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TOTAL BOUNDEDNESS IN VECTOR-VALUED F-SEMINORMED FUNCTION SPACES

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We present a compactness criterion of Vitali-type in a class of vector-valued real F-seminormed spaces, which satisfy the W-property.

1. Introduction

The definition of some quantitative characteristics in spaces of functions and their comparison with the Hausdorff measure of noncompactness has allowed many authors to generalize some classical compactness results (see [2], [5], [12], [14], for example). In this paper we introduce a quantitative characteristic, which measures the degree of non equiabsolute continuity for subsets of spaces in a class of vector-valued real F-seminormed function spaces. We compare this quantitative characteristic with the Hausdorff measure of noncompactness. By this comparison we obtain some inequalities, that give, as a special case, sufficient conditions for the total boundedness of a set of functions. From our results we derive a Vitali-type compactness criterion in spaces, of the class we have considered, which satisfy the W-property (see Definition 3.7). In our context we generalize some of the results obtained in [12]. Moreover, the results we present are a partial anticipation of those contained in [13], where the total boundedness

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of a set is considered in a wider class of vector-valued F-seminormed function spaces.

2. Preliminaries and notations

Throughout the present paper all linear spaces are real and we adopt the convention that $\inf\emptyset = +\infty$. The notions concerning the theory of F-seminormed spaces (resp. Riesz F-seminormed spaces) can be found in [8] (resp.[10] and [16]). We give here only the basic ones. Let V be a linear space. An F-seminorm on V is a function $\|\cdot\|_V : V \longrightarrow [0,+\infty[$ such that $\|u+v\|_V \leq \|u\|_V + \|v\|_V$, $\lim_{n\to\infty} \left\|\frac{1}{n}u\right\|_V = 0$, $\|\lambda u\|_V \leq \|u\|_V$, for each $u,v \in V$ and for all real number λ with $|\lambda| \leq 1$. If $\|u\|_V = 0$ only when $u = 0$, then $\|\cdot\|_V$ is called an F-norm. An F-seminorm (F-norm) is called a q-seminorm (q-norm), $0 < q \leq 1$, if $\|\lambda u\|_V = |\lambda|^q \|u\|_V$ for all $u \in V$ and for all real number λ. We observe that if $(V, \|\cdot\|_V)$ is an F-seminormed space and u is an element of V, then the function

$$[0,+\infty[\ni t \to \varphi(t) = \|tu\|_V \in \mathbb{R}$$

is continuous and non-decreasing.

In the following we assume that $(E, \|\cdot\|_E)$ is an F-normed space, Ω a nonempty set, $\mathscr{A}$ a subalgebra of the power set $\mathscr{P}(\Omega)$ of Ω, $\eta : \mathscr{A} \longrightarrow [0,+\infty]$ a submeasure, (i.e., a monotone, subadditive function with $\eta(\emptyset) = 0$), and $\tilde{\eta} : \mathscr{P}(\Omega) \longrightarrow [0,+\infty]$ the submeasure defined by $\tilde{\eta}(B) = \inf\{\eta(A) : B \subset A \text{ and } A \in \mathscr{A}\}$. Let E^Ω be the linear space of all E-valued functions on Ω. For $f \in E^\Omega$ we denote by $\|f\|_E$ the function $x \to \|f(x)\|_E$ and we put $\{\|f\|_E \geq a\} = \{x \in \Omega : \|f(x)\|_E \geq a\}$. Then

$$\|f\|_0 = \inf\{a > 0 : \tilde{\eta}(\{\|f\|_E \geq a\}) \leq a\}$$

defines a Riesz group pseudonorm on E^Ω, i.e., $\|0\|_0 = 0$, $\|f+g\|_0 \leq \|f\|_0 + \|g\|_0$ and $\|f\|_E \leq \|g\|_E \Longrightarrow \|f\|_0 \leq \|g\|_0$ for each $f,g \in E^\Omega$, (see [3], [4] for more details).
Moreover set $\mathscr{A}_0 = \{A \in \mathscr{A} : \eta(A) < +\infty\}$. We will denote by

$$S(\mathscr{A}_0, E) = span\{\chi_A y : y \in E \text{ and } A \in \mathscr{A}_0\}$$

the linear space of all E-valued $\mathscr{A}_0$-simple functions on Ω, where χ_A denotes the characteristic function of a set A defined on Ω. In the remainder of this paper $L = (L, \|\cdot\|_L)$ stands for an F-seminormed subspace of E^Ω with the following properties:

(a) if $A \in \mathscr{A}$ and $f \in L$, then $\chi_A f \in L$;

(b) $S(\mathscr{A}_0, E)$ is a linear subspace of L dense in $(L, \|\cdot\|_L)$;

(c) if $A \in \mathscr{A}_0$ and $f \in L$, then $\lim\limits_{\substack{\eta(B)\to 0 \\ A\supseteq B\in\mathscr{A}}} \|\chi_B f\|_L = 0$;

(d) there is $k \geq 1$ such that $\|f\|_E \leq \|g\|_E$ implies $\|f\|_L \leq k\|g\|_L$ for each $f, g \in L$.

If the condition (d) is satisfied for $k = 1$, L is a Riesz F-seminormed space.
Throughout we denote by $B_\varepsilon(E^\Omega) = \{f \in E^\Omega : \|f\|_0 \leq \varepsilon\}$ and by $B_\varepsilon(L) = \{f \in L : \|f\|_L \leq \varepsilon\}$ for $\varepsilon > 0$.

Definition 2.1. Let $M \subseteq E^\Omega$; then

$$\beta_0(M) := \inf\{\varepsilon > 0 : \text{there is a finite subset } F \text{ of } M \text{ such that } M \subseteq F + B_\varepsilon(E^\Omega)\}.$$

Let $M \subseteq L$; then

$$\gamma_L(M) := \inf\{\varepsilon > 0 : \text{ there is a finite subset } F \text{ of } L \text{ such that } M \subseteq F + B_\varepsilon(L)\}.$$

The set functions β_0 and γ_L are, respectively, the inner Hausdorff measure of noncompactness in $(E^\Omega, \|\cdot\|_0)$ and the classical Hausdorff measure of noncompactness in $(L, \|\cdot\|_L)$.
Clearly, $M \subseteq E^\Omega$ is $\|\cdot\|_0$-totally bounded ($\|\cdot\|_0$-tb, for short) iff $\beta_0(M) = 0$, and $M \subseteq L$ is $\|\cdot\|_L$-totally bounded ($\|\cdot\|_L$-tb, for short) iff $\gamma_L(M) = 0$.

3. Main results

In this section we introduce a quantitative characteristic which measures the degree of non equiabsolute continuity for a subset of the space L.

Definition 3.1. Let $M \subseteq L$. We define for $A \in \mathscr{A}_0$ and $\delta > 0$:

$$\Pi_L(M, A, \delta) = \max\left\{ \sup_{f\in M} \left\|\chi_{\Omega\backslash A} f\right\|_L, \sup_{f\in M} \sup_{\substack{A\supseteq B\in\mathscr{A} \\ \eta(B)\leq\delta}} \|\chi_B f\|_L \right\},$$

$$\Pi_L(M, A) = \lim_{\delta\to 0} \Pi_L(M, A, \delta),$$

$$\Pi_L(M) = \inf_{A\in\mathscr{A}_0} \Pi_L(M, A).$$

A subset M of L is called $\|\cdot\|_L$-equiabsolutely continuous ($\|\cdot\|_L$-eac, for short) if $\Pi_L(M) = 0$.

Remark 3.2. Let $s \in S(\mathscr{A}_0, E)$ and set $A_s = \{x \in \Omega : s(x) \neq 0\}$. Then $A_s \in \mathscr{A}_0$ and $\|\chi_{\Omega \setminus A_s} s\|_L = 0$. Hence, by (c), $\Pi_L(\{s\}, A_s) = 0$.

We are now in a position to prove the main results of this note.

Theorem 3.3. *Let $M \subseteq L$. Then*

$$\Pi_L(M) \leq k\gamma_L(M).$$

Proof. The inequality is trivially true if $\gamma_L(M) = \sup_{f \in L} \|f\|_L$. Assume that $\gamma_L(M) < \sup_{f \in L} \|f\|_L$. Let $\alpha > \gamma_L(M)$. By (b) there are $s_1, \ldots, s_n \in S(\mathscr{A}_0, E)$ such that $M \subseteq \bigcup_{i=1}^{n} (s_i + B_\alpha(L))$. Set $A_i = \{x \in \Omega : s_i(x) \neq 0\}$ for $i = 1, \ldots, n$. Clearly $A = \bigcup_{i=1}^{n} A_i \in \mathscr{A}_0$. Let $f \in M$. Choose $i \in \{1, \ldots, n\}$ such that $\|f - s_i\|_L \leq \alpha$. Then

$$\|\chi_{\Omega \setminus A} f\|_L = \|\chi_{\Omega \setminus A}(f - s_i)\|_L \leq k\|f - s_i\|_L \leq k\alpha. \tag{1}$$

Moreover, let $\delta > 0$. For $B \subseteq A, B \in \mathscr{A}$ and $\eta(B) \leq \delta$, we have

$$\begin{aligned} \|\chi_B f\|_L \leq\ & \|\chi_B(f - s_i)\|_L + \|\chi_B s_i\|_L \leq k\|f - s_i\|_L + \|\chi_B s_i\|_L \\ \leq\ & k\alpha + \max_{j=1,\ldots,n} \Pi_L(\{s_j\}, A, \delta). \end{aligned} \tag{2}$$

Having in mind Remark 3.2, by (1) and (2) we get

$$\Pi_L(M, A) \leq k\alpha + \max_{j=1,\ldots,n} \Pi_L(\{s_j\}, A) = k\alpha + \max_{j=1,\ldots,n} \|\chi_{\Omega \setminus A} s_j\|_L = k\alpha,$$

hence $\Pi_L(M) \leq k\alpha$, and therefore $\Pi_L(M) \leq k\gamma_L(M)$. $\square$

Theorem 3.4. *Let $M \subseteq L, A \in \mathscr{A}_0$ and suppose that $\beta_0(\chi_A M) < \sup_{y \in E} \|y\|_E$. Then*

$$\gamma_L(M) \leq 3 \lim_{\delta \to \beta_0(\chi_A M)^+} \Pi_L(M, A, \delta) + k\, \|\chi_A y_0\|_L, \tag{3}$$

for some $y_0 \in E$ such that $\|y_0\|_E = \beta_0(\chi_A M)$. In fact, for every $\bar{y} \in E$ satisfying $\|\bar{y}\|_E > \beta_0(\chi_A M)$, there is a $t_0 \in [0, 1[$ such that $\|t_0 \bar{y}\|_E = \beta_0(\chi_A M)$ and such that the inequality (3) is satisfied by $y_0 = t_0 \bar{y}$. In particular, if L is a Riesz F-seminormed space, the inequality (3) is satisfied by any $y_0 \in E$ such that $\|y_0\|_E = \beta_0(\chi_A M)$.

Proof. Fix an element $\bar{y} \in E$ such that $\|\bar{y}\|_E > \delta_0 = \beta_0(\chi_A M)$. Then, by the definition of δ_0, for every $\delta \in]\delta_0, \|\bar{y}\|_E]$ there are a positive number $\sigma \in [\delta_0, \delta[$ and functions $f_1, ..., f_n \in M$ such that

$$\chi_A M \subseteq \bigcup_{i=1}^{n} (\chi_A f_i + B_\sigma(E^\Omega)).$$

Fix $f \in M$ and let $i \in \{1, ..., n\}$ such that $\|\chi_A(f - f_i)\|_0 \leq \sigma$. Set $D_f = \{\|\chi_A(f - f_i)\|_E > \sigma\}$, then $D_f \subseteq A$ and $\tilde{\eta}(D_f) \leq \sigma$. Hence, by the definition of $\tilde{\eta}$ there exists $C_f \in \mathscr{A}$ such that $D_f \subseteq C_f$ and $\eta(C_f) \leq \delta$. Then $B_f = A \cap C_f \subseteq A$, $B_f \in \mathscr{A}$, $\eta(B_f) \leq \delta$ and $\chi_{B_f} \chi_A f_i = \chi_{B_f} f_i$. Set $a_\delta = \Pi_L(M, A, \delta)$. By the definition of $\Pi_L(M, A, \delta)$ we have $\|\chi_{\Omega \setminus A} f\|_L \leq a_\delta$, $\|\chi_{Bf} f\|_L \leq a_\delta$ and $\|\chi_{B_f} f_i\|_L \leq a_\delta$. Therefore

$$\begin{aligned} \|f - \chi_A f_i\|_L &\leq \|\chi_{\Omega \setminus A} f\|_L + \|\chi_A(f - f_i)\|_L \\ &\leq \|\chi_{\Omega \setminus A} f\|_L + \|\chi_{A \setminus B_f}(f - f_i)\|_L + \|\chi_{B_f} f\|_L + \|\chi_{B_f} f_i\|_L \\ &\leq 3a_\delta + \|\chi_{A \setminus B_f}(f - f_i)\|_L. \end{aligned} \tag{4}$$

Since the function $\varphi : [0, 1] \longrightarrow [0, +\infty[$ defined by $\varphi(t) = \|t\bar{y}\|_E$ is continuous and $\varphi([0, 1]) = [0, \|\bar{y}\|_E]$, there is $t_\delta \in [0, 1]$ such that $\|t_\delta \bar{y}\|_E = \delta$. Clearly the function $\chi_A t_\delta \bar{y} \in L$. Moreover, it is easy to see that

$$\|\chi_{A \setminus B_f}(f - f_i)\|_E \leq \|\chi_A t_\delta \bar{y}\|_E.$$

Then

$$\|\chi_{A \setminus B_f}(f - f_i)\|_L \leq k \|\chi_A t_\delta \bar{y}\|_L.$$

By (4) it follows that

$$\gamma_L(M) \leq 3a_\delta + k \; \|\chi_A t_\delta \bar{y}\|_L. \tag{5}$$

Now, since the function φ is non-decreasing, we have that

$$]\delta_0, \|\bar{y}\|_E] \ni \delta \to t_\delta \in [0, 1]$$

is a strictly increasing function and

$$\lim_{\delta \to \delta_0^+} t_\delta = t_0 = \max\{t \in [0, 1] : \varphi(t) = \delta_0\}.$$

Thus, by the continuity of the function

$$[0, 1] \ni t \to \psi(t) = \|\chi_A t \bar{y}\|_L \in \mathbb{R},$$

we have

$$\lim_{\delta \to \delta_0^+} \|\chi_A t_\delta \bar{y}\|_L = \|\chi_A y_0\|_L,$$

where $y_0 = t_0\bar{y}$. Therefore, by (5), the inequality

$$\gamma_L(M) \le 3 \lim_{\delta\to\delta_0^+} a_\delta + k \,\|\chi_A y_0\|_L,$$

that is inequality (3), holds and $\|y_0\|_E = \beta_0(\chi_A M)$. This accomplishes the proof. □

Corollary 3.5. *Let $M \subseteq L$, $A \in \mathscr{A}_0$, and suppose $\chi_A M$ $\|\cdot\|_0$-tb. Then*

$$\gamma_L(M) \le 3\Pi_L(M,A).$$

The following corollary of the Theorem 3.4 gives a sufficient condition for the total boundedness of a subset of L.

Corollary 3.6. *Let M be a $\|\cdot\|_L$-eac subset of L and suppose that $\chi_A M$ is $\|\cdot\|_0$-tb for all $A \in \mathscr{A}_0$. Then M is $\|\cdot\|_L$-tb.*

Definition 3.7. (see [15, Chapter III], [17]) A space L has the W-property if $f_n \overset{\|\cdot\|_L}{\longrightarrow} 0$ implies $\chi_A f_n \overset{\|\cdot\|_0}{\longrightarrow} 0$ for all sequences (f_n) of elements of L and for all $A \in \mathscr{A}_0$.

Proposition 3.8. *If the space L has the W-property and if M is a subset of L $\|\cdot\|_L$-tb , then $\chi_A M$ is $\|\cdot\|_0$-tb for all $A \in \mathscr{A}_0$.*

Proof. Let M be $\|\cdot\|_L$-totally bounded and let $A \in \mathscr{A}_0$. By the W-property we have that

$$\text{for all } \varepsilon > 0 \text{ there exists } \delta > 0 \text{ such that } \chi_A B_\delta(L) \subset B_\varepsilon(E^\Omega). \tag{6}$$

Now fix $\varepsilon > 0$. By (6) there exists $\delta > 0$ such that $\chi_A B_\delta(L) \subset B_\varepsilon(E^\Omega)$. Since M is $\|\cdot\|_L$-tb there exists a finite subset F of L such that $M \subset F + B_\delta(L)$. Hence $\chi_A M \subset \chi_A F + \chi_A B_\delta(L) \subset \chi_A F + B_\varepsilon(E^\Omega)$, and therefore $\chi_A M$ is $\|\cdot\|_0$-tb. □

Combining Theorem 3.3 and Proposition 3.8, and having in mind Corollary 3.6, we obtain the following Vitali-type total boundedness criterion.

Theorem 3.9. *Assume that the space L has the W-property. Then a subset M of L is $\|\cdot\|_L$-tb if and only if it is $\|\cdot\|_L$-eac and $\chi_A M$ is $\|\cdot\|_0-$tb for all $A \in \mathscr{A}_0$.*

In the setting of q-seminormed spaces, $0 < q \le 1$, we have the following corollaries of the Theorem 3.4.

Corollary 3.10. *Assume that E is a q-normed space, $0 < q \leq 1$. Let $M \subseteq L$ and $A \in \mathscr{A}_0$, then*

$$\gamma_L(M) \leq 3 \lim_{\delta \to \beta_0(\chi_A M)^+} \Pi_L(M, A, \delta) + k \, \|\beta_0(\chi_A M)^{\frac{1}{q}} \chi_A y_0\|_L,$$

where $y_0 \in E$ and $\|y_0\|_E = 1$.

Corollary 3.11. *Assume that L is a p-normed space, $0 < p \leq 1$, and E is a q-normed space, $0 < q \leq 1$. Let $M \subseteq L$ and $A \in \mathscr{A}_0$, then*

$$\gamma_L(M) \leq 3 \lim_{\delta \to \beta_0(\chi_A M)^+} \Pi_L(M, A, \delta) + k \, \beta_0(\chi_A M)^{\frac{p}{q}} \|\chi_A y_0\|_L,$$

where $y_0 \in E$ and $\|y_0\|_E = 1$.

4. Example

In this section we give an example of a class of F-seminormed spaces of type L satisfying the W-property. We consider the " Orlicz spaces " L_N introduced in [6] in the same way as Dunford and Schwartz [7, Chapter III] define the space of integrable functions and the integral for integrable functions. We briefly recall the definition of the space L_N. Let $\|\cdot\| : S(\mathscr{A}_0, \mathbb{R}) \longrightarrow [0, +\infty[$ be a Riesz F-seminorm such that $\eta(A) = \|\chi_A\|$ for all $A \in \mathscr{A}_0$ and $N : [0, +\infty) \to [0, +\infty)$ a continuous, strictly increasing function such that $N(0) = 0$ and $N(s+t) \leq \bar{n}(N(s) + N(t))$ for all $s, t \geq 0$. Assume that $(E, \|\cdot\|_E)$ is a complete F-normed space and let us denote by L_0 the closure of $S(\mathscr{A}_0, E)$ in $(E^\Omega, \|\cdot\|_0)$. For $s \in S(\mathscr{A}_0, E)$, $\|s\|_N$ is defined by $\|s\|_N = \|N \circ \|s\|_E\|$. Then L_N (see [6, p.92]) is the linear space of all functions $f \in L_0$, for which there is a $\|\cdot\|_N$- Cauchy sequence (s_n) in $S(\mathscr{A}_0, E)$ converging to f with respect to $\|\cdot\|_0$. Such a sequence (s_n) of simple functions is said to determine f. If (s_n) is a determining sequence for $f \in L_N$, $\|\cdot\|_N$ is defined by $\|f\|_N = \lim_{n \to +\infty} \|s_n\|_N$. The function $\|\cdot\|_N$ has the following properties:

$$\|f + g\|_N \leq 2\bar{n} \max\{\|f\|_N, \ \|g\|_N\}, \lim_{n \to +\infty} \|\frac{1}{n} f\|_N = 0, \|\lambda f\|_N \leq \|f\|_N,$$

for all $f, g \in L_N$ and for all real number λ with $|\lambda| \leq 1$, therefore $\|\cdot\|_N$ is a Δ-seminorm on L_N in the sense of [9, p.2]. Moreover, the space $(L_N, \|\cdot\|_N)$ satisfies the properties (a)-(c) of Section 2 and (see [6, Proposition 2.6])

$$\|f\|_E \leq \|g\|_E \text{ implies } \|f\|_N \leq \|g\|_N, \quad \text{for each } f, g \in L_N.$$

Set $L = L_N$, then by [9, Theorem 1.2], if p is choosen such that $2^{\frac{1}{p}} = 2\bar{n}$ then the formula

$$\|f\|_L = \inf\left\{ \sum_{i=1}^{n} \|f_i\|_N^p : \sum_{i=1}^{n} f_i = f \right\}$$

defines an F-seminorm on L generating the same topology of the Δ-seminorm $\|\cdot\|_N$.
Further, being $\frac{1}{4}\|f\|_N^p \leq \|f\|_L \leq \|f\|_N^p$ (see [9, Lemma 1.1]), using (4) we obtain that

$$\|f\|_E \leq \|g\|_E \text{ implies } \|f\|_L \leq 4\|g\|_L, \quad \text{for each} f, g \in L_N. \tag{7}$$

Hence the space $(L, \|\cdot\|_L)$ is a F-seminormed subspace of E^Ω which satisfies the properties (a)-(d) of Section 2. Moreover, as consequence of [6, Theorem 2.7], it has the W-property.

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