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SKEW HURWITZ SERIES OVER QUASI BAER AND *PS*-RINGS

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In this paper, we consider some properties of rings which are shared by the ring R and the ring $T = (HR, \sigma)$ of skew Hurwitz series. In particular we show that:

1) If R is a ring with $char(R) = 0$ and σ is an R -automorphism such that $\sigma(e) = e$ and the left annihilator of every left ideal is σ-invariant, then the following are equivalent:
   i) T is a quasi Baer ring.
   ii) R is a quasi Baer ring.
2) If R is a right *PS*-ring with $char(R) = 0$, then T is a right *PS*-ring.

1. Introduction

Throughout this paper R denotes an associative ring with identity and $char(R) = 0$ which means that $nx = 0$ if and only if $x = 0$ which is a stronger condition than the usual definition that no positive multiple of the identity vanishes. Recall from [5] that R is a Baer ring if the right annihilator of every nonempty subset of R is generated as a right ideal by an idempotent, this definition is left-right symmetric see [5], and it was proved in [1] that Baer rings are ubiquitous which forms a very wide class.

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The concept of Baer rings was generalized by Clark [3] in 1967 to that of quasi Baer rings. A ring R is called quasi Baer if the right annihilator of every ideal is generated as a right ideal by an idempotent. Moreover, Clark [3] showed the left-right symmetric of this condition by proving that a ring R is quasi Baer if and only if the right annihilator of every right ideal of R is generated as a right ideal by an idempotent.

A natural question for a given class of rings is, how does the given class behaves with respect to ring extensions?

Birkenmeier et al proved in [2] that a ring R is quasi Baer if and only if $R[[X]]$ is quasi Baer, where X is an arbitrary non empty set of not necessarily commuting indeterminate. In a series of papers [6–8] Keigher introduced the notion of the ring HR of Hurwitz series over a commutative ring with identity and demonstrated that it has many interesting application in differential algebra. The ring HR has been named the ring of Hurwitz series over R to credit the contribution of Hurwitz to its definition.

The motivation of this paper is two folded:

1) To extend the notion of the ring of Hurwitz series HR to the ring of skew Hurwitz series $T = (HR, \sigma)$.

2) To study when the property of being right quasi Baer (PS) ring is shared between the ring T of Skew Hurwitz series over the ring R and R itself.

For any ring R with identity and R-automorphism σ, we denote by

$$T = (HR, \sigma) = \{f : N \to R\},$$

where N is the set of natural numbers. Let the operation of addition in T be component wise and the operation of multiplication for each $f, g \in T$ be defined by

$$(fg)(n) = \sum_{k=0}^{n} \binom{n}{k} f(k)\sigma^k g(n-k),$$

for all $n \in N$, where $\binom{n}{k}$ is the binomial coefficient.

It can be easily shown that T is a ring with identity h_1, defined by $h_1(0) = 1$ and $h_1(n) = 0$ for all $n \geq 1$. It is called the ring of skew Hurwitz series over R.

We denote by $supp(f)$ the support of f, i.e.,

$$supp(f) = \{n \in N | f(n) \neq 0\},$$

and by $\pi(f)$ the smallest element in $supp(f)$. It is clear that R is canonically embedded as a subring of T via $r \in R \mapsto h_r \in T$, where $h_r(0) = r, h_r(n) = 0$ for every $n \geq 1$, hence $supp(h_r) = \{0\}$.

A right (left, two-sided) ideal I of a ring R is called *σ-invariant* if $\sigma(I) \subseteq I$. If R is a ring and σ is an R-endomorphism, R is called σ-compatible if $ab = 0$ if and only if $a\sigma(b) = 0$.

From now on let σ be an R-automorphism.

2. Quasi-Baer Rings of Skew Hurwitz Series

Theorem 2.1. *Suppose that R is a ring and $char(R) = 0$. If R is a quasi-Baer ring, then the skew Hurwitz series ring $T = (HR, \sigma)$ is a quasi-Baer ring.*

Proof. Let M be a left ideal of T. We claim that $\ell_T(M) = Th_e$, for some idempotent $h_e \in T$. Set $I_n = \{g(n) \in R \mid g \in M,\ n = \pi(g)\} \subset R$, and $I = \bigcup_{n\in\mathbb{N}} I_n$. Let J be the left ideal of R generated by I. Then there exists an idempotent e of R such that $\ell_R(J) = Re$.

First, to show that $Th_e \subseteq \ell_T(M)$, take $f \in M$, then $h_e f \in M$. If $h_e f \neq 0$, then $supp(h_e f)$ is a nonempty subset of $\mathbb{N}$. Let $t = \pi(h_e f)$. Then

$$0 \neq (h_e f)(t) = \sum_{k=0}^{t} \binom{t}{k} h_e(k)\sigma^k(f(t-k)) = ef(t) \in I_t \subseteq J,$$

but $ef(t) = e(ef(t)) = 0$, which is a contradiction. So, we inductively obtain that $(h_e f)(t) = 0$ for each $t \in supp(f)$. Hence $h_e f = 0$, which means that $Th_e \subseteq \ell_T(M)$.

Now we will show that $\ell_T(M) \subseteq Th_e$. Let $0 \neq g \in \ell_T(M)$ and let $s = \pi(g)$. For any $a \in J$, there exist

$$s_1, s_2, \cdots, s_n \in \mathbb{N},\ f_1, f_2, \cdots, f_n \in M,$$

and $r_1, r_2, .., r_n \in R$, such that

$$a = r_1 f_1(s_1) + r_2 f_2(s_2) + \cdots + r_n f_n(s_n).$$

Let $s_j = \pi(f_j)$, then $f_j(s_j) \in I_{s_j}$, $j = 1, 2, \cdots, n$. Since $h_{r_j} f_j \in M$, we have $g(h_{r_j} f_j) = 0$. Clearly, $\pi(h_{r_j} f_j) = s_j$, thus

$$\begin{aligned} 0 = g(h_{r_j} f_j)(s_j + s) &= \sum_{k=0}^{s_j+s} \binom{s_j+s}{k} g(k)\sigma^k((h_{r_j} f_j)(s_j + s - k)) \\ &= \binom{s_j+s}{s_j} g(s)\sigma^s((h_{r_j} f_j)(s_j)). \end{aligned}$$

Since $char(R) = 0$, then $(g(s)\sigma^s((r_j f_j(s_j)) = 0$, for any $j = 1, 2, \cdots, n$. Thus $g(s)\sigma^s(a) = 0$. Since σ is an automorphism, there exists $d_1 \in R$ such that

$\sigma^s(d_1) = g(s)$. Then $\sigma^s(d_1 a) = g(s)\sigma^s(a) = 0$. Consequently, $d_1 \in \ell_R(J) = Re$. Thus $d_1 = d_1 e$, and it follows that $g(s) = g(s)\sigma^s(e)$.

Suppose that $u \in supp(g)$ and $g(v) = g(v)\sigma^u(e)$ for any $v \in supp(g)$ with $v < u$. We will show that $g(u) = g(u)\sigma^u(e)$ for any $u \in supp(g)$. Denote

$$(g_u)(x) = g(x) \quad \text{when} \quad x < u and \quad (g_u)(x) = 0 \quad \text{when} \quad x \geq u.$$

Thus $\pi(g - g_u) = u$. By hypothesis $g_u = g_u h_e \in Th_e \subseteq \ell_T(M)$.

Now $g - g_u \in \ell_T(M)$. Using the same procedure above, it follows that

$$(g - g_u)(u) = (g - g_u)(u)\sigma^u(e),$$

which implies that $g(u) = g(u)\sigma^u(e)$ and our claim holds.

Now from

$$(gh_e)(t) = \sum_{k=0}^{t} \binom{t}{k} g(k)\sigma^k(h_e(t-k)) = g(t)\sigma^t(h_e(0)) = g(t),$$

it follows that $g = gh_e \in Th_e$. Therefore, $Th_e = \ell_T(M)$, and we have that T is quasi-Baer.

Recall from [2] that an idempotent $e \in R$ is called left (resp. right) semicentral in R if, $ere = re$ ($ere = er$), for all $r \in R$. Equivalently, $e^2 = e \in R$ is left (resp. right) semicentral in R if eR (Re) is an ideal of R. Since the left annihilator of a left ideal is an ideal, we see that the left annihilator of a left ideal is generated by a right semicentral idempotent in a quasi-Baer ring. □

Proposition 2.2. *Suppose that $f \in T$ is a right semicentral idempotent, then:*

1) $f(0) = e$ is a right semicentral idempotent of R.

2) If $f(0) = e$ is σ-invariant, then $Tf = Th_e$.

Proof. 1) Let $f(0) = e$, since $f \in T$ is a right semicentral idempotent, then $fh_r = fh_r f$ for any $r \in R$. Thus

$$er = f(0)r = (fh_r)(0) = (fh_r f)(0) = f(0)rf(0) = ere$$

which implies that $e = f(0)$ is a right semicentral idempotent of R.
2) If $f(0) = 0$, then $f = 0$. Otherwise, suppose that $f \neq 0$, then $supp(f) \neq \phi$. Let $t = \pi(f)$. Then

$$0 \neq f(t) = f^2(t) = \sum_{k=0}^{t} \binom{t}{k} f(k)\sigma^k(f(t-k)) = 0,$$

which is a contradiction. This shows us that $f = 0$ and $e = f(0) = 0$. Thus, $h_e = 0$ and we get that $Tf = Th_e$.

Now suppose that $f(0) \neq 0$. If $supp(f) = \{0\}$, then clearly $f = h_e$. So assume $supp(f) \neq \{0\}$. Denote the minimal element in $supp(f)\backslash\{0\}$ by t. Since $\sigma(e) = e$ and $f(s) = 0$ for any $s \in \mathbb{N}$ with $0 < s < t$, then

$$f(t)\sigma^t(r) = (fh_r)(t) = (fh_rf)(t) = \sum_{k=0}^{t} \binom{t}{k} f(k)\sigma^k(rf(t-k))$$
$$= f(0)rf(t) + f(t)\sigma^t(r)\sigma^t(f(0)) = erf(t) + f(t)\sigma^t(r)e.$$

Multiply the left-hand side by $e = f(0)$, we get

$$ef(t)\sigma^t(r) = erf(t) + ef(t)\sigma^t(r)e.$$

But $ef(t)\sigma^t(r) = ef(t)\sigma^t(r)e$. Hence

$$erf(t) = 0, \quad \text{and} \quad f(t)\sigma^t(r) = f(t)\sigma^t(r)e.$$

Suppose now that $w \in supp(f)$ is such that for any $u \in supp(f)$ with $0 < u < w$,

$$f(u)\sigma^u(r) = f(u)\sigma^u(r)e, \quad erf(u) = 0, \quad \forall r \in R.$$

Then

$$f(w)\sigma^w(r) = (fh_r)(w) = (fh_rf)(w) = \sum_{k=0}^{w} \binom{w}{k} f(k)\sigma^k(rf(w-k))$$
$$= f(0)rf(w) + \sum_{k=1}^{w-1} \binom{w}{k} f(k)\sigma^k(rf(w-k)) + f(w)\sigma^w(rf(0)).$$

Multiply the left-hand side by $f(0) = e$, we get

$$ef(w)\sigma^w(r) = erf(w) + \sum_{k=1}^{w-1} \binom{w}{k} ef(k)\sigma^k(rf(w-k)) + ef(w)\sigma^w(re).$$

But $ef(w)\sigma^w(r)e = ef(w)\sigma^w(r)$ and $\sum_{k=1}^{w-1} \binom{w}{k} ef(k)\sigma^k(rf(w-k)) = 0$. Thus $erf(w) = 0$ and it follows that

$$f(w)\sigma^w(r) = \sum_{k=1}^{w-1} \binom{w}{k} f(k)\sigma^k(rf(w-k)) + f(w)\sigma^w(r)e.$$

Multiply the right-hand side by $f(0)=e$, we get

$$\begin{aligned} f(w)\sigma^w(r)e &= \sum_{k=1}^{w-1}\binom{w}{k}f(k)\sigma^k(rf(w-k))e+f(w)\sigma^w(r)e \\ &= \sum_{k=1}^{w-1}\binom{w}{k}f(k)\sigma^k(rf(w-k))+f(w)\sigma^w(r)e. \end{aligned}$$

Thus

$$\sum_{k=1}^{w-1}\binom{w}{k}f(k)\sigma^k(rf(w-k))=0$$

and it follows that

$$f(w)\sigma^w(r)e=f(w)\sigma^w(r).$$

Therefore, we get for any $w\in supp(f)$,

$$f(w)\sigma^w(r)e=f(w)\sigma^w(r), \quad erf(w)=0, \quad \forall r\in R.$$

Hence, we can conclude that $h_e=h_ef$ and $f=fh_e$, which imply that $Tf=Th_e$. □

The following example shows us that there exists skew Hurwitz series $T=(HR,\sigma)$ which is quasi-Baer, but R isn't quasi-Baer.

Example 2.3. Consider the ring $R=\{(a,b)\in\mathbb{Z}\oplus\mathbb{Z}\mid a\equiv b\ (mod\ 2)\}$, with the usual operations of componentwise addition and multiplication R is clearly a commutative reduced ring and the only idempotent of R are $(0,0)$ and $(1,1)$. Let $\sigma:R\to R$ be defined by $\sigma(a,b)=(b,a)$, then σ is an automorphism of R. Now we claim that $T=(HR,\sigma)$ is quasi-Baer. Let I be a nonzero ideal of T and $0\neq g\in I$, let $i=\pi(g)$ and $g(i)=(a_i,b_i)$. Let $f,h\in T$ be such that

$$f(2k-i)=(1,1) \quad \text{and} \quad f(j)=0 \quad \text{otherwise},$$

$$h(2k-i+1)=(1,1) \quad \text{and} \quad h(j)=0 \quad \text{otherwise},$$

Hence, $gf\in I$ and $gh\in I$ are such that $\pi(gf)=2k$ and $(gf)(2k)=\binom{2k}{i}g(i)$ and $\pi(gh)=2k+1$ and $(gh)(2k)=\binom{2k+1}{i}g(i)$. Suppose that $0\neq q\in r_T(I)$, $j=\pi(q)$ and $q(j)=(u_j,v_j)\neq(0,0)$.

Hence

$$\begin{aligned} 0=(gfq)(2k+j) &= \binom{2k+j}{2k}\binom{2k}{i}g(i)\sigma^{2k}(q(j)) \\ &= \binom{2k+j}{2k}\binom{2k}{i}(a_i,b_i)(u_j,v_j). \end{aligned}$$

Also,

$$0=(ghq)(2k+j+1)=\binom{2k+j+1}{2k+1}\binom{2k+1}{i}(g)(i)\sigma^{2k+i}(q(i))$$
$$=\binom{2k+j+1}{2k+1}\binom{2k+1}{i}(a_i,b_i)(v_j,u_j).$$

Since $char(R)=0$, then $(a_i,b_i)(u_j,v_j)=(a_iu_j,b_iv_j)=(0,0)$ and $(a_i,b_i)(v_j,u_j)=(a_iv_j,b_iu_j)=(0,0)$. Since $(a_i,b_i)\neq(0,0)$ this means that a_i or b_i are nonzero. Consequently, $(u_j,v_j)=(0,0)$ which is a contradiction.

Therefore, $r_T(I)=\{(0,0)\}$ and T is quasi-Baer.

In the contrary, R isn't quasi-Baer. For $(2,0)\in R$ we get

$$r_R((2,0))=\{(0,2n)|n\in\mathbb{Z}\}.$$

Consequently, $r_R((2,0))$ doesn't contain any nonzero idempotent.

Hence R isn't quasi-Baer.

Theorem 2.4. *Suppose that R is a ring such that every semicentral idempotent is σ-invariant and $\sigma(l_R(I))=l_R(I)$ for each left ideal I of R. If $T=(HR,\sigma)$ is quasi-Baer, then R is quasi-Baer.*

Proof. Let I be a left ideal of a ring R and $M=TI$ be the left ideal of T generated by I. Since, T is a quasi-Baer ring, then there exists a semicentral idempotent $f\in T$ such that $l_T(M)=Tf$. Using Proposition 2.2 it follows that $l_T(M)=Tf=Th_e$ for some semicentral idempotent $e\in R$. Hence, $h_eg=0$ for each $g\in M$ and we have that $0=(h_eh_x)(0)=ex$ for each $x\in I$. Therefore $Re\subseteq l_R(I)$. Now, suppose that $y\in l_R(I)$, $g\in T$ and $x\in I$, then $gh_x\in M$ and $h_ygh_x=0$. Hence, $0=(h_ygh_x)(n)=yg(n)\sigma^n(x)$. Since, σ is an R-automorphism and $\sigma(l_R(I))=l_R(I)$, then $(h_ygh_x)(n)=\sigma^n(trx)$ where $t=\sigma^{-n}y$, $r=\sigma^n(g(n))$.

So, $t\in l_RI$ and it follows that $(h_ygh_x)(n)=\sigma^n(trx)=\sigma^n(0)=0$ for each $n\in N$. Therefore $h_y\in l_T(M)=Th_e$. Hence, $h_y=kh_e=h_yh_e$ for some $k\in T$ and it follows that $y=ye\in R$ which means that $Re=l_R(I)$ and R is a quasi-Baer ring. □

Combining Theorem 2.1 and Theorem 2.4 we get the main Theorem of this section.

Theorem 2.5. *Suppose that R is a ring with $char(R)=0$ and σ is an R-automorphism such that every left semicentral idempotent $e\in R$ is σ-invariant and $\sigma(l_R(I))=l_R(I)$ for each left ideal I of R. Then R is a quasi-Baer ring if and only if $T=(HR,\sigma)$ is quasi-Baer.*

The following Theorem shows us that the prime property can be shared between T and R under certain condition.

Theorem 2.6. *Suppose that R is a ring and σ is an R-automorphism, then*

i) If R is prime and $char(R)=0$, then $T=(HR,\sigma)$ is prime.

ii) If the left annihilator of every left ideal of R is σ-invariant and T is prime, then R is prime.

Proof. i) Suppose that R is a prime ring and $T=(HR,\sigma)$ is not prime, then there exists nonzero elements $f,g\in T$ such that $fTg=0$. Hence, $fkg=0$ for each $k\in T$ in particular $fh_rg=0$ for each $r\in R$. Suppose that $\pi(f)=n_1$ and $\pi(g)=n_2$, then

$$\begin{aligned} 0=(fh_rg)(n_1+n_2) &= \sum_{i=0}^{n_1+n_2} f(i)\sigma^i(h_rg)(n_1+n_2-i) \\ &= f(n_1)\sigma^{n_1}(h_rg)(n_2)=f(n)\sigma^{n_1}((r)g(n_2)). \end{aligned}$$

Since, σ is an R-automorphism, then $0=f(n)R\sigma^{n_1}(g(n_2))$ for nonzero elements $f(n)$ and $\sigma^{n_1}g(n_2)$ which contradicts the fact that R is a prime ring.
ii) Suppose that R isn't prime, then there exists a nonzero elements $a,b\in R$ such that $aRb=0$. Therefore, $b\in R_R(a)$ and by hypothesis $\sigma^n(b)\in R_R(a)$. So $aR\sigma^n(b)=0$ for all $n\in N$.

Hence, $0=ar\sigma^n(b)=(h_agh_b)(n)$ for each $g\in T$ and $n\in N$. So, $h_aTh_b=0$ which contradicts the fact that T is a prime ring. □

3. *PS*-Rings Of Skew Hurwitz Series

A ring R (not necessarily commutative) is called a *PS*-ring if the socle, $Soc(_R(R))$ is projective. These rings were studied by Gordon in [4] and Nicholson and Watter in [9]. In [9] Nicholson and Watter proved that if R is a left *PS*-ring, then so are $R[X]$ and $R[[X]]$. The following result is due to Nicholson and Watter [9] which gives an equivalent condition for the ring R to be *PS* and we need it in the sequel.

Lemma 3.1. *The following conditions on a ring R are equivalent:*

1) R is a left PS-ring.

2) If M is a maximal left ideal of R, then $r_R(M)=eR$, where $e^2=e\in R$ and $r_R(M)$ is the right annihilator of M in the ring R.

The following Theorem is due to L. Zhongkui [10] which shows us that the ring HR of hurwitz series inherits the PS property from the ring R.

Theorem 3.2. *Suppose that R is a commutative ring and* $char(R) = 0$. *If R is a PS-ring then so, is HR.*

The following Theorem is the main result of this section which extends the above theorem to the noncommutative case.

Theorem 3.3. *Suppose that R is a right PS-rings with* $char(R) = 0$, *then the skew Hurwitz series ring* $T = (HR, \sigma)$ *is a right PS-ring.*

Proof. Let M be a maximal right ideal of T and

$$I_n = \{g(n) \in R | g \in M, n = \pi(g)\} \subset R.$$

Hence I_n is a right ideal of R. Let $I = \cup_{n\in N} I_n$ and J be the ideal of R generated by I. It can be easily shown that J is a maximal right ideal of R. For if $J = R$, then there exists nonzero elements $f_1, f_2, \cdots, f_m$ in M and $r_1, r_2, \cdots, r_m$ in R such that $1 = f_1(n_1)r_1 + \cdots + f_m(n_m)r_m$ with $n_i = \pi(f_i)$ and $f(n_i) \in I_{n_i} \subset J$ for each $i = 1, \cdots, m$. Suppose that $0 \neq g \in l_T(M)$ and $k = \pi(g)$. If

$$\binom{m+n_i}{k} g(k)\sigma^k(f_i(n_i)) \neq 0,$$

then $\pi(gf_i) = k + n_i$ and it follows that $(gf_i)(k+n_i) \neq 0$ which contradicts the fact that $g \in l_T(M)$. Hence

$$\binom{k+n_i}{k} g(k)\sigma^k(f_i(n_i)) = 0.$$

Since $char(R) = 0$, then $g(k)\sigma^k(f_i(n_i)) = 0$ for each $i = 1, \cdots, m$.

Now,

$$1 = \sigma^k(1) = \sigma^k(f_1(n_1)r_1 + \cdots + f_m(n_m)r_m).$$

Therefore,

$$g(k) = g(k)\sigma^k(f_1(n_1)r_1 + \cdots + f_m(n_m)r_m) = 0$$

which contradicts the fact that $\pi(g) = k$. Hence $g = 0$ and $l_T(M) = 0$.

Now, suppose that $J \neq R$, we will show that J is a maximal right ideal of R. Let $r \in R \setminus J$. If $h_r \in M$, then $r = h_r(0) \in I_0 \subset J$ which is a contradiction. Hence $h_r \notin M$ and by maximality of M, $T = M + h_rT$. Therefore, there exists $f \in M$ and $g \in T$ such that $h_1 = g + h_r f$. Thus $1 = g(0) + rf(0)$. If $f(0) = 0$, then $1 \in rR$ and $R = J + rR$. If $f(0) \neq 0$, then $R = J + rR$. Consequently J is a maximal right ideal of R.

Since R is a right PS-ring, then there exists an idempotent $e \in R$ such that $l_R(J) = Re$, we will show that $l_T(M) = Th_e$. Suppose that $h_eM \nsubseteq M$ by maximality of M, $T = M + h_eM$. Hence $h_1 = f + h_eg$ for some $f, g \in M$. Therefore, $1 = f(0) + eg(0)$, if $g(0) \neq 0$, then $\pi(g) = 0$ and it follows that $g(0) \in I_0 \subset J$. Hence $0 = eeg(0) = eg(0)$. Therefore, $1 = f(0) \in I_0 \subset J$ which is a contradiction. Hence, $h_eM \subseteq M$. Suppose that $g \in M$, hence $h_eg \in M$. If $h_eg \neq 0$, let $k = \pi(h_eg)$, then $(h_eg)(k) = h_e(0)g(k) = eg(k) \in I_k \subset J$. Hence, $0 = eeg(k) = eg(k) = (h_eg)(k)$ which is a contradiction. Consequently, $h_eg = 0$ and $Th_e \subseteq l_T(M)$. Conversely, let $0 \neq g \in l_T(M) - Th_e$, then using the same argument used in Theorem 2.1 it can be easily shown that $g \in The_e$ which is a contradiction. Hence, $l_T(M) \subseteq Th_e$. Therefore, $l_T(M) = Th_e$ and T is a right PS-ring. □

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REFERENCES

[1] G. F. Birkenmeier - J. Y. Kim - J. K. Park, *Quasi-Baer Ring Extensions And Biregular Rings*, Bull. Austral. Math. Soc. 61 (2000), 39–52.

[2] G. F. Birkenmeier - J. Y. Kim - J. K. Park, *Polynomial extensions of Baer and quasi-Baer rings*, J. Pure Appl. Algebra 159 (2001), 25–42.

[3] W. E. Clark, *Twisted matrix units Semigroup algebras*, Duke Math. J. 33 (1967), 417–423.

[4] R. Gordon, *Ring in which minimal left ideal are projective*, Pacific. J. Math. 31 (1969), 679–692.

[5] I. Kaplansky, *Rings of Operators*, Benjamin, New York, 1968.

[6] W. F. Keigher, *Adjunctions and comonads in differential algebra*, Pacific. J. Math. 248 (1975), 99–112.

[7] W. F. Keigher, *On the ring of Hurwitz Series*, Comm. Algebra 25 (6) (1997), 1845–1859.

[8] W. F. Keigher - F. L. Pritchard, *Hurwitz Series as formal functions*, J. Pure Appl. Algebra 146 (2000), 291–304.

[9] K. Nicholson - J. F. Watters, *Rings with projective socle*, Proc. Amer. Math. Soc. 102 (3) (1988), 443–450.

[10] L. Zhongkui, *Hermite and PS-rings of Hurwitz Series*, Comm. Algebra 28 (1) (2000), 299–305.

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