g(w(z))$.
In particular, if g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$f(z) \prec g(z) \iff f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

The concept of subordination was mainly used in defining various classes of functions and studying their basic properties in the geometric function theory. For the functions

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \text{ and } g(z) = \sum_{k=0}^{\infty} b_k z^k,$$

we denote by $f * g$ the convolution (or Hadamard product) of f and g defined by

$$(f*g)(z) = \sum_{k=0}^{\infty} a_k b_k z^k \ (z \in \mathbb{U}).$$

Suppose $\psi : \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$, and let h be univalent in $\mathbb{U}$. If $p(z)$ is analytic in $\mathbb{U}$ and satisfies the second-order diferential subordination

$$\psi\left(p(z), zp'(z), z^2p''(z); z\right) \prec h(z), z \in \mathbb{U}, \tag{1.1}$$

then $p(z)$ is called a solution of the differential subordination. The univalent function q is called a dominant, if $p \prec q$ for all p satisfying (1.1). A dominant $\widetilde{q}$ that satisfies $\widetilde{q} \prec q$ for all dominants q of (1.1) is said to be the best dominant of (1.1). The best dominant is unique up to a rotation of $\mathbb{U}$.

In order to prove our main results, we require the following lemmas.

Lemma 1.1. (Hallenbeck and Ruscheweyh [4]; see also [5, Theorem 3.1.6, p.71]). *Let h be a convex function in $\mathbb{U}$ with $h(0) = a$, $0 \neq \gamma \in \mathbb{C}$ and $\Re\,\gamma \geqq 0$. If $p(z) \in \mathscr{H}[a,n]$ and*

$$p(z) + \frac{1}{\gamma} zp'(z) \prec h(z),$$

then

$$p(z) \prec q(z) \prec h(z),$$

where

$$q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t) t^{\frac{\gamma}{n}-1} dt.$$

The function q is convex and is the best (a,n)-dominant.

Lemma 1.2. ([6, Lemma 13.5.1, p. 375]) *Let g be a convex function in $\mathbb{U}$, and let*

$$h(z) = g(z) + n\alpha z g'(z) \ \ (z \in \mathbb{U}),$$

where $\alpha > 0$ and n a positive integer. If

$$p(z) = g(0) + p_n z^n + p_{n+1} z^{n+1} + \dots \ \ (z \in \mathbb{U}),$$

is holomorhic in $\mathbb{U}$, and

$$p(z) + \alpha z p'(z) \prec h(z) \ \ (z \in \mathbb{U}),$$

then

$$p(z) \prec g(z)$$

and this result is sharp.

For the purpose of this paper, we introduce here a new linear operator $\mathscr{I}_{\lambda}^{l,m}(a,c) : \mathscr{A} \to \mathscr{A}$, which is defined as follows:
If $f \in \mathscr{A}$ is of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.2}$$

then

$$\mathscr{I}_{\lambda}^{l,m}(a,c) f(z) = z + \sum_{k=2}^{\infty} (1 + \lambda(k-1))^l \left[\frac{(a)_{k-1}}{(c)_{k-1}} \right]^m a_k z^k. \tag{1.3}$$

$$\left(\lambda \geqq 0, a \in \mathbb{R}, c \in \mathbb{R}/\mathbb{Z}_0^-;\ \mathbb{Z}_0^- = \{0, -1, -2, \dots\};\ m, l \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\right)$$

It is obvious from (1.3) that

$$\mathscr{I}_{\lambda}^{0,0}(a,c) f(z) = f(z),$$

$$\mathscr{I}_{\lambda}^{0,1}(a,c) f(z) = \mathscr{L}(a,c) f(z)$$

and

$$\mathscr{I}_{\lambda}^{1,1}(a,c) f(z) = (1-\lambda)\mathscr{L}(a,c) f(z) + \lambda z (\mathscr{L}(a,c) f(z))',$$

where $\mathscr{L}(a,c)$ is the Carlson-Shaffer operator [3]. It is easily verified from the above definition that the operator $\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)$ satisfies a three-term recurrence relation given by

$$\mathscr{I}_{\lambda}^{l+1,m}(a,c) f(z) = (1-\lambda) \mathscr{I}_{\lambda}^{l,m}(a,c) f(z) + \lambda z \left[\mathscr{I}_{\lambda}^{l,m}(a,c) f(z) \right]'. \tag{1.4}$$

The motivation in considering a linear operator such as the one defined by (1.3) is mainly to provide a unification to various known linear operators. A special case of the linear operator (1.3) (when $l = m$) was very recently extended to include the Dziok-Srivastava linear operator in [13]. We note that the operator $\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)$ generalizes some known operators which are exhibited here by the following relationships:

(i) $\mathscr{J}_{\lambda}^{l,m}(a,a) \equiv D_{\lambda}^{l}$ (Al-Oboudi differential operator [2])

(ii) $\mathscr{J}_{\lambda}^{l,l}(a,c) \equiv I_{a,c;\lambda}^{l}$ (Prajapat and Raina [9])

(iii) $\mathscr{J}_{1}^{l,m}(a,a) \equiv D^{l}$ (Sâlâgean differential operator [10])

(iv) $\mathscr{J}_{\lambda}^{l,1}(a,c) \equiv D_{\lambda}^{l}(a,c)$ (Selvaraj and Karthikeyan [11]).

In this paper we obtain certain subordination properties (Theorems 1-5 below) involving the linear operator (1.3). Some corollaries and examples are also deduced from the main results exhibiting the usefulness and relevant connections with other results.

2. Main Results

Theorem 2.1. *Let*

$$h(z) = \left(\frac{1+Az}{1+Bz}\right)^{r} \; (z \in \mathbb{U};\; |A| \leqq 1;\; |B| \leqq 1;\; A \neq B;\; 0 < r \leq 1). \tag{2.1}$$

If $\lambda > 0, l, m \in \mathbb{N}_0$ *and* $f \in \mathscr{A}$ *satisfies the differential subordination:*

$$[\mathscr{J}_{\lambda}^{l+1,m}(a,c)f(z)]' \prec h(z) \; (z \in \mathbb{U}), \tag{2.2}$$

then

$$[\mathscr{J}_{\lambda}^{l,m}(a,c)f(z)]' \prec q(z), \tag{2.3}$$

where

$$q(z) = \begin{cases} \left(\frac{A}{B}\right)^{r} \sum\limits_{i \geq 0} \frac{(-r)_i}{i!} \left(\frac{A-B}{A}\right)^{i} (1+Bz)^{-i} {}_2\mathrm{F}_1\left(i,1;1+\frac{1}{\lambda};\frac{Bz}{1+Bz}\right) \; (B \neq 0), \\ {}_2F_1\left(-r,\frac{1}{\lambda};1+\frac{1}{\lambda};-Az\right), \; (B = 0), \end{cases} \tag{2.4}$$

and the function q(z) is convex and is the best (1,1)- dominant.

Proof. We first observe ([12, p. 16]; see also [8, p. 132]) that the function $h(z)$ defined by (2.1) is analytic and convex univalent in $\mathbb{U}$, since

$$\Re\left(1+\frac{zh''(z)}{h'(z)}\right) = -1+(1-r)\,\Re\left(\frac{1}{1+Az}\right)+(1+r)\,\Re\left(\frac{1}{1+Bz}\right)$$

$$> -1+\frac{1-r}{1+|A|}+\frac{1+r}{1+|B|} \;\geq\; 0 \qquad (z \in \mathbb{U}).$$

Differentiating the recurrence relation (1.4) (which is satisfied by the operator $\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)$) with respect to z, we get

$$[\mathscr{I}_{\lambda}^{l+1,m}(a,c)f(z)]' = [\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)]' + \lambda z[\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)]'' \ (z \in \mathbb{U}), \tag{2.5}$$

and (2.5) in view of (2.2) gives

$$[\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)]' + \lambda z[\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)]'' \prec h(z) = \left(\frac{1+Az}{1+Bz}\right)^r. \tag{2.6}$$

Let

$$p(z) = \left[\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)\right]' = 1 + p_1 z + p_2 z^2 + \ldots \ (z \in \mathbb{U}, p \in \mathscr{H}[1,1]), \tag{2.7}$$

then (2.6) and (2.7) yield the differential subordination

$$p(z) + \lambda z p'(z) \prec h(z) = \left(\frac{1+Az}{1+Bz}\right)^r \ (z \in \mathbb{U}).$$

Applying now Lemma 1.1, we conclude that

$$p(z) \prec q(z) = \frac{1}{\lambda z^{1/\lambda}} \int_0^z h(t) t^{\frac{1}{\lambda}-1} dt \ = \frac{1}{\lambda z^{1/\lambda}} \int_0^z \left(\frac{1+At}{1+Bt}\right)^r t^{\frac{1}{\lambda}-1} dt \ .$$

To evaluate the integral (see [8]), we first express the integrand in the form

$$t^{\frac{1}{\lambda}-1} \left(\frac{1+At}{1+Bt)}\right)^r = \left(\frac{A}{B}\right)^r t^{\frac{1}{\lambda}-1} \left(1 - \frac{A-B}{A(1+Bt)}\right)^r,$$

expanding the binomial expression and using the following well known integral and transformation formulas ([1]; see also [5, p.7]):

$$\int_0^1 t^{b-1} \, (1-t)^{c-b-1} \, (1-zt)^{-a} \, dt = \frac{\Gamma(b)\,\Gamma(c-b)}{\Gamma(c)} \, {}_2F_1 \, (a,b;\, c;z)$$

and

$${}_2F_1 \, (a,b;\, c;z) = (1-z)^{-a} \, {}_2F_1 \, \left(a, c-b; c; \frac{z}{z-1}\right)$$

in the steps of evaluation of the integral, we finally obtain

$$q(z) = \left(\frac{A}{B}\right)^r \sum_{i \geq 0} \frac{(-r)_i}{i!} \left(\frac{A-B}{A}\right)^i (1+Bz)^{-i} \, {}_2\mathrm{F}_1 \left(i, 1; 1 + \frac{1}{\lambda}; \frac{Bz}{1+Bz}\right) \ (B \neq 0).$$

On the other hand, if $B=0$, then

$$q(z)=\frac{1}{\lambda z^{1/\lambda}}\int_0^z (1+At)^r t^{\frac{1}{\lambda}-1}dt,$$

which upon integrating similarly (as above) gives

$$q(z)={}_2F_1\left(-r,\frac{1}{\lambda};1+\frac{1}{\lambda};-Az\right).$$

In view of Lemma 1 (for $\gamma=\frac{1}{\lambda}, n=1$), we assert that

$$[\mathscr{I}_{\lambda}^{l,m}(a,c)f(z)]'\prec q(z)\prec h(z),$$

where

$$q(z)=\frac{1}{\lambda z^{1/\lambda}}\int_0^z h(t)t^{\frac{1}{\lambda}-1}dt=\frac{1}{\lambda z^{1/\lambda}}\int_0^z\left(\frac{1+At}{1+Bt}\right)^r t^{\frac{1}{\lambda}-1}dt\ \ (z\in\mathbb{U}),$$

whose value is given by (2.4) is convex and is the best (1,1)-dominant (see [5, p.72]), which completes the proof. $\square$

Remark 2.2. We note that for $r=1, A=2\alpha-1$ $(0\leqq\alpha<1)$ and $B=0$:

$${}_2F_1\left(-1,\frac{1}{\lambda};1+\frac{1}{\lambda};(1-2\alpha)z\right)=1+\frac{(2\alpha-1)}{\lambda+1}z,$$

therefore, $q(z)$ given by (2.4) becomes

$$q(z)=\begin{cases} 2\alpha-1-2(\alpha-1)(1+z)^{-1}{}_2F_1(1,1;\frac{\lambda+1}{\lambda};\frac{z}{z+1}) & \text{for} \quad B=1\\ 1+\frac{(2\alpha-1)}{\lambda+1}z & \text{for} \quad B=0\end{cases}$$

Evidently then, for $c=a$ and $r=1, A=2\alpha-1$ $(0\leqq\alpha<1)$ and $B=1$, Theorem 2.1 corresponds to a simplified form of the known result of Oros and Oros [7, Theorem 1, p. 872]. We deem it proper here to point out a correction in one of the main results of [8]. The subordinated function mentioned in [8, Theorem 3.1, p. 131] is expressed as a series with summation index from 0 to m. This series, however, should have the same summation index as mentioned in (2.4) above.

Example 2.3. If $l=0$, $m=r=1$, $\lambda>0$, $A=1$ and $B=0$, then from Theorem 2.1, we easily deduce the following assertion:

$$\left[(1-\lambda)\mathscr{L}(a,c)f(z)+\lambda z(\mathscr{L}(a,c)f(z))'\right]'\prec z+1\ \ (f(z)\in\mathscr{A}, z\in\mathbb{U})$$

implies that

$$[\mathscr{L}(a,c)f(z)]'\prec 1+\frac{z}{\lambda+1}\ (z\in\mathbb{U}).$$

Theorem 2.4. *Let q be a convex function in $\mathbb{U}$ with $q(0) = 1$, and let*

$$h(z) = q(z) + \lambda z q'(z) \ \ (z \in \mathbb{U}).$$

If $\lambda > 0$, $l, m \in \mathbb{N}_0, f \in \mathscr{A}$ satisfies the differential subordination:

$$\left[\mathscr{I}_{\lambda}^{l+1,m}(a,c) f(z)\right]' \prec h(z), \tag{2.8}$$

then

$$\left[\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)\right]' \prec q(z) \ \ (z \in \mathbb{U}) \tag{2.9}$$

and this result is sharp.

Proof. Making use of (2.7) in (2.5), then the differential subordination (2.8) becomes

$$p(z) + \lambda z p'(z) \prec h(z) = q(z) + \lambda z q'(z).$$

Applying Lemma 1.2, we obtain at once that $p(z) \prec q(z)$, which implies that

$$\left[\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)\right]' \prec q(z)$$

and the result is sharp. □

Example 2.5. On putting $l = 0$, $m = 1$, $\lambda > 0$ and $q(z) = \frac{1-z}{1+z}$ in Theorem 2.4, we get the following result:

$$\left[(1-\lambda)\mathscr{L}(a,c) f(z) + \lambda z (\mathscr{L}(a,c) f(z))'\right]' \prec \frac{1 - 2\lambda z - z^2}{(1+z)^2} \ \ (f(z) \in \mathscr{A}, z \in \mathbb{U})$$

implies that

$$[\mathscr{L}(a,c) f(z)]' \prec \frac{1-z}{1+z} \ (z \in \mathbb{U}).$$

Theorem 2.6. *Let q be a convex function in $\mathbb{U}$, with $q(0) = 1$, and let*

$$h(z) = q(z) + z q'(z) \ \ (z \in \mathbb{U}).$$

If $\lambda > 0$, $l, m \in \mathbb{N}_0, f \in \mathscr{A}$ satisfies the differential subordination:

$$\left[\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)\right]' \prec h(z), \tag{2.10}$$

then

$$\frac{\mathscr{I}_{\lambda}^{l,m}(a,c) f(z)}{z} \prec q(z)$$

and this result is sharp.

Proof. Let

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} = \theta(z). \tag{2.11}$$

Differentiating with respect to z, we obtain

$$\left[\mathscr{I}_\lambda^{l,m}(a,c)f(z)\right]' = \theta(z) + z\theta'(z), \tag{2.12}$$

and upon using (2.10), we get the differential subordination relation

$$\theta(z) + z\theta'(z) \prec h(z) = q(z) + zq'(z).$$

Using Lemma 1.2, we infer that $\theta(z) \prec q(z)$, which implies that

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} \prec q(z).$$

$\square$

Example 2.7. In the special case, when $l = 0$, $m = 0$, $\lambda > 0$ and $q(z) = \frac{1-z}{1+z}$, then Theorem 2.6 yields the result:

$$[f(z)]' \prec \frac{1-2z-z^2}{(1+z)^2} \quad (f(z) \in \mathscr{A}, z \in \mathbb{U})$$

implies that

$$\frac{f(z)}{z} \prec \frac{1-z}{1+z} \; (z \in \mathbb{U}).$$

Theorem 2.8. *Let*

$$h(z) = \left(\frac{1+Az}{1+Bz}\right)^r \; (z \in \mathbb{U};\ |A| \leqq 1; |B| \leqq 1; A \neq B; 0 < r \leq 1).$$

If $\lambda \geqq 0, l, m \in \mathbb{N}_0, f \in \mathscr{A}$ *satisfies the differential subordination:*

$$[\mathscr{I}_\lambda^{l,m}(a,c)f(z)]' \prec h(z), \tag{2.13}$$

then

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} \prec \phi(z),$$

where

$$\phi(z) = \begin{cases} \left(\frac{A}{B}\right)^r \sum\limits_{i \geq 0} \frac{(-r)_i}{i!} \left(\frac{A-B}{A}\right)^i (1+Bz)^{-i} {}_2\mathrm{F}_1\left(i,1;2;\frac{Bz}{1+Bz}\right) \; (B \neq 0), \\ {}_2F_1\left(-r,1;2;-Az\right), \; (B = 0), \end{cases} \tag{2.14}$$

and $\phi(z)$ *is the best dominant.*

Proof. Using (2.12), the differential subordination (2.13) becomes

$$\theta(z)+z\theta'(z) \prec h(z) = \left(\frac{1+Az}{1+Bz}\right)^r,$$

and applying Lemma 1.1, we get

$$\theta(z) \prec \phi(z) = \frac{1}{z}\int_0^z \left(\frac{1+At}{1+Bt}\right)^r dt,$$

which upon integration gives (2.14), and hence it follows that

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} \prec \phi(z).$$

□

Theorem 2.9. *Let h be a convex function with $h(0)=1$. If $f \in \mathscr{A}$, $\lambda > 0, l, m \in \mathbb{N}_0$ satisfies the differential subordination*

$$\left[\mathscr{I}_\lambda^{l,m}(a,c)f(z)\right]' \prec h(z), \tag{2.15}$$

then

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} \prec \tau(z) = \frac{1}{z}\int_0^z h(t)dt$$

and τ is convex and is the best dominant.

Proof. Using (2.12) in (2.15), we have

$$p(z)+zp'(z) \prec h(z).$$

From Lemma 1.1, we obtain

$$p(z) \prec \tau(z) = \frac{1}{z}\int_0^z h(t)dt,$$

and using (2.11), we get the desired result:

$$\frac{\mathscr{I}_\lambda^{l,m}(a,c)f(z)}{z} \prec \tau(z) = \frac{1}{z}\int_0^z h(t)dt.$$

□

Example 2.10. For $l=0,\ m=0,$ and $h(z)=\frac{1-z}{1+z}$, Theorem 2.9 yields the result:

$$[f(z)]' \prec \frac{1-z}{1+z} \quad (f(z)\in\mathscr{A}, z\in\mathbb{U})$$

implies that

$$\frac{f(z)}{z} \prec \frac{2\log(1+z)-z}{z} \quad (z\in\mathbb{U}).$$

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