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COMMON FIXED POINT THEOREMS FOR WEAKLY COMPATIBLE MAPPINGS IN FUZZY METRIC SPACES

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The aim of this paper is to prove a common fixed point theorem for a pair of weakly compatible mappings in fuzzy metric space by using the (CLRg) property. An example is also furnished which demonstrates the validity of our main result. As an application to our main result, we present a fixed point theorem for two finite families of self mappings in fuzzy metric space by using the notion of pairwise commuting. Our results improve the results of Sedghi, Shobe and Aliouche [31].

1. Introduction

The concept of a fuzzy set is investigated by Zadeh [40] in his seminal paper. In 1975, Kramosil and Michalek [14] introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces. Further, George and Veeramani [8] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [14] with a view to obtain a Hausdoroff topology which has very important applications in quantum particle physics, particularly in connection with both string and ε^{∞} theory (see, [21–23]). Fuzzy set theory also has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory,

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engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc. Consequently in due course of time some metric fixed point results were generalized to fuzzy metric spaces by various authors viz Grabiec [9], Cho [6, 7], Subrahmanyam [38] and Vasuki [39].

In 2002, Aamri and El-Moutawakil [2] defined the notion of (E.A) property for self mappings which contained the class of non-compatible mappings in metric spaces. It was pointed out that (E.A) property allows replacing the completeness requirement of the space with a more natural condition of closedness of the range as well as relaxes the compleness of the whole space, continuity of one or more mappings and containment of the range of one mapping into the range of other which is utilized to construct the sequence of joint iterates. Many authors have proved common fixed point theorems in fuzzy metric spaces for different contractive conditions. For details, we refer to [5, 10, 11, 15–17, 20, 24–26, 29, 30, 32–34, 36, 37]. Recently, Sintunavarat and Kumam [35] defined the notion of (CLRg) property in fuzzy metric spaces and improved the results of Miheţ [18] without any requirement of the closedness of the subspace.

In this paper, we prove a common fixed point theorem for a pair of weakly compatible mappings by using (CLRg) property in fuzzy metric space. We also present a common fixed point theorem for two finite families of self mappings in fuzzy metric space by using the notion of pairwise commuting due to Imdad, Ali and Tanveer [12]. Our results improve the results of Sedghi, Shobe and Aliouche [31].

2. Preliminaries

Definition 2.1. [28] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $a * 1 = a$ for all $a \in [0,1]$,
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1]$.

Examples of continuous t-norms are $a * b = ab$ and $a * b = \min\{a,b\}$.

Definition 2.2. [8] A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: for all $x,y,z \in X$, $t,s > 0$,

1. $M(x,y,t) > 0$,
2. $M(x,y,t) = 1$ if and only if $x = y$,
3. $M(x,y,t) = M(y,x,t)$,
4. $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
5. $M(x,y,\cdot) : [0,\infty) \to [0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Let $(X,M,*)$ be a fuzzy metric space. For $t > 0$, the open ball $\mathcal{B}(x,r,t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$\mathcal{B}(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}.$$

Now let $(X,M,*)$ be a fuzzy metric space and τ the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $\mathcal{B}(x,r,t) \subset A$. Then τ is a topology on X induced by the fuzzy metric M.

In the following example (see [8]), we know that every metric induces a fuzzy metric:

Example 2.3. Let (X,d) be a metric space. Denote $a * b = ab$ (or $a * b = \min\{a,b\}$) for all $a,b \in [0,1]$ and let M_d be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$$M_d(x,y,t) = \frac{t}{t + d(x,y)}.$$

Then $(X,M_d,*)$ is a fuzzy metric space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Lemma 2.4. *[9] Let $(X,M,*)$ be a fuzzy metric space. Then $M(x,y,t)$ is non-decreasing for all $x,y \in X$.*

Definition 2.5. [13] Two self mappings f and g of a non-empty set X are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $fz = gz$ some $z \in X$, then $fgz = gfz$.

Remark 2.6. [13] Two compatible self mappings are weakly compatible, but the converse is not true. Therefore the concept of weak compatibility is more general than that of compatibility.

Definition 2.7. [3] A pair of self mappings f and g of a fuzzy metric space $(X,M,*)$ is said to satisfy the (E.A) property, if there exists a sequence $\{x_n\}$ in X for some $z \in X$ such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z.$$

Remark 2.8. It is noted that weak compatibility and (E.A) property are independent to each other (see [27], Example 2.1, Example 2.2).

Definition 2.9. [3] Two self mappings f and g of a fuzzy metric space $(X,M,*)$ are non-compatible if and only if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$, but for some $t > 0$, $\lim_{n\to\infty} M(fgx_n, gfx_n, t)$ is either less than 1 or nonexistent.

Remark 2.10. From Definition 2.9, it is easy to see that any non-compatible self mappings of a fuzzy metric space $(X,M,*)$ satisfy the (E.A) property. But two mappings satisfying the (E.A) property need not be non-compatible (see [27], Remark 4.8).

Definition 2.11. [35] A pair of self mappings f and g of a fuzzy metric space $(X,M,*)$ is said to satisfy the (CLRg) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$.

Inspired by Sintunavarat and Kumam [35], we show examples of self mappings f and g which are satisfying the (CLRg) property.

Example 2.12. Let $(X,M,*)$ be a fuzzy metric space with $X = [0,\infty)$ and

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

for all $x,y \in X$. Define self mappings f and g on X by $f(x) = x+3$ and $g(x) = 4x$ for all $x \in X$. Let a sequence $\{x_n\} = \{1+\frac{1}{n}\}_{n\in\mathbb{N}}$ in X, we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 4 = g(1) \in X,$$

which shows that f and g satisfy the (CLRg) property.

Example 2.13. The conclusion of Example 2.12 remains true if the self mappings f and g is defined on X by $f(x) = \frac{x}{7}$ and $g(x) = \frac{x}{8}$ for all $x \in X$. Let a sequence $\{x_n\} = \{\frac{1}{n}\}_{n\in\mathbb{N}}$ in X. Since

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 0 = g(0) \in X,$$

therefore f and g satisfy the (CLRg) property.

Definition 2.14. [12] Two families of self mappings $\{f_i\}_{i=1}^m$ and $\{g_k\}_{k=1}^n$ are said to be pairwise commuting if

1. $f_i f_j = f_j f_i$ for all $i, j \in \{1,2,\dots,m\}$,

2. $g_k g_l = g_l g_k$ for all $k,l \in \{1,2,\dots,n\}$,

3. $f_i g_k = g_k f_i$ for all $i \in \{1,2,\dots,m\}$ and $k \in \{1,2,\dots,n\}$.

Throughout this paper, $(X,M,*)$ is considered to be a fuzzy metric space with condition $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y \in X$.

3. Results

In 2010, Sedghi, Shobe and Aliouche [31] proved a common fixed point theorem for a pair of weakly compatible mappings with (E.A) property in fuzzy metric space by using the following function:

Let Φ is a set of all increasing and continuous functions $\phi : (0,1] \to (0,1]$, such that $\phi(t) > t$ for every $t \in (0,1)$.

Example 3.1. [31] Let $\phi : (0,1] \to (0,1]$ defined by $\phi(t) = t^{\frac{1}{2}}$.

Theorem 3.2. (*Theorem 1, [31]*) *Let $(X,M,*)$ be a fuzzy metric space and f and g be self mappings of X satisfying the following conditions:*

1. $f(X) \subseteq g(X)$ and $f(X)$ or $g(X)$ is a closed subset of X,

2.

$$M(fx,fy,t) \geq \phi\left(\min\left\{\begin{array}{c} M(gx,gy,t), \\ \sup_{t_1+t_2=\frac{2}{k}t} \min\left\{\begin{array}{c} M(gx,fx,t_1), \\ M(gy,fy,t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\left\{\begin{array}{c} M(gx,fy,t_3), \\ M(gy,fx,t_4) \end{array}\right\} \end{array}\right\}\right) \quad (1)$$

for all $x,y \in X$, $t > 0$ and for some $1 \leq k < 2$. Suppose that the pair (f,g) satisfies the (E.A) property and (f,g) is weakly compatible. Then f and g have a unique common fixed point in X.

Now we prove our main result:

Theorem 3.3. *Let $(X,M,*)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Further let f, g be mappings from X into itself and satisfying the inequality (1) of Theorem 3.2. If the pair (f,g) satisfies the (CLRg) property then f and g have a unique common fixed point provided the pair (f,g) is weakly compatible.*

Proof. Since the pair (f,g) satisfies the (CLRg) property, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu,$$

for some $u \in X$. Now we assert that $fu = gu$. Let, on the contrary, $fu \neq gu$, then there exists $t_0 > 0$ such that

$$M\left(fu,gu,\frac{2}{k}t_0\right) > M(fu,gu,t_0). \tag{2}$$

To support the claim, let it be untrue. Then we have

$$M\left(fu,gu,\tfrac{2}{k}t\right) > M(fu,gu,t), \text{ for all } t > 0.$$

Repeatedly using this equality, we obtain

$$M(fu,gu,t) = M\left(fu,gu,\frac{2}{k}t\right) = \ldots = M\left(fu,gu,\left(\frac{2}{k}\right)^n t\right) \to 1,$$

as $n \to \infty$. This shows that $M(fu,gu,t) = 1$ for all $t > 0$ which contradicts $fu \neq gu$ and hence (2) is proved. On using inequality (1), with $x = x_n$, $y = u$, we get

$$M(fx_n,fu,t_0) \geq \phi\left(\min\left\{\begin{array}{c} M(gx_n,gu,t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{c} M(gx_n,fx_n,t_1), \\ M(gu,fu,t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{c} M(gx_n,fu,t_3), \\ M(gu,fx_n,t_4) \end{array}\right\} \end{array}\right\}\right)$$

for all $\varepsilon \in \left(0,\frac{2}{k}t_0\right)$. As $n \to \infty$, it follows that

$$\begin{aligned} M(gu,fu,t_0) &\geq \phi\left(\min\left\{\begin{array}{c} M(gu,gu,t_0), \\ \min\left\{M(gu,gu,\varepsilon), M\left(gu,fu,\frac{2}{k}t_0-\varepsilon\right)\right\}, \\ \max\left\{M\left(gu,fu,\frac{2}{k}t_0-\varepsilon\right), M(gu,gu,\varepsilon)\right\} \end{array}\right\}\right) \\ &= \phi\left(M\left(gu,fu,\frac{2}{k}t_0-\varepsilon\right)\right) > M\left(gu,fu,\frac{2}{k}t_0-\varepsilon\right), \end{aligned}$$

as $\varepsilon \to 0$, we have

$$M(gu,fu,t_0) \geq M\left(gu,fu,\frac{2}{k}t_0\right),$$

which contradicts (2), we have $gu = fu$. Next, we let $z = fu = gu$. Since the pair (f,g) is weakly compatible, $fgu = gfu$ which implies that $fz = fgu = gfu =$

gz. Now we show that $z = fz$. Suppose that $z \neq fz$, then on using (1) with $x = z$, $y = u$, we get, for some $t_0 > 0$,

$$M(fz, fu, t_0) \geq \phi\left(\min\left\{\begin{array}{c} M(gz, gu, t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{c} M(gz, fz, t_1), \\ M(gu, fu, t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{c} M(gz, fu, t_3), \\ M(gu, fz, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$M(fz, z, t_0) \geq \phi\left(\min\left\{\begin{array}{c} M(fz, z, t_0), \\ \min\left\{M(fz, fz, \varepsilon), M\left(z, z, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M(fz, z, \varepsilon), M\left(z, fz, \frac{2}{k}t_0 - \varepsilon\right)\right\} \end{array}\right\}\right),$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $\varepsilon \to 0$, we have

$$\begin{aligned} M(fz, z, t_0) &\geq \phi\left(\min\left\{M(fz, z, t_0), M\left(z, fz, \frac{2}{k}t_0\right)\right\}\right) \\ &= \phi\left(M(fz, z, t_0)\right) > M(fz, z, t_0), \end{aligned}$$

which is a contradiction. Hence $fz = gz = z$. Therefore z is a common fixed point of f and g.

Uniqueness: Let $w(\neq z)$ be another common fixed point of f and g. On using inequality (1) with $x = z$, $y = w$, we get, for some $t_0 > 0$,

$$M(fz, fw, t_0) \geq \phi\left(\min\left\{\begin{array}{c} M(gz, gw, t_0), \\ \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{\begin{array}{c} M(gz, fz, t_1), \\ M(gw, fw, t_2) \end{array}\right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{\begin{array}{c} M(gz, fw, t_3), \\ M(gw, fz, t_4) \end{array}\right\} \end{array}\right\}\right)$$

$$M(z, w, t_0) \geq \phi\left(\min\left\{\begin{array}{c} M(z, w, t_0), \\ \min\left\{M(z, z, \varepsilon), M\left(w, w, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \\ \max\left\{M(z, w, \varepsilon), M\left(w, z, \frac{2}{k}t_0 - \varepsilon\right)\right\} \end{array}\right\}\right),$$

for all $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$. As $\varepsilon \to 0$, we have

$$\begin{aligned} M(z, w, t_0) &\geq \phi\left(\min\left\{M(z, w, t_0), M\left(w, z, \frac{2}{k}t_0\right)\right\}\right) \\ &= \phi\left(M(z, w, t_0)\right) > M(z, w, t_0), \end{aligned}$$

which is a contradiction. Therefore $Bz = z = Tz$. It implies that f and g have a unique a common fixed point. □

Remark 3.4. Theorem 3.3 improves the main result of Sedghi, Shobe and Aliouche ([31], Theorem 1) without any requirement on containment of ranges amongst the involved mappings and closedness of one or more subspaces.

The following examples illustrates Theorem 3.3.

Example 3.5. Let $(X,M,*)$ be a fuzzy metric space, where $X=[3,19)$, with t-norm $*$ is defined by $a*b=ab$ for all $a,b\in[0,1]$ and

$$M(x,y,t)=\begin{cases}\frac{t}{t+|x-y|}, & \text{if } t>0;\\ 0, & \text{if } t=0.\end{cases}$$

for all $x,y\in X$. Let the function $\phi:(0,1]\to(0,1]$ defined by $\phi(t)=t^{\frac{1}{2}}$. Define the self mappings f and g by

$$f(x)=\begin{cases}3, & \text{if } x\in\{3\}\cup(5,19);\\ 12, & \text{if } x\in(3,5].\end{cases}\quad g(x)=\begin{cases}3, & \text{if } x=3;\\ 11, & \text{if } x\in(3,5];\\ \frac{x+1}{2}, & \text{if } x\in(5,19).\end{cases}$$

Taking $\{x_n\}=\{5+\frac{1}{n}\}_{n\in\mathbb{N}}$ or $\{x_n\}=\{3\}$, it is clear that the pair (f,g) satisfies the (CLRg) property.

$$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=3=g(3)\in X.$$

It is noted that $f(X)=\{3,12\}\nsubseteq[3,10)\cup\{11\}=g(X)$. Thus, all the conditions of Theorem 3.3 are satisfied and 3 is a unique common fixed point of the pair (f,g). Also, all the involved mappings are even discontinuous at their unique common fixed point 3. Here, it may be pointed out that $g(X)$ is not a closed subspace of X.

Now we utilize the notion of commuting pairwise due to Imdad, Ali and Tanveer [10] and extend Theorem 3.3 to two finite families of self mappings in fuzzy metric space.

Corollary 3.6. *Let $(X,M,*)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Further let $\{f_1,f_2,\dots,f_p\}$ and $\{g_1,g_2,\dots,g_q\}$ be two finite families of mappings from X into itself such that $f=f_1f_2\dots f_p$ and $g=g_1g_2\dots g_q$ and satisfy inequality (1) of Theorem 3.2. Suppose that the pair (f,g) shares the (CLRg) property.*

Moreover, if the family $\{f_i\}_{i=1}^{p}$ commutes pairwise with the family $\{g_i\}_{j=1}^{q}$, then (for all $i\in\{1,2,\dots,p\}$ and $j\in\{1,2,\dots,q\}$) f_i and g_j have a unique common fixed point.

Proof. The proof of this theorem is similar to that of Theorem 3.1 contained in Imdad, Ali and Tanveer [12], hence details are avoided. □

Remark 3.7. Corollary 3.6 improve the result of Sedghi, Shobe and Aliouche ([31], Theorem 2).

By setting $f_1 = f_2 = \ldots = f_p = f$ and $g_1 = g_2 = \ldots = g_q = g$ in Corollary 3.6, we deduce the following:

Corollary 3.8. *Let $(X, M, *)$ be a fuzzy metric space, where $*$ is a continuous t-norm. Further let f and g be mappings from X into itself such that the pair (f^p, g^q) satisfies the (CLRg) property such that*

$$M(f^p x, f^p y, t) \geq \phi \left(\min \left\{ \begin{array}{c} M(g^q x, g^q y, t), \\ \sup_{t_1+t_2=\frac{2}{k}t} \min \left\{ \begin{array}{c} M(g^q x, f^p x, t_1), \\ M(g^q y, f^p y, t_2) \end{array} \right\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max \left\{ \begin{array}{c} M(g^q x, f^p y, t_3), \\ M(g^q y, f^p x, t_4) \end{array} \right\} \end{array} \right\} \right) \quad (3)$$

holds for all $x, y \in X$, $t > 0$, for some $1 \leq k < 2$ and p, q are fixed positive integers. Then f and g have a unique common fixed point provided that the pair (f^p, g^q) commutes pairwise.

Open problem.

Can the above mentioned theorems be proved for intuitionistic fuzzy metric spaces?

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