LE MATEMATICHE
Vol. LXV (2010) – Fasc. II, pp. 49–51
doi: 10.4418/2010.65.2.6

ERRATUM TO: SECOND ORDER EVOLUTION INCLUSIONS GOVERNED BY SWEEPING PROCESS IN BANACH SPACES

AHMED G. IBRAHIM - FERYAL A. ALADSANI

We show that a condition imposed in the assumptions of the main results of our paper leads to a problem. Then we explain how to overcome this problem.

In our paper "Second order evolution inclusions governed by sweeping process in Banach spaces"(published in Le Matematiche 64 (2) (2009), 17–39), we imposed the following condition in the assumptions of theorems (3.1) and (3.2):

(C_1)There are positive real numbers $\lambda, \gamma, a', b', c'$ such that $a', b', c' \in [q', \infty[$ with $q' = \frac{q}{q-1}$ and that for all φ, $\psi \in X^*$, all x, $y \in X$ and all t, $t' \in I$,

$$|d^V_{C(t,x)}(\psi) - d^V_{C(t',y)}(\varphi)| \leq \lambda(|t-t'|^{a'} + \|\psi - \varphi\|^{b'}) + \gamma\|y-x\|^{c'},$$

where X is a separable $p-$uniformly convex and $q-$uniformly smooth Banach space $(p,q > 1)$.

First, we show that this condition leads to the following problem. Suppose that X is a Hilbert space. Then $q' = 2$, and $d^V_{C(t,x)}(\varphi) = (d_{C(t,x)}(\varphi))^2$, where $d_{C(t,x)}(\varphi)$ is the distance from φ to the subset $C(t,x)$. Consequently $a', b', c'. \in [2, \infty[$. Hence, by the condition (C_1), for each $(t,x,\varphi), \in I \times X \times X$ the following relations are satisfied:

Entrato in redazione: 9 febbraio 2011

AMS 2010 Subject Classification: 34A60

Keywords: Sweeping process, Differential inclusion, Normal cone, Duality mapping, Smooth Banach space, Generalized projection.

$$\begin{cases} (1)\lim\limits_{t'\to t}\frac{|d^V_{C(t,x)}(\varphi)-d^V_{C(t',x)}(\varphi)|}{|t'-t\;|}=0, \\ (2)\lim\limits_{y\to x}\frac{|d^V_{C(t,,y)}(\varphi)-d^V_{C(t,x)}(\varphi)|}{\|y-x\|}=0, \\ (3)\lim\limits_{\psi\to\varphi}\frac{|d^V_{C(t,x)}(\psi)-d^V_{C(t,x))}(\varphi)|}{\|\psi-\varphi\;\|}=0. \end{cases}$$

This means that the function $(t,x,\varphi)\to d^V_{C(t,x)}(\varphi) \;=(d_{C(t,x)}(\varphi))^2$ is constant. Therefore,

$$d_{C(t,x)}(\varphi)=0, \text{for all } (t,x,\varphi)\in I\times X\times X.$$

Then, $C(t,x)=X$, for all $(t,x)\in I\times X$. This case does not have any importance.

Secondly, based on this requirement, the condition (C_1) in the theorems (3.1) and (3.2) will take the following form:

(C_1) for all φ, $\psi\in X^*$, all x, $y\in X$ and all t, $t'\in I$,

$$|\left(d^V_{C(t,x)}(\psi)\right)^{\frac{1}{q'}}-\left(d^V_{C(t',y)}(\varphi)\right)^{\frac{1}{q'}}|\leq\lambda|t-t'|+\beta\|\psi-\varphi\|+\gamma\|y-x\|,$$

where $q'=\frac{q}{q-1}$ and λ, β, γ are positive real numbers.

Therefore, some modifications will occur in the proof of the theorems (3.1) and (3.2). These modifications are in the following pages:

(1) Replace the lines from No. 6 to No. 24 Page 29 by the following:

$$\begin{aligned} &\alpha\|J(x^n_{i+1})-(J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i)))\|^{q'} \\ \leq\;& V_*(J^*(J(x^n_{i+1})),J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i)))) \\ =\;& \|x^n_{i+1}\|^2-2\langle x^n_{i+1},J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i))\rangle \\ &+\|J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i))\|^2 \\ =\;& V(J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i)),x^n_{i+1}) \\ =\;& d^V_{C(t^n_{i+1},v_n(t^n_{i+1}))}(J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i))) \end{aligned}$$

and so

$$\begin{aligned} &\alpha^{\frac{1}{q'}}\|J(x^n_{i+1})-(J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i)))\| \\ &\leq\left(d^V_{C(t^n_{i+1},v_n(t^n_{i+1}))}(J(x^n_i)-e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i))\right)^{\frac{1}{q'}} \\ &-\left(d^V_{C(t^n_i,v_n(t^n_i))}(J(x^n_i))\right)^{\frac{1}{q'}}\leq \quad \text{(by (Theorem 2.5))} \\ &\leq\lambda|t^n_{i+1}-t^n_i|+\beta\|e_nh(t^n_i,v_n(t^n_i),u_n(t^n_i))\| \\ &+\gamma\|v_n(t^n_{i+1})-v_n(t^n_i)\|\leq \quad \text{(by condition (C}_1\text{))} \\ &\leq\lambda e_n+\beta e_n\mu+\gamma e_n\|u_n(t^n_i)\| \\ &=\lambda e_n+\beta e_n\mu+\gamma e_nk=e_n\left[\lambda+\beta\mu+\gamma k\right]. \end{aligned}$$

Then,

$$\|J(x_{i+1}^n) - J(x_i^n) + e_n h(t_i^n, v_n(t_i^n), u_n(t_i^n))\| \leq \left[\frac{\lambda + \beta\mu + \gamma k}{\alpha^{\frac{1}{q'}}}\right] e_n,$$

which gives us

$$\|\frac{J(x_{i+1}^n) - J(x_i^n)}{e_n} + h(t_i^n, v_n(t_i^n), u_n(t_i^n))\| \leq \left[\frac{\lambda + \beta\mu + \gamma k}{\alpha^{\frac{1}{q'}}}\right] = \delta.$$

(2) Replace the lines from No. 10 to No. 18 Page 31 by the following:

Let $t \in I$ and n be a fixed positive integer, by (17) and condition (C_1), we have

$$\begin{aligned}
& \left(d^V_{C(t,v(t))}(J(u_n(\theta_n(t))))\right)^{\frac{1}{q'}} \\
= \; & \left(d^V_{C(t,v(t))}(J(u_n(\theta_n(t))))\right)^{\frac{1}{q'}} - \left(d^V_{C(\theta_n(t),v_n(\theta_n(t)))}(J(u_n(\theta_n(t))))\right)^{\frac{1}{q'}} \\
\leq \; & \lambda\, |\theta_n(t) - t| + \gamma\, \|v_n(\theta_n(t)) - v(t)\|.
\end{aligned}$$

Then,

$$\begin{aligned}
& \left(d^V_{C(t,v(t))}(J(u(t)))\right)^{\frac{1}{q'}} \leq | \left(d^V_{C(t,v(t))}(J(u(t)))\right)^{\frac{1}{q'}} - \\
& - \left(d^V_{C(t,v(t))}(J(u_n(\theta_n(t))))\right)^{\frac{1}{q'}} | + \left(d^V_{C(t,v(t))}(J(u_n(\theta_n(t))))\right)^{\frac{1}{q'}} \leq \\
\leq \; & \gamma\, \|J(u(t)) - J(u_n(\theta_n(t)))\| + \lambda |\theta_n(t) - t| + \gamma\, \|v_n(\theta_n(t)) - v(t)\|.
\end{aligned}$$

By passing to the limit when $n \to \infty$ in the preceding inequality, we get $u(t) \in C(t, v(t))$.

AHMED G. IBRAHIM
Mathematics Department
Factuly of Science
King Faisal University
e-mail: `agamal2000@yahoo.com`

FERYAL A. ALADSANI
Mathematics Department
Factuly of Science
King Faisal University
e-mail: `faladsani@hotmail.com`